



OVERTURNING CRITERIA FOR NON-ANCHORED NON-SYMMETRIC RIGID BODIES

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SUMMARY

Several analytical procedures have been previously developed to evaluate the overturning potential of non-anchored rigid bodies, but all of them consider that their center of gravity is equally distant from their two base corners. This situation is not always present in the real world. This work extends the existing analytical development to include the horizontal asymmetry of the center of gravity of a planar rigid body under horizontal and vertical excitations and to evaluate the accuracy of existing overturning criteria for this case. Two motion states are considered: rest and rocking without sliding. Criteria for sliding with and without rocking are also presented but analyses are performed for the rocking body only.

To include different demand characteristics, several actual earthquake records from Taiwan 1999, Kern County 1952, Northridge 1994, Chile 1985, Kobe 1995, Kocaeli 1999, Landers 1992, Loma Prieta 1989, Imperial Valley 1979, Morgan Hill 1984, San Fernando 1971, Borrego Mountain 1968, Palm Springs 1986, Sierra Madre 1991 and Whittier 1987 are considered. Some of these records present localized high motions, permanent displacement and others show long-duration oscillatory motions. For the analysis, acceleration as well as displacement are considered. Records are used in their original form, so results are valid mainly for a rigid body located at ground level.

The existing overturning criteria for symmetric bodies are evaluated and limits are presented where these criteria are valid for the non-symmetric body. Also a procedure that gives a good prediction of the expected response for non-symmetric bodies is indicated.

INTRODUCTION

A fundamental topic in the functional seismic protection in hospitals and other facilities is to limit the excessive motion of non-anchored bodies. As mentioned by Ishiyama [1], since late XIX century, the behavior of regular non-anchored bodies has been studied responding to different base excitations. The motion of a non-anchored body can be partitioned in 6 basic conditions: rest, slide, rock, slide-rock, free

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flight and impact. There are an important number of publications that evaluate the different aspects of these motions, Ishiyama [1], Housner [2], Shenton [3], Yim [4] and others.

Housner [2] proposed in the 60's an energy balance formula that estimates the pseudo-velocities, S_v , necessary to predict a 50% chance of overturning of a symmetric homogeneous rigid block.

$$S_v = \frac{1}{\cos \alpha} \sqrt{\frac{8}{3} gr(1 - \cos \alpha)} \quad (1)$$

Where α is the angle between the vertical and the line between a corner and the center of gravity and r is the distance between one corner and the center of gravity.

Ishiyama [1] proposes a maximum velocity value, v_{\max} , of the seismic record that generates overturning on regular rectangular rigid bodies, and it is given by

$$v_{\max} > 0.4 \left\{ \frac{1}{\cos \alpha} \sqrt{\frac{8}{3} gr(1 - \cos \alpha)} \right\} \text{ and } a_{\max} \geq \{B/H\}g \quad (2)$$

Where B is the width of the base and H is the height of the block or double the center of gravity height.

Additionally Uematsu et al [5] propose a criterion based in normalized motion values.

In this work, we present formulae to evaluated the overturning potential associated with pure rocking motion of a planar body with a center of mass located non-symmetric with respect to its base corners, under horizontal and vertical seismic actions.

Model Geometry

The model used in this analysis corresponds to a rigid body with its center of gravity not necessarily located in the horizontal middle of its base. The contact conditions are determined considering the Coulomb's static friction law. The model and the geometric parameters of the body used in this work are shown in Figure 1.

In this figure h is the height of the body's center of gravity and B , the width of its base. Coefficient ν_1 measures the horizontal eccentricity of the center of gravity with respect to the base of the body and it can adopt values between 0 and 1; and ν_2 corresponds to the complement of ν_1 , therefore

$$\nu_2 = 1 - \nu_1 \quad (3)$$

The angles defined in Figure 1 are related to the geometry of the body by the following expressions:

$$\alpha_1 = \arctan \left(\frac{\nu_1 B}{h} \right) \quad (4)$$

$$\alpha_2 = \arctan\left(\frac{v_2 B}{h}\right) \quad (5)$$

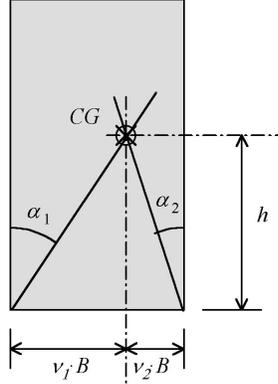


Figure 1. Body geometric parameters

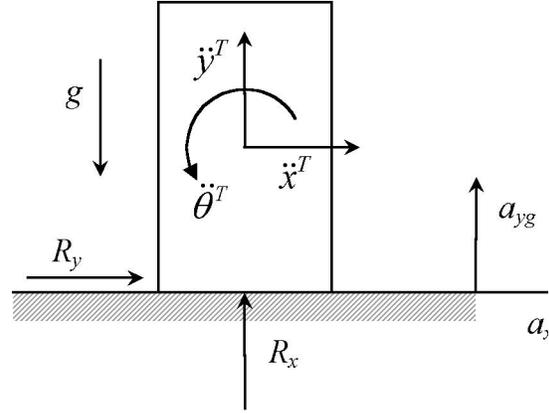


Figure 2: System accelerations diagram.

MOTION LIMITS

Six basic motion types have been defined. They are sliding, rocking, slide-rocking, free flight and impact. Those motion types are related between each other, each one of them structuring a basic phase of the total motion of the body, so the dynamic and kinetic conditions of the body at the end of one phase will be the initial conditions for the next phase.

Transition Conditions from Rest

We define a body at rest as a body going through a motion along its support. The body will keep this state while certain conditions of the ground motion, the body geometry and the Coulomb law friction coefficient are met. Shenton [2] presents the conditions for a rectangular symmetric block to initiate different motion modes from rest, assuming Coulomb friction and horizontal ground acceleration. Developing a similar analysis, the criteria for initiating motions from rest can be extended to include two new variables: the vertical ground acceleration component and the possible mass asymmetry of the body.

Limits to avoid free flight

For the solution, the location of the center of mass is represented by x^T, y^T (dependency on time is considered but not indicated), Figure 2; the angle of rotation of the body is represented by θ^T and the ground acceleration as continuous function of time by a_{xg} and a_{yg} . The total acceleration of the center of mass in the case of rest is

$$\ddot{x}^T = a_{xg} + \ddot{x} \quad (6)$$

$$\ddot{y}^T = a_{yg} + g + \ddot{y} \quad (7)$$

where a_{xg} and a_{yg} correspond to the horizontal and vertical ground accelerations. Therefore, the equilibrium equations are, Figure 2

$$0 = mR_x - ma_{xg} \quad (8)$$

$$0 = mR_y - mg - ma_{yg} \quad (9)$$

where R_x and R_y are the horizontal and vertical ground reactions (per unit of mass), and m is the total mass of the body. Isolating the vertical reaction

$$R_y = a_{yg} + g \quad (10)$$

The reaction R_y can take positive values or zero, but never negative values, since the body is not anchored, therefore the motion restrain is only in one direction. It can be concluded that the body will keep in contact with the ground if the following condition is satisfied.

$$a_{yg} + g \geq 0 \quad (11)$$

It can be easily shown that this relation also should be maintain to avoid free flight during rocking.

Requirements for Rocking Motion

To analyze the rocking motion of a body that is initially at rest, two essential conditions must be satisfied: the external moment acting on the body must be sufficient to lift the opposite base corner, and under this action, the body must rotate before it initiates sliding. If it is considered that the body is just before the initiation of a rocking motion, then, it can be assumed that the vertical reaction corresponds to a concentrated force applied at the base corner that will be the center of rotation of the body (relative to the ground). In practice the force will be applied in a relative small area capable of resisting the applied loads.

Due to the eccentricity of the center of mass, the torque equilibrium equations will be different depending on which base corner is the center of rotation of the body, Figure 3. Since the orientation of the rotation depends on the sign of the horizontal ground acceleration, the following coefficient is defined:

$$v_s = v_1; \text{ if } a_{xg} \geq 0 \quad (12)$$

$$v_s = 1 - v_1; \text{ if } a_{xg} < 0 \quad (13)$$

For symmetric bodies, the v_s coefficient will adopt a value of 0.5, independent on the sign of a_{xg} .

The sum of the torque around the center of gravity on the onset of rotation is

$$0 = mR_x h - mR_y v_s B \operatorname{sgn}(a_{xg}) - I_g \ddot{\theta} \quad (14)$$

Given the resting condition of the body, the angular acceleration of the body will be null until a rocking motion is initiated, then

$$ma_{xg}h = m(a_{yg} + g)v_s B \operatorname{sgn}(a_{xg}) \quad (15)$$

Organizing the terms of this last equation,

$$\frac{|a_{xg}|}{a_{yg} + g} = \frac{v_s B}{h} \quad (16)$$

The ratio between the center of mass height and the horizontal distance between the base border and the center of mass is defined as γ i.e.

$$\gamma = \frac{h}{v_s B} \quad (17)$$

and defining A as the ratio between the effective horizontal and vertical accelerations acting on the body, then

$$A = \frac{a_{xg}}{a_{yg} + g} \quad (18)$$

Therefore, the condition (16) can be written as

$$|A| = \frac{1}{\gamma} \quad (19)$$

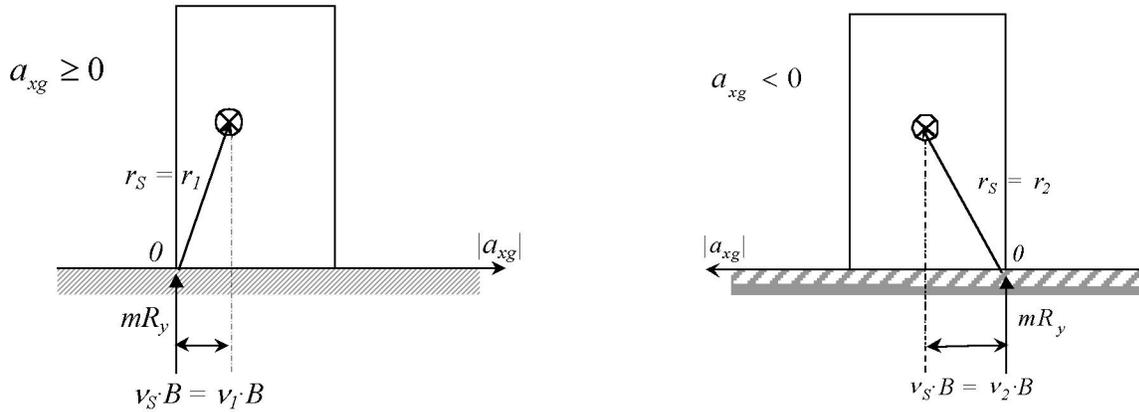


Figure 3: Location of the vertical reaction and value of v_s depending on the sign of the horizontal ground acceleration.

Limits to Avoid Sliding

As mentioned by Ishiyama [1] and Shenton [3], the transition can be to rocking, to rocking and sliding or to sliding. In the case of transition from rest to a rocking motion, the friction force must be sufficient to ensure the necessary adherence in the standing point to avoid a slide

$$|R_x| \leq \mu_e R_y \quad (20)$$

where μ_e corresponds to the static coefficient of friction between the body and the ground. The ground reactions can be determined from equations (8) and (9)

$$\frac{|a_{xg}|}{a_{yg} + g} \leq \mu_e \quad (21)$$

and

$$|A| \leq \mu_e \quad (22)$$

For the case of symmetric bodies with no vertical ground acceleration, as mentioned by Shenton [3], the condition to ensure a pure rocking motion from rest is

$$|A| \leq \frac{(4\gamma^2 + 1)\mu_e - 3\gamma}{(\gamma^2 + 4) - 3\mu_e} \quad (23)$$

Following a procedure similar to that presented by Shenton [3], it can be demonstrated that this condition for the case of asymmetric bodies is

$$|A| \leq \frac{\gamma^2 \mu_e + i_g^2 / (v_s^2 B^2) \mu_e - \gamma}{i_g^2 / (v_s^2 B^2) + 1 - \mu_e} \quad (24)$$

or

$$\mu_e \geq \frac{|A| i_g^2 / (v_s^2 B^2) + |A| + \gamma}{i_g^2 / (v_s^2 B^2) + |A| \gamma + \gamma^2} \quad (25)$$

The limit value of $|A|$ to initiate rocking is given in (19), therefore, at the instant when the body is initiating a rocking motion, the expression in (25) becomes

$$\mu_e \geq \frac{1}{\gamma} \quad (26)$$

In order to avoid sliding, equation (20) is the required condition from rest and during pure rocking.

Energy Loss in Rocking Motion with Null Impact Restitution Effect

The energy loss for a pure rocking motion is evaluated for the case of a perfect plastic impact, and similar to the symmetric case, the energy loss can be evaluated preserving the angular momentum.

Following Housner [2] and assuming a total conservation of angular momentum during the impact with positive rotational motion,

$$I_{o_1} \dot{\theta}' = -\dot{\theta} m B (v_2 B) + I_{o_2} \dot{\theta} \quad (27)$$

Where $\dot{\theta}'$ is the angular velocity at the instant after the impact, v_2 is the eccentricity coefficient before the impact, I_{o_1} is the moment of inertia about the centre of rotation after the impact and I_{o_2} is the moment of inertia about the centre of rotation before the impact.

$$I_{o_2} = I_g + m r_2^2 \quad (28)$$

$$I_{o_1} = I_g + m r_1^2 \quad (29)$$

Therefore (27) can be written as

$$\frac{\dot{\theta}'}{\dot{\theta}} = \frac{i_g^2 + r_2^2 - B^2 v_2}{i_g^2 + r_1^2} \quad (30)$$

In the case that $\dot{\theta} < 0$, the analysis results

$$\frac{\dot{\theta}'}{\dot{\theta}} = \frac{i_g^2 + r_1^2 - B^2 v_1}{i_g^2 + r_2^2} \quad (31)$$

Then, in the general case of impact, v_s can be defined as

$$v_s = v_1 \text{ if } \dot{\theta} > 0 \quad (32)$$

$$v_s = v_2 \text{ if } \dot{\theta} < 0 \quad (33)$$

Finally, the expression for the velocity variation is

$$\frac{\dot{\theta}'}{\dot{\theta}} = \frac{i_g^2 + h^2 - (1 - v_s) v_s B^2}{i_g^2 + h^2 + v_s^2 B^2} \quad (34)$$

On the other hand, energy before and after the impact is described by

$$E = \frac{1}{2} \left[i_g^2 + h^2 + (1 - v_s)^2 B^2 \right] \dot{\theta}^2 \quad (35)$$

$$E' = \frac{1}{2} \left[i_g^2 + h^2 + v_s^2 B^2 \right] \dot{\theta}'^2 \quad (36)$$

And defining δE as

$$\delta E = \frac{E'}{E} \quad (37)$$

Replacing (34), (35) and (36) in (37)

$$\delta E = \frac{\left[i_g^2 + h^2 - (1 - \nu_s) \nu_s B^2 \right]^2}{\left[i_g^2 + h^2 + (1 - \nu_s)^2 B^2 \right] \left(i_g^2 + h^2 + \nu_s^2 B^2 \right)} \quad (38)$$

LIMITING CONDITION

Finally, the conditions that must be satisfied to limit the motion of the body to obtain a pure rocking motion are:

Limit to avoid free flight

$$R_y > 0$$

Limit to avoid sliding from rest

$$\frac{1}{\gamma} \leq \mu_e$$

Limit to avoid sliding from rocking

$$\frac{|R_x|}{R_y} \leq \mu_e$$

Overturning Criteria Validation for “Symmetric” Bodies

To validate Ishiyama’s criteria [1], a total of 48 symmetric regular bodies were considered. Their center of gravity height varies from 0.25 m to 4.0 m and widths from 0.08 to 3.5 depending on body height.

Each body was analyzed using a specially developed computer program with 288 different pair of earthquake records from Taiwan 1999, Kern County 1952, Northridge 1994, Chile 1985, Kobe 1995, Kocaeli 1999, Landers 1992, Loma Prieta 1989, Imperial Valley 1979, Morgan Hill 1984, San Fernando 1971, Borrego Mountain 1968, Palm Springs 1986, Sierra Madre 1991 and Whittier 1987. All these records have been integrated with standard procedures but those that present permanent displacements have been integrated by a procedure described in Boroschek et al [6].

Figure 4 represents the extreme response of symmetrically located center of mass bodies: rest or overturning of the studied bodies. These figures show that Ishiyama's criterion is satisfied in 99.4% of the cases. The ones where the criteria are slightly not satisfied are Chile 1985, Iloca N90E records, and Northridge 1994, Los Angeles City Terrace 090 records.

Overturning Criteria for Non – Symmetric Bodies

The same set of accelerograms was used to analyze the bodies, with the following eccentricities (ν): 0.4; 0.3; and 0.2. In these cases the bodies are considered with the same moment of inertia, but with the center of mass non-equidistant from the base corners. Results are presented in Figure 5 to 8. Figure 78 presents the percentage of bodies that overturned as a function of eccentricity. It clearly indicates that with greater eccentricity, a larger number of bodies overturned and this depends on the height of the body center of mass and its dimensions. Figure 8 presents the ratio of non-symmetric to symmetric overturned bodies. Traditionally, a maximum eccentricity of 0.3 can be expected. According to Figure 8, this implies that approximate 2 to 3.5 times more bodies will overturning than in the symmetrical case.

In Figures 5 and 6, in addition to the red circles indicating overturning and the black dots indicating rest, several overturning limits criteria are plotted: The dashed line represents the original Ishiyama's equation where the eccentricity is assumed as half the base length ($\nu = 0.5$); the solid line, Ishiyama's equation but using a base width of double the minimum eccentricity length; and the dash-dotted line represents Ishiyama's equation reducing the 0.4 coefficient by half and considering the original base length (i.e. "ignoring" the eccentricity).

From these figures and limits, two important results can be obtained:

1. If the eccentricity of the body is known, then the criteria presented by Ishiyama [1] is valid if it is considered that the body has the same inertia properties of a symmetric body, but its base width equals the double of its minimum eccentricity ($B' = 2 * \nu * B$ with $\nu < 0.5$).
2. In case the eccentricity is unknown, a conservative procedure, if the body has a maximum eccentricity of 0.3, will be to replace the coefficient 0.4 used by Ishiyama in equation (2) by 0.2 and assume a symmetric body.

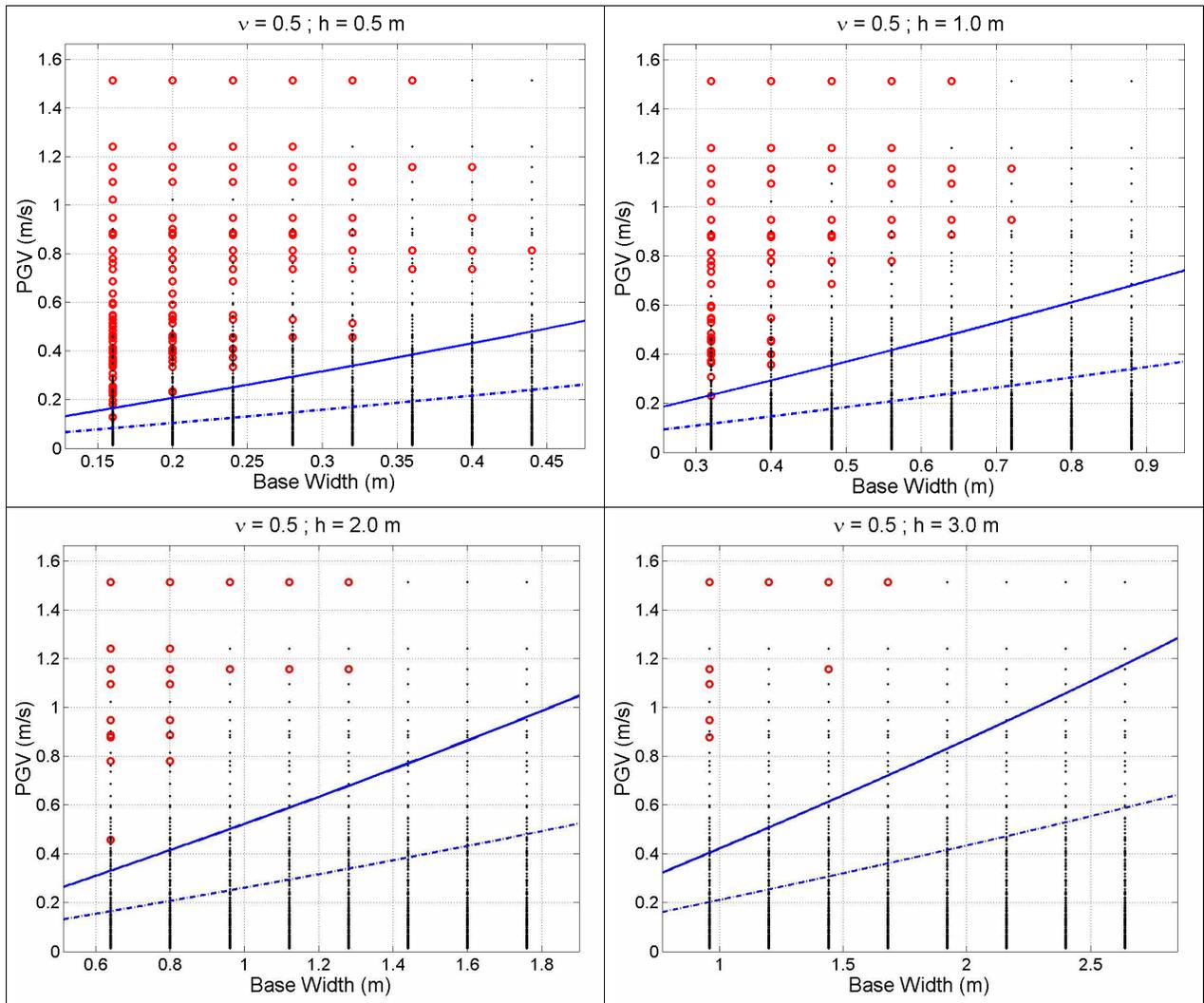


Figure 4: Extreme response of bodies and overturning criteria. Symmetric case and 4 heights. The red circles represent overturned bodies and the black dots show non-overturned bodies. The solid line represents equation (2) and the dot-dotted line the proposed criteria for non-symmetric bodies (shown as reference only).

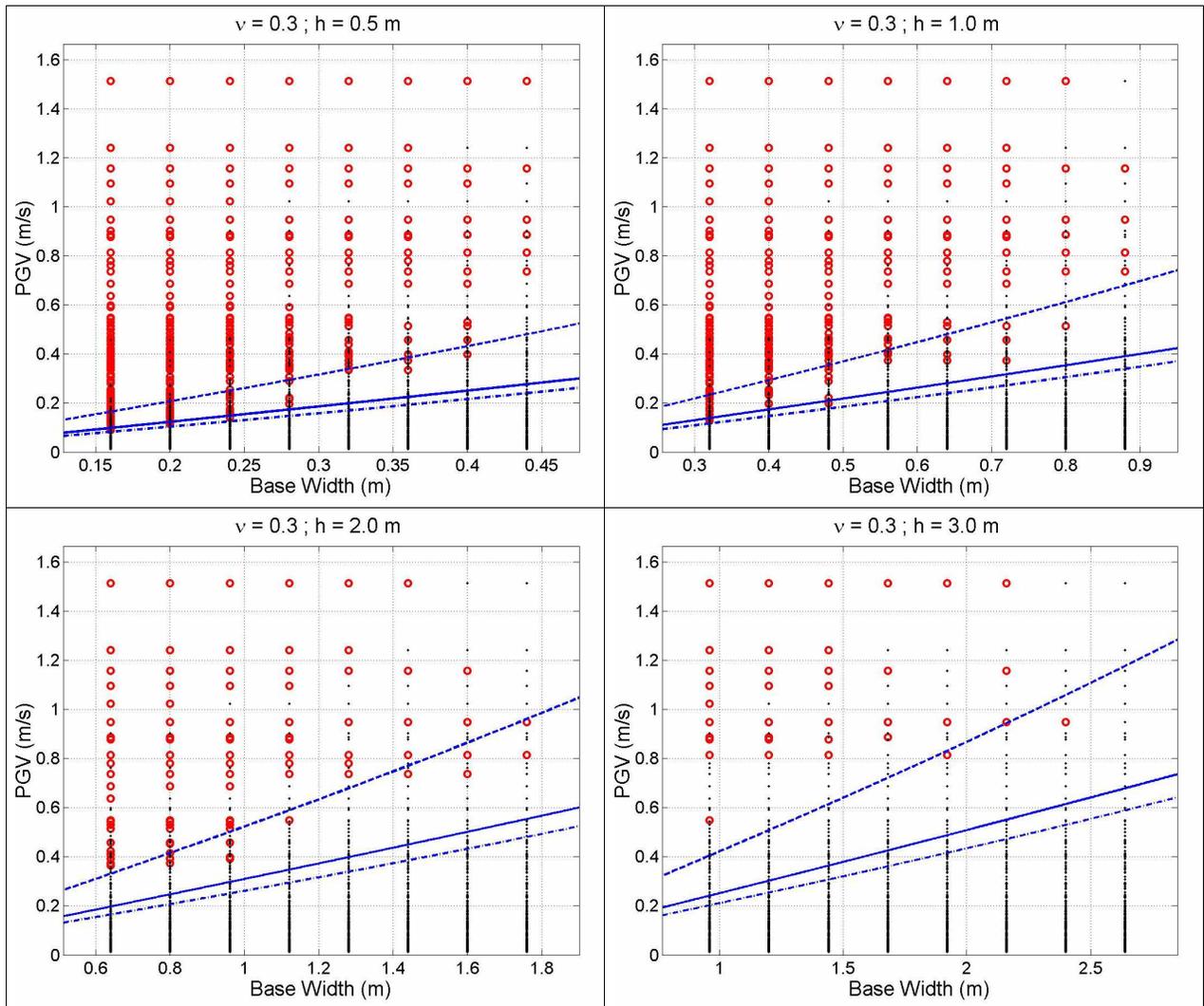


Figure 5: Extreme response of bodies and overturning criteria. Non-symmetric case $v = 0.3$ and 4 heights. The red circles represent overturned bodies and the black dots show non-overturned bodies. The solid line represents equation (2) with B as double the minimum eccentric base length, the dotted line represents equation (2) assuming a symmetric body and the dash-dotted line the proposed criteria for non-symmetric bodies.

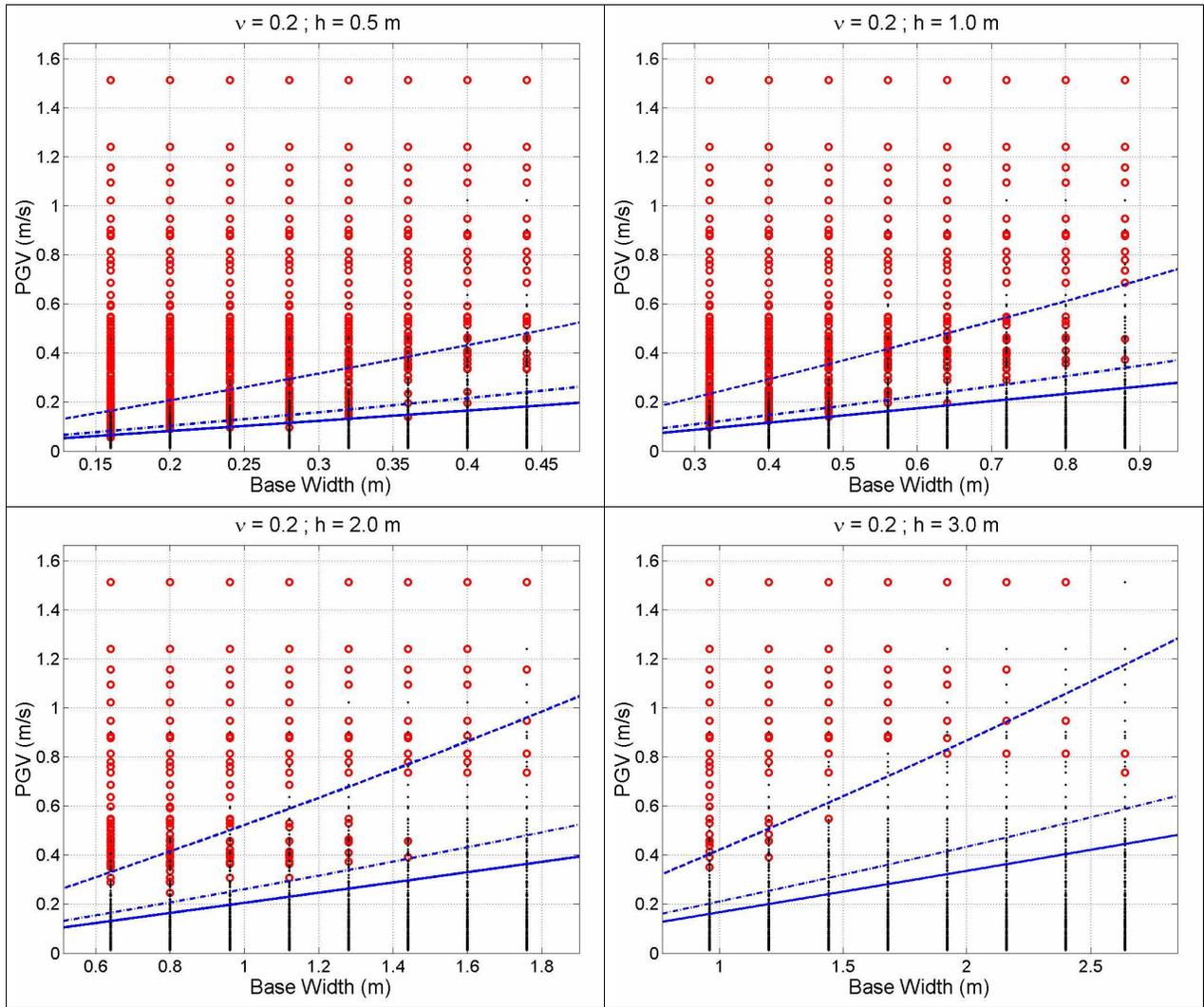


Figure 6: Extreme response of bodies and overturning criteria. Non-symmetric case $v = 0.2$ and 4 heights. The red circles represent overturned bodies and the black dots show non-overturned bodies. The solid line represents equation (2) with B as double the minimum eccentric base length, the dotted line represents equation (2) assuming a symmetric body and the dash-dotted line the proposed criteria for non-symmetric bodies.

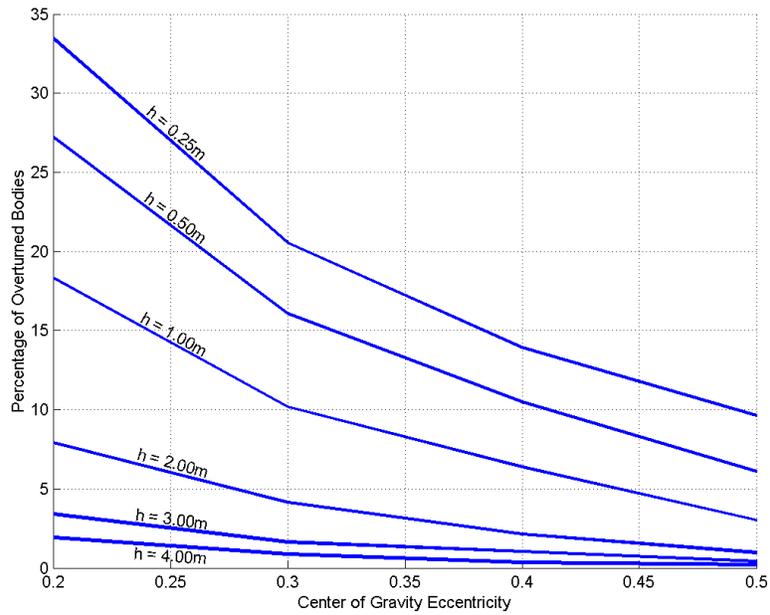


Figure 7: Percentage of Overturned Bodies

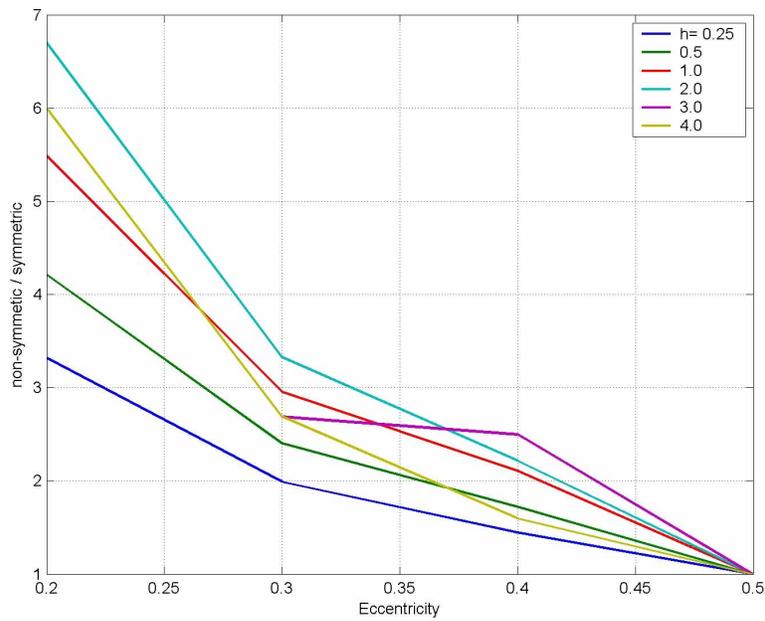


Figure 8. Ratio of overturned non-symmetric / symmetric bodies

Conclusions

Motion equations have been generated for an asymmetric plane body for a pure rocking motion type. Existing criteria for estimating overturning of symmetric bodies are validated and modified to consider non-symmetric ones.

In addition, the following criterion for overturning is proposed for bodies that have an eccentricity of less than 0.3 B:

$$v_{\max} \geq 0.2 \left\{ \frac{1}{\cos \alpha} \sqrt{\frac{8}{3} gr (1 - \cos \alpha)} \right\}$$

Where

$$r = \frac{1}{2} \sqrt{B^2 + (2h)^2}$$

$$\alpha = \arctan \left(\frac{B}{2h} \right)$$

In general, more conservative results are obtained for bigger bodies and/or for smaller eccentricities.

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