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Adaptive passivation with time-varying gains of mimo nonlinear systems

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Abstract *This paper addresses the adaptive passivation of multi-input multi-output (MIMO) non-linear systems,with unknown parameters. The class of MIMO non-linear systems considered here has an explicit linear parametric uncertainty and it is made equivalent to a passive system by means of an adaptive controller with adaptive laws specially designed, which include suitable time-varying gains. The solution presented here is an extension of that obtained by the authors for single-input single-output (SISO) systems. The proposed algorithm was applied, at simulation level, to models of dynamical MIMO systems, to exemplify the controller design methodology and to observe the adaptive system behavior.*

1. Introduction

Over the last years, passivity of dynamical systems has been extensively studied and discussed for the case of non-linear systems when plant parameters are known (Byrnes *et al.*, 1991; Hill and Moylan, 1977; Lin, 1995). Currently, the interest has switched to passivity when the model of a system contains uncertain elements such as constant or time-varying parameters that are not known or imperfectly known. Several results have been reported in control literature to solve the adaptive passivation of single-input single-output (SISO) systems using different kind of methods (Castro and Duarte, 1998; Fradkov and Hill, 1998; Kanellakopoulos, 1993; Kaufman *et al.*, 1994; Kokotovic *et al.*, 1997; Krstic and Kanellakopoulos, 1995; Lin and Shen, 1999; Sepulchre *et al.*, 1997; Seron *et al.*, 1995; Su and Xie, 1996). The most interesting case is certainly the multi-input multi-output (MIMO) which has not been studied in such detail.

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In this paper, a MIMO non-linear system with a Lyapunov stable zero dynamics is considered. The system structure is assumed to have explicit linear parametric uncertainties. An adaptive controller is designed for this class of systems, so that it is made feedback equivalent to a passive system. The controller is a static state feedback whose parameters are updated using adaptive laws suitably obtained. The adaptive laws contain time-varying gains, which allow to handle the transient system behavior. It is shown that, under mild assumptions, the resultant closed-loop system is equivalent to a passive system; hence asymptotic stabilization of the adaptive system can also be obtained by output feedback (Byrnes *et al.*, 1991). The algorithm presented here is an extension of the results given by Duarte *et al.* (2001), for the case of non-linear SISO systems with unity relative degree.

The proposed methodology was applied to models of dynamical MIMO systems, to analyze the adaptive system behavior and to compare with the case of constant adaptive gains (Duarte *et al.*, 2002). The results obtained by simulation indicate that transient behavior of the controlled system can be improved by using time-varying gains.

The paper is organized as follows. In Section 2, some basic definitions and results about MIMO passivity are recalled. In Section 3, under mild assumptions, an adaptive controller with adaptive laws specially designed (including time-varying adaptive gains) is presented. This controller transforms the uncertain non-linear system into a passive system. The adaptive control scheme is applied and evaluated through some numerical simulations in Section 4. Finally, some conclusions and remarks are given in Section 5.

2. Basic concepts of MIMO passive systems

In what follows we will give some basic definitions and concepts concerning passivity of MIMO systems, following the ideas and notation presented by Byrnes *et al.* (1991). Let us consider a MIMO non-linear system of the form

$$\dot{x} = f(x) + G(x)u, \quad y = H(x) \tag{1}$$

where $x \in \mathscr{R}^n$ is the state, $u \in \mathscr{R}^m$ is the input, $y \in \mathscr{R}^m$ is the output, $f \in \mathscr{R}^m$ and $G, H \in \mathscr{R}^{mxm}$. Function f and the m columns of G are C^∞ vector fields, and H is a smooth mapping (i.e. C^∞). It is assumed that the vector field f has at least one equilibrium and without loss of generality we can assume that $f(0) = 0$ and $H(0) = 0$ (in case this is not true, a state shift will be needed). Let $\mathscr{U}$ be the set of admissible inputs that consists of all $\mathbb{R}^m$-valued piecewise continuous functions defined on $\mathbb{R}$. Let us also assume that for any $u \in \mathscr{U}$ and $x^0 = x(0) \in \mathbb{R}^n$, the output $y(t) = H(\Phi(t, x^0, u))$ of system (1) is such that

$$\int_0^t |y^{\mathrm{T}}(s)u(s)|\mathrm{d}s < \infty, \text{ for all } t \geq 0,$$

i.e. the energy stored in system (1) is bounded. $\Phi(t,x^0,u)$ denotes the flow of $f(x)+G(x)u$ for any initial condition $x^0 \in \mathbb{R}^n$ and for any $u \in \mathcal{U}$. Such a property is present in many practical systems and has been widely exploited for control purposes.

We will recall some passivity properties as well as some stability properties of MIMO passive systems in accordance with the results of Byrnes *et al.* (1991) and Hill and Moylan (1977).

The system (1) is said to be *C^r-passive* if there exists a C^r non-negative function $V:\mathbb{R}^n \rightarrow R$, called the *storage function*, which satisfies $V(0)=0$, such that for all $u \in \mathcal{U}$, $x^0 \in \mathbb{R}^n$, $t \geq 0$

$$V(x) - V(x^0) \leq \int_0^t y^{\mathrm{T}}(s)u(s)\,\mathrm{d}s. \tag{2}$$

Accordingly, system (1) is said to be *locally feedback equivalent to a C^r-passive system*, or just *locally feedback C^r-passive*, if there exists a state feedback of the form $u=\alpha(x)+\beta(x)w$, where $\alpha(x)$ and $\beta(x)$ are smooth functions defined either locally near $x=0$ or globally and $\beta(x)$ is invertible for all x, such that, for the closed-loop system, the following inequality is satisfied:

$$\dot{V}(x) = \left(\frac{\partial V(x)}{\partial x}\right)^{\mathrm{T}}[f(x)+G(x)\alpha(x)+G(x)\beta(x)w] \leq y^{\mathrm{T}}w \tag{3}$$

where V is a C^r storage function. Condition (3) is the differential version of condition (2) for passive systems.

If one makes the assumption that $L_G H(x) = [\partial H/\partial x]G(x) \neq 0$ in a neighborhood of $x=0$, then system (1) has a so-called *relative degree* $\rho = 1,1,\ldots,1\}$ around $x=0$, and there exists a new set of local coordinates $\zeta(x)=(z(x), y=H(x))$, with $z(x)=[z_1(x)\cdots z_{n-1}(x)]^{\mathrm{T}}$, under which system (1) can be represented in the *normal form* (Byrnes *et al.*, 1991)

$$\dot{y} = a(y,z)+B(y,z)u, \quad \dot{z}=c(y,z) \tag{4}$$

where $B(y,z)$ is invertible for all (y,z) near (0, 0). The *zero dynamics* of a system describes the internal dynamics which is consistent with the external constraint $y=0$. For system (4), the zero dynamics is given by

$$\dot{z} = c(0,z) := f_0(z). \tag{5}$$

It is shown by Byrnes *et al.* (1991) that system (4) can be written as

$$\dot{y} = a(y,z)+B(y,z)u, \quad \dot{z}=f_0(z)+P(y,z)y \tag{6}$$

where $P(y,z)$ is a C^∞ matrix function of dimension $m \times (n-1)$.

Another two important concepts on the zero dynamics of system (1) are *locally minimum phase* and *locally weakly minimum phase*. Suppose that $L_G H(x) = [\partial H/\partial x]G(x) \neq 0$ in a neighborhood of $x = 0$. Then system (1) is said to be *locally minimum phase* if its zero dynamics is asymptotically stable in a neighborhood of $z = 0$. On the other hand, system (1) is said to be *locally weakly minimum phase* if there exists a positive differentiable function $W_0(z)$, with $W_0 = 0$, such that $L_{f_0} W_0(z) = [\partial W_0/\partial z] f_0(z) \leq 0$ for all z in a neighborhood of $z = 0$.

A geometric characterization of systems that can be made C^2-passive was given by Byrnes *et al.* (1991). They proved that a system (1) can be made locally feedback equivalent to a passive system with a C^2-positive definite storage function, if and only if system (1) has a relative degree $\rho = \{1, 1, \ldots, 1\}$ at $x = 0$ and is locally weakly minimum phase (see Byrnes *et al.* (1991), Theorem 4.7) The *global feedback equivalence* of system (1) to a C^2-passive system is based on the existence of a global defined diffeomorphism which transforms this system into another one having the so-called *globally defined normal form* (see section IV of Byrnes *et al.* (1991) for a more detailed discussion of this equivalence).

It is also useful to characterize the stability properties of a passive system. In this sense, suppose that system (1) has been made locally feedback equivalent to the C^2-passive system

$$\dot{x} = f(x) + G(x)\alpha(x) + G(x)\beta(x)w := \bar{f}(x, w), \quad y = H(x). \tag{7}$$

If, in addition, system (7) is *locally zero-state detectable* [1], then its equilibrium $x = 0$ can be made locally asymptotically stable via the output feedback

$$w = -\varphi(y) \tag{8}$$

where $\varphi : \mathscr{R}^m \rightarrow \mathscr{R}^m$ is any smooth function such that $\varphi(0) = 0$ and $y^{\mathrm{T}}\varphi(y) > 0$ for each $y \in \mathscr{R}^m$ (Byrnes *et al.*, 1991).

3. MIMO adaptive feedback passivity

Motivated by equations (4) and (6) and the geometric characterization of feedback passive systems recalled in the previous section, we will consider a MIMO non-linear system with linear explicit parametric dependence expressed in the normal form, i.e.

$$\dot{y} = \Lambda_a a(y, z) + \Lambda_b B(y, z)u, \quad \dot{z} = \Lambda_0 f_0(z) + P^{\mathrm{T}}(y, z)\Lambda_p y \tag{9}$$

with

$$\Lambda_a = \begin{bmatrix} \lambda_{a_{11}} & \lambda_{a_{12}} & \cdots & \lambda_{a_{1n}} \\ \lambda_{a_{21}} & \lambda_{a_{22}} & \cdots & \lambda_{a_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{a_{n1}} & \lambda_{a_{n2}} & \cdots & \lambda_{a_{nn}} \end{bmatrix} \in \mathscr{R}^{mxm}, \Lambda_b = \begin{bmatrix} \lambda_{b_{11}} & \lambda_{b_{12}} & \cdots & \lambda_{b_{1m}} \\ \lambda_{b_{21}} & \lambda_{b_{22}} & \cdots & \lambda_{b_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{b_{m1}} & \lambda_{b_{m2}} & \cdots & \lambda_{b_{mm}} \end{bmatrix} \in \mathscr{R}^{mxm},$$

$$\Lambda_0 = \begin{bmatrix} \lambda_{0_{11}} & \lambda_{0_{12}} & \cdots & \lambda_{0_{1n}} \\ \lambda_{0_{21}} & \lambda_{0_{22}} & \cdots & \lambda_{0_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{0_{n1}} & \lambda_{0_{n2}} & \cdots & \lambda_{0_{nn}} \end{bmatrix} \in \mathscr{R}^{nxn}, \Lambda_p = \begin{bmatrix} \lambda_{p_{11}} & \lambda_{p_{12}} & \cdots & \lambda_{p_{1n}} \\ \lambda_{p_{21}} & \lambda_{p_{22}} & \cdots & \lambda_{p_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{p_{n1}} & \lambda_{p_{n2}} & \cdots & \lambda_{p_{nn}} \end{bmatrix} \in \mathscr{R}^{mxm},$$

where $z \in \mathscr{R}^n$, $y \in \mathscr{R}^m$, $u \in \mathscr{R}^m$, $a(y,z) \in \mathscr{R}^m$, $B(y,z) \in \mathscr{R}^{mxm}$, $f_0(z) \in \mathscr{R}^n$, $P(y,z) \in \mathscr{R}^{mxn}$. Matrices Λ_a, Λ_b, Λ_0, Λ_p, represent constant but unknown parameters, belonging to a compact set Ω.

The following assumption regarding matrix $B(y, z)$ is introduced.

Assumption 1. The term $B(y,z)$ in system (9) is such that $B^{-1}(y,z)$ exists in a neighborhood of (0,0).Let us assume that the true values of plant parameters are $\Lambda_a^* \in \mathscr{R}^{mxm}$, $\Lambda_b^* \in \mathscr{R}^{mxm}$, $\Lambda_0^* \in \mathscr{R}^{nxn}$, $\Lambda_p^* \in \mathscr{R}^{mxm}$. The nominal system considered in this study is system (9) under Assumption 1, with true values Λ_a^*, Λ_b^*, Λ_0^*, Λ_p^*, i.e.

$$\begin{aligned} \dot{y} &= \Lambda_a^* a(y,z) + \Lambda_b^* B(y,z)u, \\ \dot{z} &= \Lambda_0^* f_0(z) + P^{\mathrm{T}}(y,z)\Lambda_p^* y \end{aligned} \tag{10}$$

Assuming that the parameters are unknown, the problem of designing a controller whose parameters are adaptively adjusted, and that renders the system (10) passive (equivalent to a passive system), is now considered. To do this, a basic assumption is introduced in the stability properties of the zero dynamics $\dot{z} = \Lambda_0^* f_0(z)$.

Assumption 2. System (10) is a locally weakly minimum phase, i.e. there exists a positive definite differentiable function $W_0(z)$ that satisfies

$$\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) \leq 0, \quad \forall \Lambda_0^* \in \Omega' \subset \mathscr{R}^{nxn}$$

in a neighborhood of $z = 0$.

When the parameter true values are known, one can straightforwardly verify, under Assumptions 1 and 2, that system (10) can be made locally feedback equivalent to a C^2-passive system with storage function (Byrnes *et al.*, 1991)

$$V(y,z) = W_0(z) + (1/2)y^{\mathrm{T}}y$$

by means of the feedback

$$u(y,z) = B^{-1}(y,z)\Lambda_b^{*-1}\left[-\Lambda_a^* a(y,z) - \Lambda_p^{*\mathrm{T}} P(y,z)\frac{\partial W_0(z)}{\partial z} + w\right]$$

where $p_{ij}(y, z)$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$; and $\partial W_0(z)/\partial z_i$ for $i = 1, 2, \ldots, n$, the entries of the matrices $P(y, z)$ and $\partial W_0(z)/\partial z$, respectively, are smooth functions. The signal w is the new input.

Recognizing the difficulties in knowing the exact values of the true plant parameters, an adaptive solution to the passivation problem is presented. Since the proposed solution for the adaptive case relies on some knowledge on Λ_b^*, three cases will be considered in what follows.

3.1 Case of Λ_{b}^* diagonal

We first address the problem of adaptive passivation of system (10) when parameters are unknown and $\Lambda_b^* \in \mathscr{R}^{mxm}$ is diagonal and invertible. The following assumption regarding the knowledge of the sign of Λ_b^* is made.

Assumption 3. The sign of all the elements of matrix Λ_b^* are known and has the following form:

$$\Lambda_b^* = \begin{bmatrix} \lambda_{b_1}^* & 0 & \cdots & 0 \\ 0 & \lambda_{b_2}^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{b_m}^* \end{bmatrix} \in \mathscr{R}^{mxm}.$$

Then, the solution of the feedback adaptive passivity problem stated earlier is given in the following theorem.

Theorem 1. Consider the non-linear system (10) with unknown matrices and suppose that Assumptions 1-3 hold. Then, the controller

$$
\begin{aligned}
u(y,z,\theta) &= B^{-1}(y,z)\left[\theta_1(t)a(y,z) + \theta_2(t)P(y,z)\frac{\partial W_0(z)}{\partial z} + \theta_3(t)w(t)\right], \\
\dot{\Phi}_{\theta_1}(t) &= \dot{\theta}_1(t) = -\frac{1}{\alpha(t)}\Gamma_1^{-1}(t)y(t)a^{\mathrm{T}}(y,z)\mathrm{sign}(\Lambda_b^*), \\
\dot{\Phi}_{\theta_2}(t) &= \dot{\theta}_2(t) = -\frac{1}{\alpha(t)}\Gamma_2^{-1}(t)y(t)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z)\mathrm{sign}(\Lambda_b^*), \\
\dot{\Phi}_{\theta_3}(t) &= \dot{\theta}_3(t) = -\frac{1}{\alpha(t)}\Gamma_3^{-1}(t)y(t)w^{\mathrm{T}}(t)\mathrm{sign}(\Lambda_b^*),
\end{aligned} \tag{11}
$$

where,

$$
\begin{aligned}
\dot{\Gamma}_1(t) &= -\Gamma_1(t)a(y,z)a^{\mathrm{T}}(y,z)\Gamma_1(t), \quad \Gamma_1(t_0) > 0 \\
\dot{\Gamma}_2(t) &= -\Gamma_2(t)P(y,z)\frac{\partial w}{\partial z}\left(\frac{\partial w}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z)\Gamma_2(t), \quad \Gamma_2(t_0) > 0 \\
\dot{\Gamma}_3(t) &= -\Gamma_3(t)w(t)w^{\mathrm{T}}(t)\Gamma_3(t), \quad \Gamma_3(t_0) > 0
\end{aligned} \tag{12}
$$

makes the system locally feedback equivalent to a C^2-passive system from the input $w(t)$ to the output $y(t)$. Matrices $\theta_1(t) \in \mathscr{R}^{mxm}$, $\theta_2(t) \in \mathscr{R}^{mxm}$, $\theta_3(t) \in \mathscr{R}^{mxm}$ are the controller adjustable parameters. The matrix sign $(\Lambda_b^*) \in \mathscr{R}^{mxm}$ denotes a matrix containing the sign of each element of Λ_b^*. The normalization factor

$$
\alpha(t) = \sqrt{1 + \mathrm{Trace}\left[\Gamma_1^{-2}(t) + \Gamma_2^{-2}(t) + \Gamma_3^{-2}(t)\right]} > 1,
$$

was defined based on system stability. The deviation variables (parameter errors) are defined as $\Phi_{\theta_1}(t) \equiv \theta_1(t) - \theta_1^*$, $\Phi_{\theta_2}(t) \equiv \theta_2(t) - \theta_2^*$ and $\Phi_{\theta_3}(t) \equiv \theta_3(t) - \theta_3^*$. The ideal values are defined as $\theta_1^* = -\Lambda_b^{*-1}\Lambda_a^*$, $\theta_2^* = -\Lambda_b^{*-1}\Lambda_p^{*\mathrm{T}}$ and $\theta_3^* = \Lambda_b^{*-1}$, as will become clear in the proof.

Proof. Let us choose the following storage function candidate for systems (10)-(12):

$$
\begin{aligned}
V(y,z,\Phi_{\theta_1},\Phi_{\theta_2},\Phi_{\theta_3}) = \frac{1}{\alpha(t)}W_0(z) + \frac{1}{2}\frac{1}{\alpha(t)}y^{\mathrm{T}}y + \frac{1}{2}\mathrm{Trace}\Big[|\Lambda_b^*|\Phi_{\theta_1}^{\mathrm{T}}\Gamma_1\Phi_{\theta_1} \\
+ |\Lambda_b^*|\Phi_{\theta_2}^{\mathrm{T}}\Gamma_2\Phi_{\theta_2} + |\Lambda_b^*|\Phi_{\theta_3}^{\mathrm{T}}\Gamma_3\Phi_{\theta_3}\Big]
\end{aligned} \tag{13}
$$

where matrix $|\Lambda_b^*| \in \mathscr{R}^{mxm}$ denotes the matrix containing the absolute values of each element of matrix Λ_b^*. The time derivative of $V(y,z,\Phi_{\theta_1},\Phi_{\theta_2},\Phi_{\theta_3})$ along the trajectories of systems (10)-(12) is

$$\dot{V} = \frac{\partial}{\partial t}\left(\frac{1}{\alpha(t)}\right)W_0 + \frac{1}{\alpha(t)}\dot{W}_0 + \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\alpha(t)}\right)y^{\mathrm{T}}y + \frac{1}{\alpha(t)}y^{\mathrm{T}}\dot{y}$$
$$+ \mathrm{Trace}\left[|\Lambda_b^*|\Phi_{\theta_1}^{\mathrm{T}}\Gamma_1\Phi_{\theta_1} + \frac{1}{2}|\Lambda_b^*|\Phi_{\theta_1}^{\mathrm{T}}\dot{\Gamma}_1\Phi_{\theta_1}\right]$$
$$+ \mathrm{Trace}\left[|\Lambda_b^*|\dot{\Phi}_{\theta_2}^{T}\Gamma_2\Phi_{\theta_2} + \frac{1}{2}|\Lambda_b^*|\Phi_{\theta_2}^{T}\dot{\Gamma}_2\Phi_{\theta_2}\right]$$
$$+ \mathrm{Trace}\left[|\Lambda_b^*|\dot{\Phi}_{\theta_3}^{\mathrm{T}}\Gamma_3\Phi_{\theta_3} + \frac{1}{2}|\Lambda_b^*|\Phi_{\theta_3}^{\mathrm{T}}\dot{\Gamma}_3\Phi_{\theta_3}\right]$$

First we replace

$$\dot{W}_0 = \left(\frac{\partial}{\partial z}W_0(z)\right)^{\mathrm{T}}\dot{z}$$

and substitute $\dot{z}$ and $\dot{y}$ from equation (10). Next we replace the adaptive laws given by equation (11) and the time-varying gains given by equation (12). Thus we get

$$\dot{V} = \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left(W_0 + \frac{1}{2}y^{\mathrm{T}}y\right) + \frac{1}{\alpha}\left(\frac{\partial W_0}{\partial z}\right)^{\mathrm{T}}\Lambda_0^* f_0(z) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}P^{\mathrm{T}}(y,z)\Lambda_p^* y$$
$$+ \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_a^* a(y,z) + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^* B(y,z)u - \frac{1}{\alpha}\mathrm{Trace}\Big[|\Lambda_b^*|\mathrm{sign}(\Lambda_b^*)^{\mathrm{T}}ay^{\mathrm{T}}\Gamma_1^{-1\mathrm{T}}\Gamma_1\Phi_{\theta_1}$$
$$+ |\Lambda_b^*|\mathrm{sign}(\Lambda_b^*)^{\mathrm{T}}P\frac{\partial W_0}{\partial z}y^{\mathrm{T}}\Gamma_2^{-1\mathrm{T}}\Gamma_2\Phi_{\theta_2} + |\Lambda_b^*|\mathrm{sign}(\Lambda_b^*)^{\mathrm{T}}wy^{\mathrm{T}}\Gamma_3^{-1\mathrm{T}}\Gamma_3\Phi_{\theta_3}\Big]$$
$$- \frac{1}{2}\mathrm{Trace}\Big[|\Lambda_b^*|\Phi_{\theta_1}^{\mathrm{T}}\Gamma_1 aa^{\mathrm{T}}\Gamma_1\Phi_{\theta_1} + |\Lambda_b^*|\Phi_{\theta_2}^{\mathrm{T}}\Gamma_2 P\frac{\partial W_0(z)}{\partial z}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}P^{\mathrm{T}}\Gamma_2\Phi_{\theta_2}$$
$$+ |\Lambda_b^*|\Phi_{\theta_3}^{\mathrm{T}}\Gamma_3 ww^{\mathrm{T}}\Gamma_3\Phi_{\theta_3}\Big]$$

Since $\Gamma_i(t) = \Gamma_i^{\mathrm{T}}(t) > 0$ for $i = 1, 2, 3$, we have

$$|\Lambda_b^*|\Phi_{\theta_1}^{\mathrm{T}}\Gamma_1 aa^{\mathrm{T}}\Gamma_1\Phi_{\theta_1} > 0, |\Lambda_b^*|\Phi_{\theta_2}^{\mathrm{T}}\Gamma_2 P\frac{\partial W_0(z)}{\partial z}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}P^{\mathrm{T}}\Gamma_2\Phi_{\theta_2} > 0$$

and

$$|\Lambda_b^*|\Phi_{\theta_3}^{\mathrm{T}}\Gamma_3 ww^{\mathrm{T}}\Gamma_3\Phi_{\theta_3} > 0.$$

Replacing the control input $u(t)$ from equation (11) then we can write

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left(W_0 + \frac{1}{2}y^{\mathrm{T}}y\right) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}\Lambda_0^* f_0(z) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z)\Lambda_p^* y$$
$$+ \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_a^* a(y,z) + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_1(t)a(y,z) + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_2(t)P(y,z)\frac{\partial W_0(z)}{\partial z}$$
$$+ \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_3(t)w - \frac{1}{\alpha}\mathrm{Trace}\left[\Lambda_b^* a y^{\mathrm{T}}\Phi_{\theta_1} + \Lambda_b^* P\frac{\partial W_0}{\partial z}y^{\mathrm{T}}\Phi_{\theta_2} + \Lambda_b^* w y^{\mathrm{T}}\Phi_{\theta_3}\right]$$

Using the property of two vector $a^{\mathrm{T}}b = b^{\mathrm{T}}a = \mathrm{Trace}(ab^{\mathrm{T}}) = \mathrm{Trace}(ba^{\mathrm{T}})$, we can write the terms inside the trace function as follows:

$$\mathrm{Trace}\left[\Lambda_b^* a y^{\mathrm{T}}\Phi_{\theta_1}\right] = y^{\mathrm{T}}\Phi_{\theta_1}\Lambda_b^* a,\ \mathrm{Trace}\left[\Lambda_b^* P\frac{\partial W_0}{\partial z}y^{\mathrm{T}}\Phi_{\theta_2}\right]$$
$$= y^{\mathrm{T}}\Phi_{\theta_2}\Lambda_b^* P\frac{\partial W_0}{\partial z} \ \text{ and }\ \mathrm{Trace}\left[\Lambda_b^* w y^{\mathrm{T}}\Phi_{\theta_3}\right] = y^{\mathrm{T}}\Phi_{\theta_3}\Lambda_b^* w.$$

Replacing these terms in the previous equation, we have

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left(W_0 + \frac{1}{2}y^{\mathrm{T}}y\right) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}\Lambda_0^* f_0(z)$$
$$+ \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z)\Lambda_p^* y + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_a^* a(y,z) + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_1 a(y,z)$$
$$+ \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_2 P(y,z)\frac{\partial W_0(z)}{\partial z} + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_3 w - \frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_1}\Lambda_b^* a$$
$$- \frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_2}\Lambda_b^* P\frac{\partial W_0(z)}{\partial z} - \frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_3}\Lambda_b^* w$$

Since $\theta_i^* = \theta_i(t) - \Phi_{\theta_i}(t)$, for $i = 1, 2, 3$, we can write

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left(W_0 + \frac{1}{2}y^{\mathrm{T}}y\right)$$
$$+ \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}\Lambda_0^* f_0(z) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z)\Lambda_p^* y + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_a^* a(y,z)$$
$$+ \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_1^* a(y,z) + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_2^* P(y,z)\frac{\partial W_0(z)}{\partial z} + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_3^* w$$

Since

$$\frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z)\Lambda_p^* y = \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_p^{*\mathrm{T}} P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right),$$

we can regroup the terms as follows

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left(W_0 + \frac{1}{2} y^{\mathrm{T}} y\right) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) + \frac{1}{\alpha} y^{\mathrm{T}}\left(\Lambda_a^* + \Lambda_b^* \theta_1^*\right) a$$

$$+ \frac{1}{\alpha} y^{\mathrm{T}}\left(\Lambda_p^{*\mathrm{T}} + \Lambda_b^* \theta_2^*\right) P \frac{\partial W_0(z)}{\partial z} + \frac{1}{\alpha} y^{\mathrm{T}}\left(I - I + \Lambda_b^* \theta_3^*\right) w$$

Let us define the ideal values θ_i^* for $i = 1, 2, 3$ as those values satisfying the following three equations; $\Lambda_a^* + \Lambda_b^* \theta_1^* = 0$, i.e. $\theta_1^* = -\Lambda_b^{*-1}$; Λ_a^*, $\Lambda_p^{*\mathrm{T}} + \Lambda_b^* \theta_2^* = 0$, i.e. $\theta_2^* = -\Lambda_b^{*-1}\Lambda_p^{*\mathrm{T}}$, and $-I + \Lambda_b^* \theta_3^* = 0$, i.e. $\theta_3^* = \Lambda_b^{*-1}$. Replacing these definitions in the previous equation, we have

$$\dot{V} \leq -\frac{\dot{\alpha}}{\alpha^2}\left(W_0 + \frac{1}{2} y^{\mathrm{T}} y\right) + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) + \frac{1}{\alpha} y^{\mathrm{T}} w$$

By Assumption 2, $(\partial W_0(z)/\partial z)^{\mathrm{T}}\, \Lambda_0^* f_0(z) \leq 0$, and since W_0 is a positive definite function, $\alpha(t)$ should satisfy the following two conditions: $\dot{\alpha}/\alpha^2 \geq 0$ and $1/\alpha > 0$. The normalization factor should also be useful to bound $\Gamma_i^{-1}(t)$ (since $\Gamma_i(t) > 0$ and tends to zero as time goes to infinity, $\Gamma_i^{-1}(t)$ diverges). One interesting choice of the normalization factor $\alpha(t)$ is motivated by the recursive least squares (RLS) method (Ljung and Söderström, 1985) and it is given by

$$\alpha(t) = \sqrt{1 + \mathrm{Trace}\left[\Gamma_1^{-2}(t) + \Gamma_2^{-2}(t) + \Gamma_3^{-2}(t)\right]}$$

From equation (12), we know that $\Gamma_i(t) > 0$ for $i = 1, 2, 3$, then the normalization factor is such that $0 < \alpha^{-1}(t) < 1$. Finally, we can write

$$\dot{V} \leq \frac{1}{\alpha} y^{\mathrm{T}} w \leq y^{\mathrm{T}} w$$

Thus, the closed-loop system is C^2-passive from w to y. □

Remark 1. From the proof of Theorem 1, it happens that $\dot{V} \leq 0$ for the closed-loop system (10)-(12) when $w = 0$. Thus, since $V(y, z, \Phi_{\theta_1}, \Phi_{\theta_2}, \Phi_{\theta_3})$ is positive definite, one has that this closed-loop system is locally stable (in the sense of Lyapunov) when the input is zero. Also notice that, though the above result is local, a global version can be obtained if Assumptions 1 and 3 hold globally.

Remark 2. Notice that if in the closed-loop system (10)-(12) the state variables (y, z) are, in addition, locally zero-state detectable, then its equilibrium can be made locally asymptotically stable via the output feedback (8), e.g. $w = -Ky$, with $K > 0$ (positive definite), while the controller parameters $\theta_i(t)$, $i = 1, 2, 3$, converge to bounded constant values. Also, in the proposed solution, the unknown matrices parameter $\Lambda_a^*, \Lambda_a^*, \Lambda_0^*, \Lambda_p^*$ are not estimated, instead the controller parameters $\theta_i(t)$, $i = 1, 2, 3$ are directly adjusted.

Remark 3. In order to avoid matrix inversion, time-varying gains are implemented using the following matrix property

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Gamma^{-1}(t) = \Gamma^{-1}(t)\Gamma(t)\Gamma^{-1}(t).$$

Thus, the adaptive gains are implemented as

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_1^{-1}(t)\right) = -a(y,z)a^{\mathrm{T}}(y,z), \quad \Gamma_1^{-1}(t_0) > 0 \\
&\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_2^{-1}(t)\right) = -P(y,z)\frac{\partial w}{\partial z}\left(\frac{\partial w}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z), \quad \Gamma_2^{-1}(t_0) > 0 \\
&\frac{\mathrm{d}}{\mathrm{d}t}\left(\Gamma_3^{-1}(t)\right) = -w(t)w^{\mathrm{T}}(t), \quad \Gamma_3^{-1}(t_0) > 0
\end{aligned} \tag{14}$$

3.2 Case of Λ_{b}^* General

The solution of the feedback passivity problem for the case when Λ_b^* is a general and invertible matrix, is given in the following theorem. The idea of having this particular control law for $u(t)$ was taken from Narendra and Annaswamy (1989).

Theorem 2. Let us consider the non-linear system (10) with unknown matrices and suppose that Assumptions 1 and 2 hold. Then, the controller

$$\begin{aligned}
&u(y,z,\theta) = B^{-1}(y,z)\left[\theta_3(t)\theta_1(t)a(y,z) + \theta_3(t)\theta_2(t)P(y,z)\frac{\partial W_0(z)}{\partial z} + \theta_3(t)w\right], \\
&\dot{\Phi}_{\theta_1}(t) = \dot{\theta}_1(t) = -\frac{1}{\alpha(t)}\Gamma_1^{-1}y(t)a^{\mathrm{T}}(y,z), \\
&\dot{\Phi}_{\theta_2}(t) = \dot{\theta}_2(t) = -\frac{1}{\alpha(t)}\Gamma_2^{-1}y(t)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z), \\
&\dot{\Phi}_{\theta_3}(t) = -\frac{1}{\alpha(t)}\Gamma_3^{-1}y(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}, \quad \text{or} \\
&\dot{\theta}_3(t) = -\frac{1}{\alpha(t)}\Gamma_3^{-1}\theta_3(t)y(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}\theta_3(t)
\end{aligned} \tag{15}$$

with

$$\dot{\Gamma}_1 = -\Gamma_1 a a^{\mathrm{T}} \Gamma_1, \quad \Gamma_1(t_0) > 0$$
$$\dot{\Gamma}_2 = -\Gamma_2 P \left(\frac{\partial W_0(z)}{\partial z}\right)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}} \Gamma_2, \quad \Gamma_2(t_0) > 0 \tag{16}$$
$$\dot{\Gamma}_3 = -\Gamma_3 B u u^{\mathrm{T}} B^{\mathrm{T}} \Gamma_3, \quad \Gamma_3(t_0) > 0$$

makes the system locally feedback equivalent to a C^2-passive system from the input $w(t)$ to the output $y(t)$. Matrices $\theta_1(t) \in \mathscr{R}^{mxm}$, $\theta_2(t) \in \mathscr{R}^{mxm}$, $\theta_3(t) \in \mathscr{R}^{mxm}$ are the adjustable controller parameters. As in the previous case, the normalization factor

$$\alpha(t) = \sqrt{1 + \mathrm{Trace}\left[\Gamma_1^{-2}(t) + \Gamma_2^{-2}(t) + \Gamma_3^{-2}(t)\right]} > 1,$$

was defined based on system stability. The deviation variables (parameter errors) are defined as $\Phi_{\theta_1}(t) \equiv \theta_1(t) - \theta_1^*$, $\Phi_{\theta_2}(t) \equiv \theta_2(t) - \theta_2^*$ and $\Phi_{\theta_3}(t) \equiv \theta_3^{*-1} - \theta_3^{-1}(t)$. The ideal values are defined as $\theta_1^* = -\Lambda_b^{*-1}$, $\theta_2^* = -\Lambda_b^{*-1}\Lambda_p^{*\mathrm{T}}$ and, $\theta_3^* = \Lambda_b^{*-1}$ or $\left(\theta_3^{*-1} = \Lambda_b^*\right)$ as will become clear in the proof.

Proof. Let us choose the following storage function candidate for systems (10), (15) and (16):

$$V(y, z, \Phi_{\theta_1}, \Phi_{\theta_2}, \Phi_{\theta_3}) = \frac{1}{\alpha} W_0(z) + \frac{1}{2}\frac{1}{\alpha} y^{\mathrm{T}} y + \frac{1}{2}\mathrm{Trace}\left[\Phi_{\theta_1}^{\mathrm{T}} \Gamma_1 \Phi_{\theta_1} + \Phi_{\theta_2}^{\mathrm{T}} \Gamma_2 \Phi_{\theta_2} + \Phi_{\theta_3} \Gamma_3 \Phi_{\theta_3}^{\mathrm{T}}\right]. \tag{17}$$

The time derivative of $V(y, z, \Phi_{\theta_1}, \Phi_{\theta_2}, \Phi_{\theta_3})$ along the trajectories of systems (10), (15) and (16) is

$$\dot{V} = \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right) W_0 + \frac{1}{\alpha}\dot{W}_0 + \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right) y^{\mathrm{T}} y + \frac{1}{\alpha} y^{\mathrm{T}} \dot{y}$$
$$+ \mathrm{Trace}\left[\dot{\Phi}_{\theta_1}^{\mathrm{T}} \Gamma_1 \Phi_{\theta_1} + \dot{\Phi}_{\theta_2}^{\mathrm{T}} \Gamma_2 \Phi_{\theta_2} + \dot{\Phi}_{\theta_3}^{\mathrm{T}} \Gamma_3 \Phi_{\theta_3}\right]$$
$$+ \frac{1}{2}\mathrm{Trace}\left[\Phi_{\theta_1}^{\mathrm{T}} \dot{\Gamma}_1 \Phi_{\theta_1} + \Phi_{\theta_2}^{\mathrm{T}} \dot{\Gamma}_2 \Phi_{\theta_2} + \Phi_{\theta_3}^{\mathrm{T}} \dot{\Gamma}_3 \Phi_{\theta_3}\right]$$

Substituting

$$\dot{W}_0 = \left(\frac{\partial}{\partial z} W_0(z)\right)^{\mathrm{T}} \dot{z},$$

replacing $\dot{y}$ from equation (10) and replacing the adaptive laws and adaptive gains from (15) and (16), respectively, one obtains

$$\dot{V} = \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right) W_0 + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \dot{z} + \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right) y^{\mathrm{T}} y + \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_a^* a + \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_b^* Bu$$
$$+ \mathrm{Trace}\left[-\frac{1}{\alpha} a y^{\mathrm{T}} \Gamma_1^{-1\mathrm{T}} \Gamma_1 \Phi_{\theta_1} - \frac{1}{\alpha} P \left(\frac{\partial W_0(z)}{\partial z}\right) y^{\mathrm{T}} \Gamma_2^{-1\mathrm{T}} \Gamma_2 \Phi_{\theta_2}\right.$$
$$\left. - \frac{1}{\alpha} \mathrm{Buy}^{\mathrm{T}} \Gamma_3^{-1\mathrm{T}} \Gamma_3 \Phi_{\theta_3}\right] - \frac{1}{2}\mathrm{Trace}\left[\Phi_{\theta_1}^{\mathrm{T}} \Gamma_1 a a^{\mathrm{T}} \Gamma_1 \Phi_{\theta_1} + \Phi_{\theta_2}^{\mathrm{T}} \Gamma_2 P \left(\frac{\partial W_0(z)}{\partial z}\right)\right.$$
$$\left. \times \left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}} \Gamma_2 \Phi_{\theta_2} + \Phi_{\theta_3}^{\mathrm{T}} \Gamma_3 B u u^{\mathrm{T}} B^{\mathrm{T}} \Gamma_3 \Phi_{\theta_3}\right]$$

Replacing $\dot{z}$ from equation (10), substituting the control input $u(t)$ from equation (15) and considering that

$$\Phi_{\theta_1}^{\mathrm{T}} \Gamma_1 a a^{\mathrm{T}} \Gamma_1 \Phi_{\theta_1} > 0, \Phi_{\theta_2}^{\mathrm{T}} \Gamma_2 P \left(\frac{\partial W_0(z)}{\partial z}\right)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}} \Gamma_2 \Phi_{\theta_2}$$
$$> 0 \text{ and } \Phi_{\theta_3}^{\mathrm{T}} \Gamma_3 B u u^{\mathrm{T}} B^{\mathrm{T}} \Gamma_3 \Phi_{\theta_3} > 0,$$

we can write

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left[W_0 + \frac{1}{2} y^{\mathrm{T}} y\right] + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) +$$
$$\frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y,z) \Lambda_p^{*\mathrm{T}} y$$
$$+ \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_a^* a(y,z) + \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_b^* \theta_3 \theta_1 a(y,z)$$
$$+ \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_b^* \theta_3 \theta_2 P(y,z) \frac{\partial W_0(z)}{\partial z} + \frac{1}{\alpha} y^{\mathrm{T}} \Lambda_b^* \theta_3 w$$
$$- \frac{1}{\alpha}\mathrm{Trace}\left[a y^{\mathrm{T}} \Phi_{\theta_1} + P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right) y^{\mathrm{T}} \Phi_{\theta_2} + B u y^{\mathrm{T}} \Phi_{\theta_3}\right].$$

Using the property that two vectors $a^{\mathrm{T}} b = b^{\mathrm{T}} a = \mathrm{Trace}(a b^{\mathrm{T}}) = \mathrm{Trace}(b a^{\mathrm{T}})$, we can write

$$\text{Trace}[ay^{\mathrm{T}}\Phi_{\theta_1}] = y^{\mathrm{T}}\Phi_{\theta_1}a$$

$$\text{Trace}\left[P\left(\frac{\partial W_0(z)}{\partial z}\right)y^{\mathrm{T}}\Phi_{\theta_2}\right] = y^{\mathrm{T}}\Phi_{\theta_2}P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right)$$

$$\text{Trace}[Buy^{\mathrm{T}}\Phi_{\theta_3}] = y^{\mathrm{T}}\Phi_{\theta_3}Bu$$

Replacing these terms in the previous equation and noting that $\Phi_{\theta_1} = \theta_1 - \theta_1^*$, $\Phi_{\theta_2} = \theta_2 - \theta_2^*$ and $\Phi_{\theta_3} = \theta_3^{*-1} - \theta_3^{-1}$, we can regroup the terms to obtain

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left[W_0 + \frac{1}{2}y^{\mathrm{T}}y\right] + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}\Lambda_0^* f_0(z)$$

$$+\frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}P^{\mathrm{T}}(y,z)\Lambda_p^* y \quad + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_a^* a(y,z)$$

$$+\frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_3\theta_1 a(y,z) + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_3\theta_2 P(y,z)\frac{\partial W_0(z)}{\partial z} + \frac{1}{\alpha}y^{\mathrm{T}}\Lambda_b^*\theta_3 w$$

$$-\frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_1}a - \frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_2}P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right) - \frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_3}Bu$$

The term $1/\alpha(\partial W_0(z)/\partial z)^{\mathrm{T}}\ P^{\mathrm{T}}(y,z)\Lambda_p^* y$ can be written as $1/\alpha y^{\mathrm{T}}\Lambda_p^{*\mathrm{T}}P(y,z)$ $(\partial W_0(z)/\partial z)$. Therefore, we have

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left[W_0 + \frac{1}{2}y^{\mathrm{T}}y\right] + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}\Lambda_0^* f_0(z)$$

$$+\frac{1}{\alpha}y^{\mathrm{T}}[\Lambda_a^* + \Lambda_b^*\theta_3\theta_1 - \Phi_{\theta_1}]a$$

$$+\frac{1}{\alpha}y^{\mathrm{T}}\left[\Lambda_p^{*\mathrm{T}} + \Lambda_b^*\theta_3\theta_2 - \Phi_{\theta_2}\right]P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right)$$

$$+\frac{1}{\alpha}y^{\mathrm{T}}[\Lambda_b^*\theta_3 + I - I]w - \frac{1}{\alpha}y^{\mathrm{T}}\Phi_{\theta_3}Bu$$

Let us define the ideal values

$$\Lambda_a^* + \Lambda_b^* \theta_3^* \theta_1^* = 0, \quad \text{i.e.} \quad \theta_1^* = -\theta_3^{*-1} \Lambda_b^{*-1} \Lambda_a^* = -\Lambda_a^*,$$
$$\Lambda_p^{*\mathrm{T}} + \Lambda_b^* \theta_3^* \theta_2^* = 0, \quad \text{i.e.} \quad \theta_2^* = -\theta_3^{*-1} \Lambda_b^{*-1} \Lambda_p^{*\mathrm{T}} = -\Lambda_p^{*\mathrm{T}},$$
$$\Lambda_b^* \theta_3^* - I = 0, \quad \text{i.e.} \quad \theta_3^* = \Lambda_b^{*-1} \quad \text{or} \quad \left(\theta_3^{*-1} = \Lambda_b^*\right).$$

We also define the deviation variables as $\Phi_{\theta_1} \equiv \theta_1 - \theta_1^*$, $\Phi_{\theta_2} \equiv \theta_2 - \theta_2^*$ and $\Phi_{\theta_3} \equiv \theta_3^{*-1} - \theta_3^{-1}$. Replacing these definitions in the previous equation and regrouping the terms we have

$$\begin{aligned}\dot{V} \leq & \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left[W_0 + \frac{1}{2} y^{\mathrm{T}} y\right] + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) \\ & + \frac{1}{\alpha} y^{\mathrm{T}}\left[\Phi_{\theta_1} + \left(\theta_3^{*-1}\theta_3 - I\right)\theta_1 - \Phi_{\theta_1}\right] a(y,z) \\ & + \frac{1}{\alpha} y^{\mathrm{T}}\left[\Phi_{\theta_3} + \left(\theta_3^{*-1}\theta_3 - I\right)\theta_2 - \Phi_{\theta_2}\right] P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right) \\ & + \frac{1}{\alpha} y^{\mathrm{T}}\left(\theta_3^{*-1}\theta_3 - I\right) w + \frac{1}{\alpha} y^{\mathrm{T}} w - \frac{1}{\alpha} y^{\mathrm{T}} \Phi_{\theta_3} B u\end{aligned}$$

Since $\theta_3^{*-1}\theta_3 - I = \Phi_{\theta_3}\theta_3$, we can write

$$\begin{aligned}\dot{V} \leq & \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left[W_0 + \frac{1}{2} y^{\mathrm{T}} y\right] + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z} 1\right)^{\mathrm{T}} \Lambda_0^* f_0(z) + \frac{1}{\alpha} y^{\mathrm{T}} w \\ & + \frac{1}{\alpha} y^{\mathrm{T}} \Phi_{\theta_3}\left[\theta_3 \theta_1 a(y,z) + \theta_3 \theta_2 P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right) + \theta_3 w\right] - \frac{1}{\alpha} y^{\mathrm{T}} \Phi_{\theta_3} B u\end{aligned}$$

Recognizing the term Bu in the above equation, we have

$$\dot{V} \leq \frac{\partial}{\partial t}\left(\frac{1}{\alpha}\right)\left[W_0 + \frac{1}{2} y^{\mathrm{T}} y\right] + \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) + \frac{1}{\alpha} y^{\mathrm{T}} w$$

Choosing the same normalization factor

$$\alpha(t) = \sqrt{1 + \mathrm{Trace}\left[\Gamma_1^{-2}(t) + \Gamma_2^{-2}(t) + \Gamma_3^{-2}(t)\right]}$$

as in Section 3.1 and using its properties, we obtain

$$\dot{V} \leq \frac{1}{\alpha}\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} \Lambda_0^* f_0(z) + \frac{1}{\alpha} y^{\mathrm{T}} w$$

From Assumption 2, we obtain

$$\dot{V} \leq \frac{1}{\alpha} y^{\mathrm{T}} w.$$

Finally, since $0 < \alpha < 1$, we obtain

$$\dot{V} \leq y^{\mathrm{T}} w.$$

Thus, the closed-loop system is C^2-passive from w to y. □

Remark 4. Notice that Remarks 1-3 are also valid in this case, as in the previous case of Λ_b^* diagonal.

Remark 5. It is important to notice that the adaptive law for parameter $\theta_3(t)$ is implemented using the property of the inverse matrix as follows:

$$\frac{\mathrm{d}\Phi_{\theta_3}(t)}{\mathrm{d}t} \equiv \frac{\mathrm{d}}{\mathrm{d}t}\left(\theta_3^{*-1} - \theta_3^{-1}(t)\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(-\theta_3^{-1}(t)\right) = \theta_3^{-1}(t)\dot{\theta}_3(t)\theta_3^{-1}(t)$$
$$= -\frac{1}{\alpha}\Gamma_3^{-1}(t)y(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}(t),$$

therefore, we adjust the parameter $\theta_3(t)$ as

$$\dot{\theta}_3(t) = -\frac{1}{\alpha}\theta_3(t)\Gamma_3^{-1}(t)y(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}\theta_3(t).$$

Thus, no inversion of $\theta_3(t)$ is needed.

4. Simulation examples

In what follows, the methodology proposed in Section 3 is applied to the case of multivariable plants. First, the diagonal case is considered and then the general case. In both cases, a simple controller of the form $u(t) = -Ky(t)$ is applied where K is a positive definite matrix.

4.1 Plant with Λ_{b} diagonal

Let us consider the multivariable plant defined by equations

$$\begin{aligned} \dot{y} &= \Lambda_a^* a(y,z) + \Lambda_b^* B(y,z)u, \\ \dot{z} &= \Lambda_0^* f_0(z) + P^{\mathrm{T}}(y,z)\Lambda_p^* y \end{aligned} \tag{18}$$

with

$$\Lambda_a^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in \mathscr{R}^{2x2}, \quad \Lambda_b^* = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \in \mathscr{R}^{2x2},$$

$$\Lambda_0^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in \mathscr{R}^{2x2}, \quad \Lambda_p^* = \begin{bmatrix} -2 & -3 \\ -4 & -1 \end{bmatrix} \in \mathscr{R}^{2x2},$$

$$a(y,z) = \begin{bmatrix} -y_1(t)z_1(t) \\ -(y_2(t)z_2(t))^2 \end{bmatrix} \in \mathscr{R}^{2x1}, \quad P(y,z) = \begin{bmatrix} z_1(t)^2 y_1(t) & y_2(t) \\ y_2(t)*y_1(t) & z_2(t)^2 y_2(t) \end{bmatrix} \in \mathscr{R}^{2x2},$$

$$B(y,z) = \begin{bmatrix} z_1(t)^2 y_1(t) y_2(t) + (1 - z_2(t)^5) & (3\sin(y_1(t)) - 1)^3 \\ \sin(e^{-y_2(t)}) + z_2(t)^2 & z_2(t)^2 y_2(t) z_1(t) + 1 \end{bmatrix} \in \mathscr{R}^{2x2},$$

$$f_0(z) = \begin{bmatrix} -z_1(t) \\ -z_2(t) \end{bmatrix} \in \mathscr{R}^{2x1}$$

This is the same system used by Duarte *et al.* (2002) for the case of constant adaptive gains, and it was chosen for comparison purposes.

Following the methodology proposed in Section 3.1, where Λ_b^* is diagonal, the controller for this case has the form:

$$u(y, z, \theta) = B^{-1}(y, z)\left[\theta_1(t)a(y, z) + \theta_2(t)P(y, z)\frac{\partial W_0(z)}{\partial z} + \theta_3(t)w(t)\right],$$

$$\begin{aligned} \dot{\Phi}_{\theta_1}(t) &= \dot{\theta}_1(t) = -\frac{1}{\alpha(t)}\Gamma_1^{-1}(t)y(t)a^{\mathrm{T}}(y, z)\mathrm{sign}\left(\Lambda_b^*\right), \\ \dot{\Phi}_{\theta_2}(t) &= \dot{\theta}_2(t) = -\frac{1}{\alpha(t)}\Gamma_2^{-1}(t)y(t)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y, z)\mathrm{sign}\left(\Lambda_b^*\right), \\ \dot{\Phi}_{\theta_3}(t) &= \dot{\theta}_3(t) = -\frac{1}{\alpha(t)}\Gamma_3^{-1}(t)y(t)w^{\mathrm{T}}(t)\mathrm{sign}\left(\Lambda_b^*\right), \end{aligned} \tag{19}$$

where

$$\begin{aligned} \dot{\Gamma}_1^{-1}(t) &= -a(y, z)a^{\mathrm{T}}(y, z), \quad \Gamma_1^{-1}(t_0) > 0 \\ \dot{\Gamma}_2^{-1}(t) &= -P(y, z)\left(\frac{\partial W_0(z)}{\partial z}\right)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}} P^{\mathrm{T}}(y, z), \quad \Gamma_2^{-1}(t_0) > 0 \\ \dot{\Gamma}_3^{-1}(t) &= -w(t)w^{\mathrm{T}}(t), \quad \Gamma_3^{-1}(t_0) > 0 \end{aligned} \tag{20}$$

$$\alpha(t) = \sqrt{1 + \text{Trace}\left[\Gamma_1^{-2}(t) + \Gamma_2^{-2}(t) + \Gamma_3^{-2}(t)\right]} > 1,$$

with $\theta_1(t) \in \mathcal{R}^{2x2}$, $\theta_2(t) \in \mathcal{R}^{2x2}$, $\theta_3(t) \in \mathcal{R}^{2x2}$, $\Gamma_1(t) \in \mathcal{R}^{2x2}$, $\Gamma_2(t) \in \mathcal{R}^{2x2}$, $\Gamma_3(t) \in \mathcal{R}^{2x2}$. The ideal controller parameters are in this case

$$\theta_1^* = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.333 \end{bmatrix}, \quad \theta_2^* = \begin{bmatrix} -0.05 & 0.2 \\ 0.1 & -0.0667 \end{bmatrix},$$

$$\theta_3^* = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.333 \end{bmatrix}.$$

The plant initial conditions were chosen as $y_1(0) = -1$, $y_2(0) = 1$, $z_1(0) = -1.5$ and $z_2(0) = 0.5$. Controller parameters initial conditions were all set to unity and the best values for I.C. adaptive gains were sought by trial and error. These turn out to be

$$\Gamma_1^{-1}(t_0) = \Gamma_2^{-1}(t_0) = \Gamma_3^{-1}(t_0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Figure 1 shows the evolution of the plant outputs and plant states when feedback matrix gain $K = -I$. Figure 2 shows all the controller parameters as time elapses. In Figure 3, the evolution of time-varying gains is presented. It was observed that I.C. for $\Gamma_i(t_0)$ for $i = 1, 2, 3$, can improve the transient behavior of the system, conversely to the case where constant adaptive gains are used.

Later, under the same conditions of previous simulations, the feedback gain was increased to $K = -10I$, observing an important improvement in the system behavior. This is shown in Figures 4-6, where the control range is reduced and speed of converge is diminished.

Note that the methodology presented in Section 3.2 where the sing of Λ_b^* is not known, can also be used in this case. If so, the controller will have the form shown in equations (15) and (16).

4.2 Plant with a general Λ_b

Let us consider the same plant as in Section 4.1 but with a non-diagonal matrix Λ_b^*. Let this matrix be defined as

$$\Lambda_b^* = \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix} \in \mathcal{R}^{2x2},$$

From Section 3.2, the controller becomes:

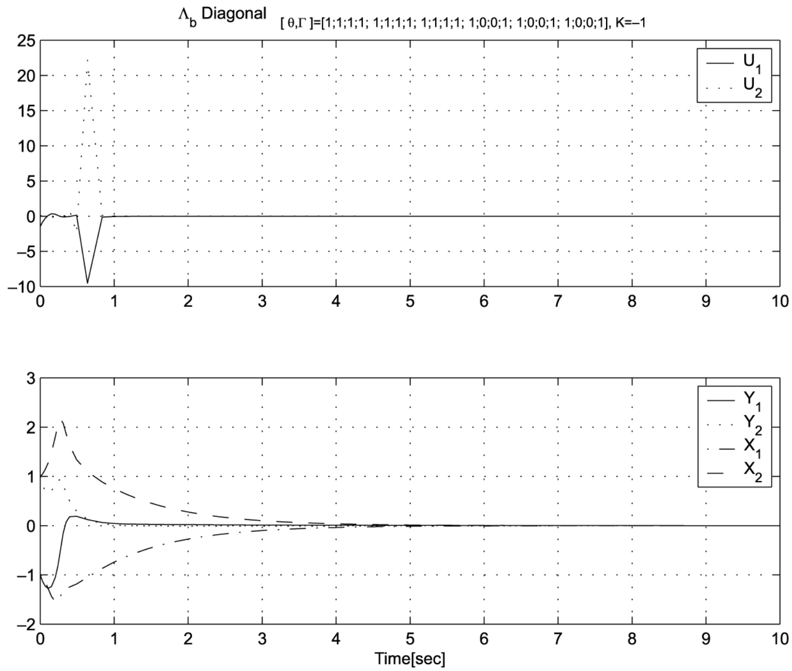

Figure 1.
States and outputs for the diagonal case (non-zero initial conditions and unity feedback)

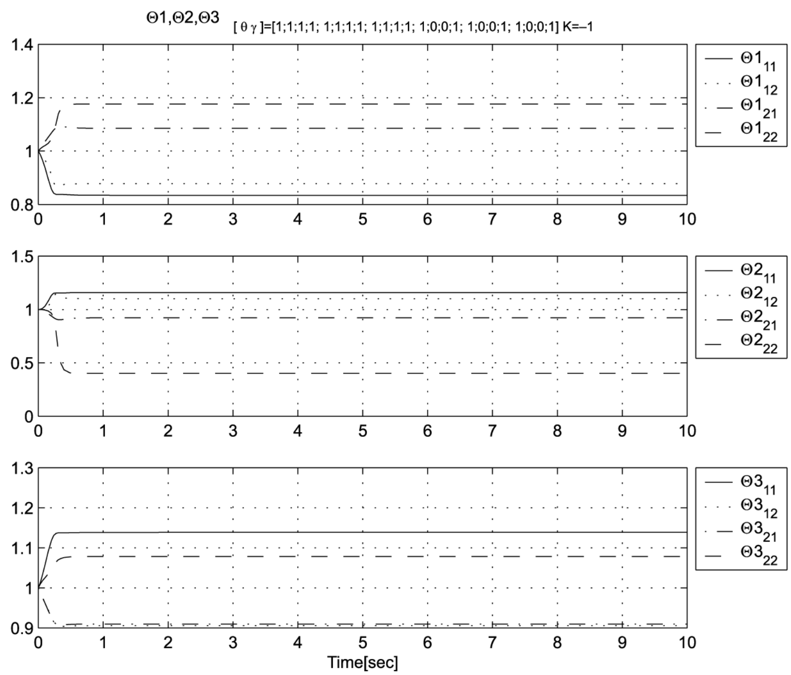

Figure 2.
Controller parameters for the diagonal case (non-zero initial conditions and unity feedback)

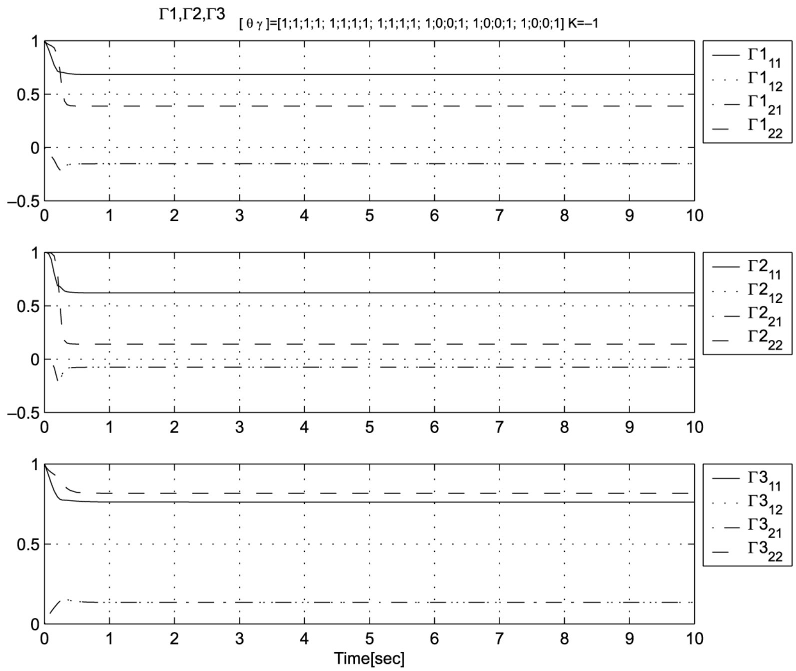

Figure 3.
Evolution of the time-varying gains for the diagonal case (non-zero initial conditions and unity feedback)

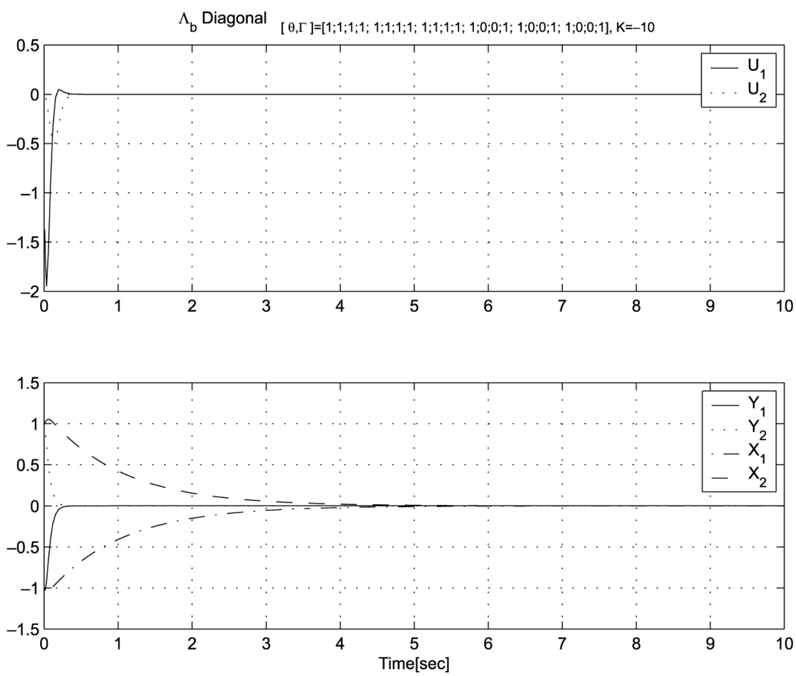

Figure 4.
States and outputs for the diagonal case (non-zero initial conditions and feedback gain $K = -10I$)

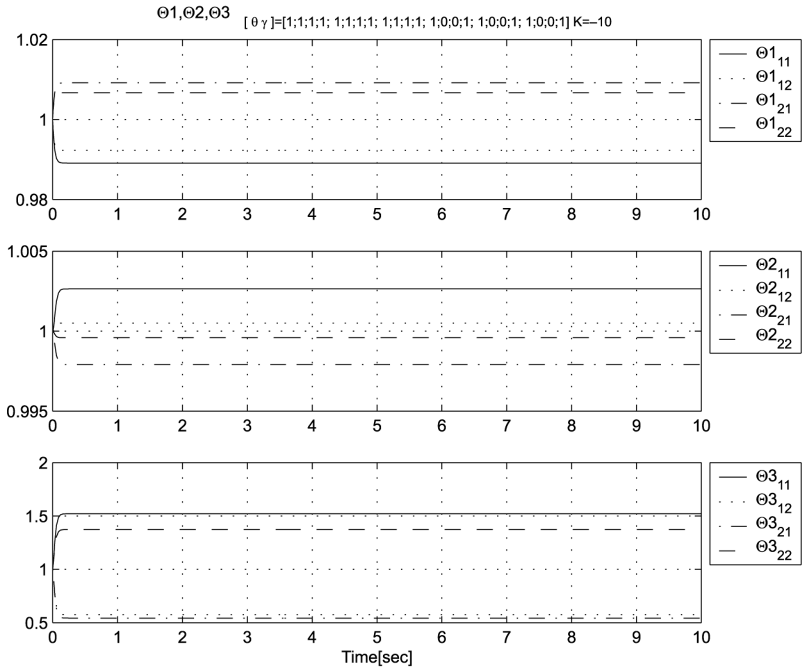

Figure 5.
Controller parameters for the diagonal case (non-zero initial conditions and feedback gain $K = -10I$)

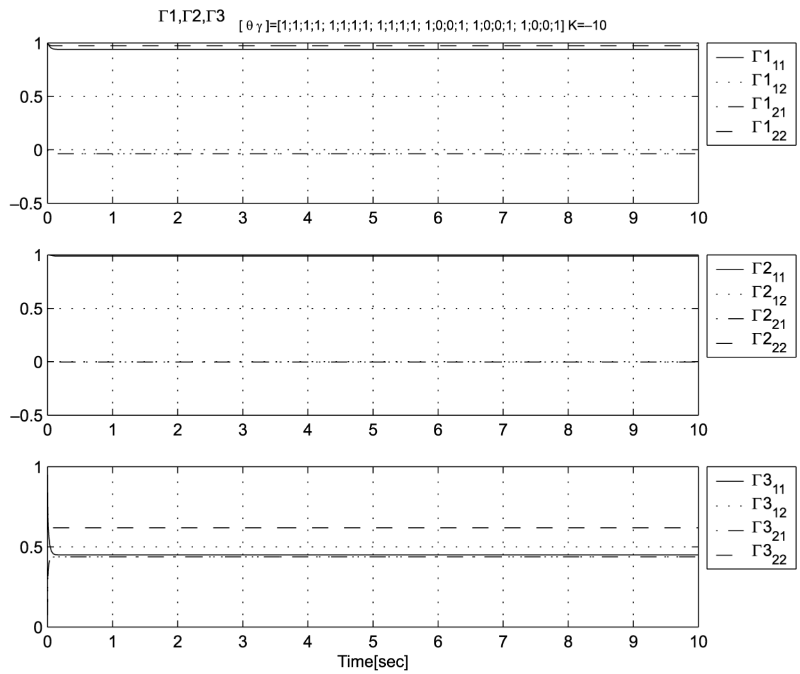

Figure 6.
Evolution of the time-varying gains for the diagonal case (non-zero initial conditions and feedback gain $K = -10I$)

$$u(y,z,\theta) = B^{-1}(y,z)\left[\theta_3(t)\theta_1(t)a(y,z) + \theta_3(t)\theta_2(t)P(y,z)\frac{\partial W_0(z)}{\partial z} + \theta_3(t)w\right],$$

$$\dot{\Phi}_{\theta_1}(t) = \dot{\theta}_1(t) = -\frac{1}{\alpha(t)}\Gamma_1^{-1}y(t)a^{\mathrm{T}}(y,z),$$

$$\dot{\Phi}_{\theta_2}(t) = \dot{\theta}_2(t) = -\frac{1}{\alpha(t)}\Gamma_2^{-1}y(t)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}P^{\mathrm{T}}(y,z), \tag{21}$$

$$\dot{\Phi}_{\theta_3}(t) = -\frac{1}{\alpha(t)}\Gamma_3^{-1}y(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}, \quad \text{or}$$

$$\dot{\theta}_3(t) = -\frac{1}{\alpha(t)}\Gamma_3^{-1}\theta_3(t)y(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}\theta_3(t)$$

$$\dot{\Gamma}_1^{-1}(t) = -a(y,z)a^{\mathrm{T}}(y,z), \quad \Gamma_1^{-1}(t_0) > 0$$

$$\dot{\Gamma}_2^{-1}(t) = -P(y,z)\left(\frac{\partial W_0(z)}{\partial z}\right)\left(\frac{\partial W_0(z)}{\partial z}\right)^{\mathrm{T}}P^{\mathrm{T}}(y,z), \quad \Gamma_2^{-1}(t_0) > 0 \tag{22}$$

$$\dot{\Gamma}_3^{-1}(t) = -w(t)w^{\mathrm{T}}(t), \quad \Gamma_3^{-1}(t_0) > 0$$

where matrices $\theta_1(t) \in \mathscr{R}^{2x2}$, $\theta_2(t) \in \mathscr{R}^{2x2}$, $\theta_3(t) \in \mathscr{R}^{2x2}$, $\Gamma_1(t) \in \mathscr{R}^{2x2}$, $\Gamma_2(t) \in \mathscr{R}^{2x2}$, $\Gamma_3(t) \in \mathscr{R}^{2x2}$. The normalization factor

$$\alpha(t) = \sqrt{1 + \mathrm{Trace}\left[\Gamma_1^{-2}(t) + \Gamma_2^{-2}(t) + \Gamma_3^{-2}(t)\right]} > 1.$$

The ideal values becomes in this case

$$\theta_1^* = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \theta_2^* = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}, \quad \theta_3^* = \begin{bmatrix} 0.2727 & -0.4545 \\ 0.0909 & 0.1818 \end{bmatrix}.$$

A similar study to the one conducted in Section 4.1 was performed here for the general case.

Figures 7 and 9 show the behavior of the controlled system using the proposed design methodology for the general case, for plant initial conditions $y_1(0) = -1$, $y_2(0) = 1$, $z_1(0) = -1.5$ and $z_2(0) = 0.5$. Controller parameters initial conditions were all set to unity and the best values for I.C. adaptive gains were sought by trial and error. These turn out to be

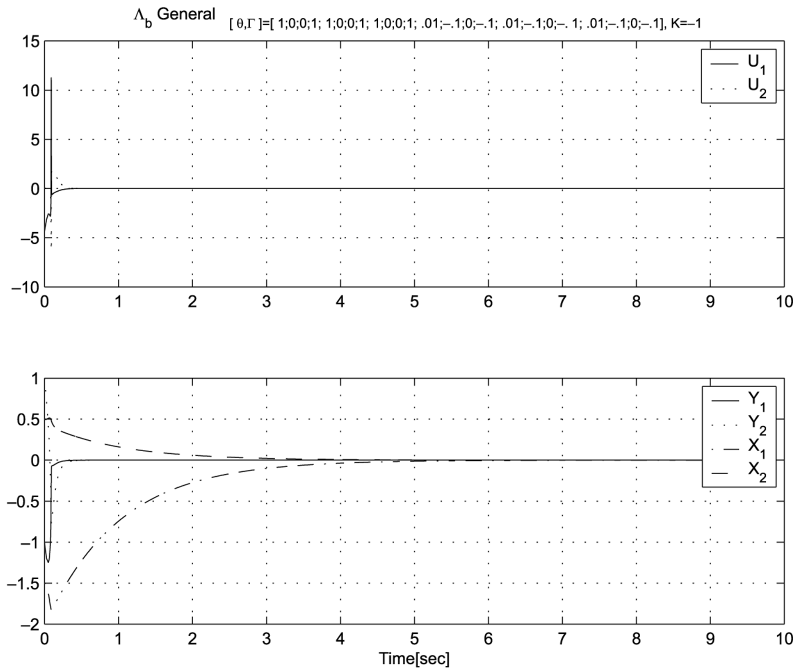

Figure 7.
States and outputs for the general case (non-zero initial conditions and unity feedback)

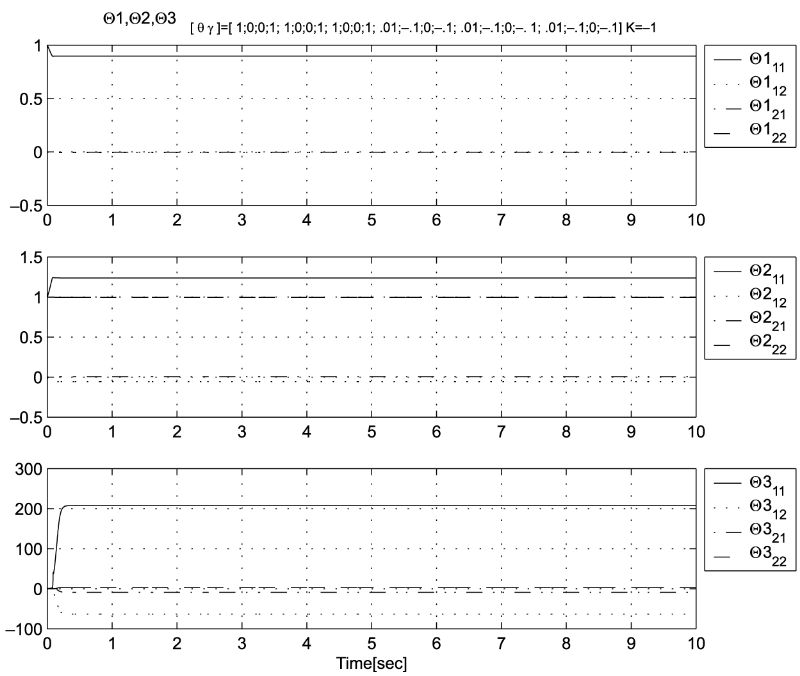

Figure 8.
Controller parameters for the general case (non-zero initial conditions and unity feedback)

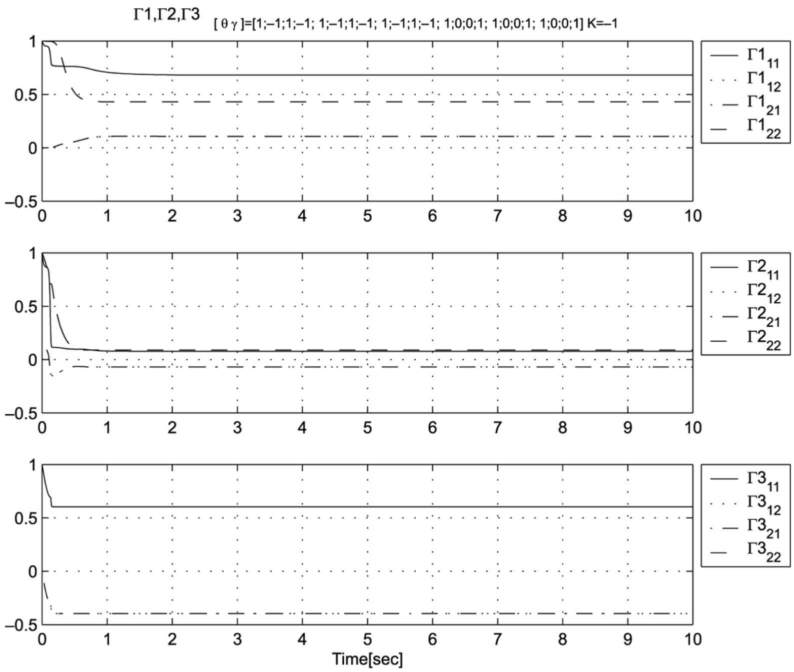

Figure 9.
Evolution of the time-varying gains for the general case (non-zero initial conditions and unity feedback)

$$\Gamma_1^{-1}(t_0) = \Gamma_2^{-1}(t_0) = \Gamma_3^{-1}(t_0) = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}.$$

The corresponding results are presented in Figures 7-9.

Again, the influence of the feedback matrix gain was explored, increasing it to $K = 10 - I$. The results are shown in Figures 10-12, where an important improvement in the system behavior is observed since the control range is reduced. No important variation in speed convergence is reached.

In all the simulations, states as well as outputs go to zero as t goes to infinity. Controller parameters tend to some constant values.

5. Conclusions

In this paper, we introduce time-varying gains in an adaptive controller that renders a MIMO non-linear system, with linear explicit parametric dependence and expressed in the normal form, locally feedback equivalent to a passive system. Time variations of adaptive gains are designed in such a way that local stability is preserved for the cases of matrix Λ_b^* diagonal and of general form. This feature allows the designer to improve the transient behavior of the resulting adaptive system, conversely to the case where constant adaptive gains are used. The methodology proposed to design the controller makes use

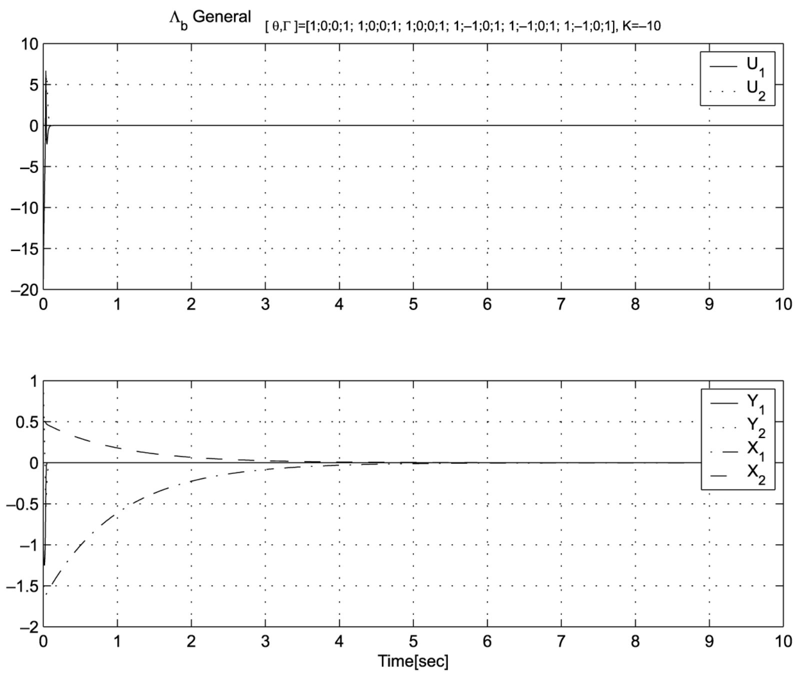

Figure 10.
States and outputs for the general case (non-zero initial conditions and feedback gain $K = -10I$)

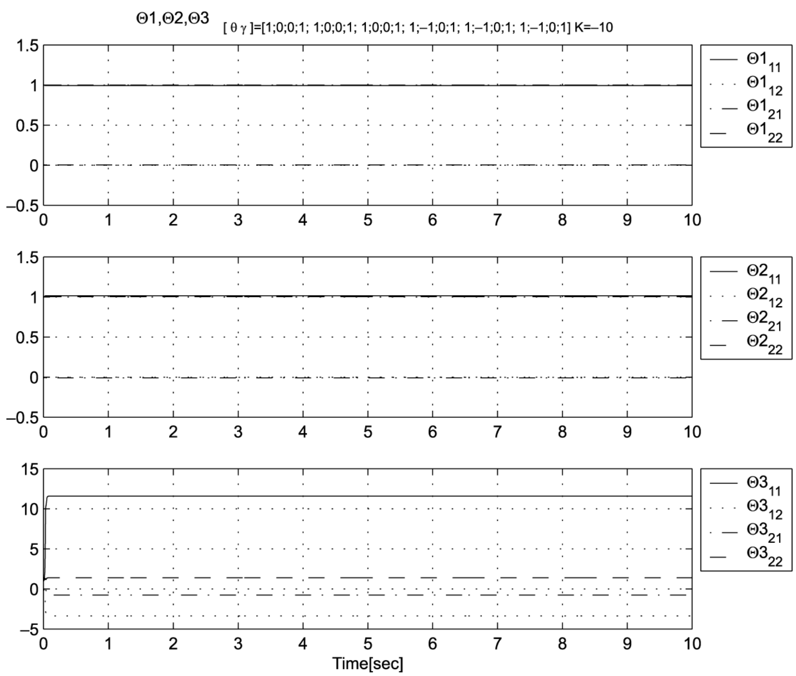

Figure 11.
Controller parameters for the general case (non-zero initial conditions and feedback gain $K = -10I$)

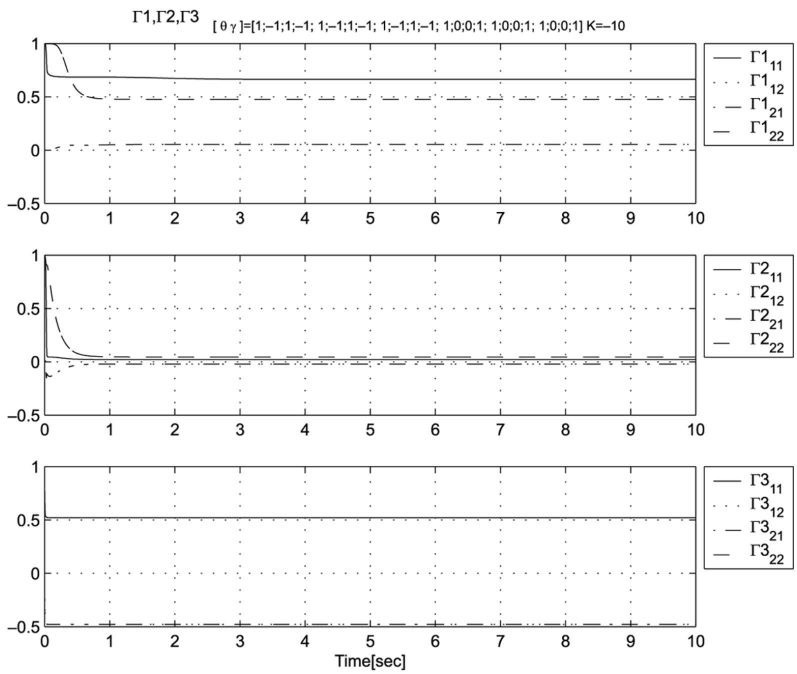

Figure 12. Evolution of the time-varying gains for the general case (non-zero initial conditions and feedback gain $K = -10I$)

of passivity and geometric concepts, and it is an extension of the results proposed by the authors in the work of Duarte *et al.* (2002). The proposed controller was also applied and evaluated in models of MIMO dynamical systems. The controller proved to have a good performance, when compared with other approaches reported in the literature.

Note

1. A system (7) is locally zero-state detectable if there exists a neighborhood X of 0 such that, for all $x \in X$, $h(\Phi(t, x, 0)) = 0$ for all $t \geq 0 \Rightarrow \lim_{t\to\infty}\Phi(t, x, 0) = 0$. It is said to be locally zero-state observable if there exists a neighborhood X of 0 such that, for all $x \in X$, $h(\Phi(t, x, 0) = 0$ for all $t \geq 0 \Rightarrow x = 0$.

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