APPLICATION OF ROBUST ADAPTIVE CONTROL USING COMBINED DIRECT AND INDIRECT METHODS

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SUMMARY

A new approach, combining direct and indirect control methods, recently proposed by the authors is used to adaptively control plants with unknown parameters. The plant parameters are known to lie in a specified compact set in parameter space and the output of the plant has to follow a desired output in the presence of a piecewise-constant input disturbance. Simulation results indicate that the combined method performs better than either the direct or the indirect method.

1. INTRODUCTION

Since the very beginning, direct and indirect control methods have been recognized as two distinct approaches to adaptive control of plants with unknown parameters. Recently\(^1,2\) it was suggested by the authors that the two methods can be combined judiciously to improve the performance of the adaptive system. The motivation for such a combined approach is based on the intuitive assumption that improved control may be possible if more information regarding the plant is available. The main contribution of References 1 and 2 is that the adaptive laws for identification and control parameters can be coupled in such a manner that the overall system is globally uniformly stable.

In this paper we apply the above technique to the problem of controlling a first- and second-order plant with unknown parameters (example problems) in the presence of an input disturbance. While the aim of the combined approach is to achieve the advantages of both direct and indirect approaches in different adaptive situations, the degree of improvement achieved over the two methods naturally depends upon the nature of the specific problem considered. Hence, before the combined approach is attempted, it is essential to determine how the direct and indirect methods fare separately.

In Section 2 we describe the adaptive laws resulting from the new approach, which assure global stability. In particular, the manner in which the direct and indirect approaches are combined for plants of first and second order is explained in detail. Section 3 describes the procedure for implementing the new adaptive controller for the two cases described in Section 2. Simulation studies using the combined approach are presented in Section 4. The performance resulting from such a controller is found to be better than those obtained using exclusively either direct or indirect control. Owing to space limitations, only the simulations using the new approach are included here.

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2. THE COMBINED APPROACH

In References 1 and 2 the combined direct–indirect method has been derived for the adaptive control of a general $n$th-order plant with relative degree $n^* \geq 1$. We shall concentrate in this section on first- and second-order systems and derive the corresponding adaptive laws. For further details regarding the method, the reader is referred to References 1 and 2.

2.1. First-order plant (ideal case)

Let a plant be described by the first-order differential equation

$$\dot{r}_p(t) = -a_p r_p(t) + k_p u(t)$$

where $a_p$ and $k_p$ are unknown constants. Let a reference model be described by the differential equation

$$\dot{r}_m(t) = -a_m r_m(t) + k_m c(t)$$

where the reference input $c(\cdot) : \mathbb{R}^+ \to \mathbb{R}$ is a piecewise-continuous bounded function of time. The aim is to generate a control input $u(\cdot)$ without using differentiators so that $\lim_{t \to \infty} [r_p(t) - r_m(t)] = \lim_{t \to \infty} e_c(t) = 0$. The simplest controller which can achieve this has the form

$$u(t) = \theta(t) r_p(t) + k_c(t) c(t)$$

where the control parameters $\theta(t)$ and $k_c(t)$ are adjustable parameters.

If indirect control is used, the plant parameter estimates have to be known at every instant. To obtain such estimates, the following identification model is used:

$$\dot{\hat{r}}_p(t) = -a_i e_i(t) - \hat{a}_p(t) r_p(t) + \hat{k}_p(t) u(t) \quad a_i > 0$$

We shall refer to $r_p(t) - r_m(t) = e_c(t)$ as the control (tracking) error and $\dot{\hat{r}}_p(t) - r_p(t) = e_i(t)$ as the identification (output) error.

The aim of adaptive control is then to use all the information available at any instant of time to adjust the parameters $\theta(t)$ and $k_c(t)$ of the controller. In the following, we indicate how this is accomplished using either direct control or indirect control or a combination of the two.

(a) Direct control. Figure 1(a) shows the plant together with the controller and the reference model, whose output is $r_m(t)$. If only direct control is used, the adaptive laws for adjusting the
parameters are given by
\[
\begin{align*}
\dot{\theta}(t) &= -\text{sgn}(k_p) \gamma_1 e_c(t) r_p(t) \\
\dot{k}_c(t) &= -\text{sgn}(k_p) \gamma_2 e_c(t) c(t)
\end{align*}
\]
where \(\gamma_1\) and \(\gamma_2\) are positive constants, referred to as adaptive gains.

The choice of the values of \(\gamma_i, i = 1, 2\), are at the discretion of the designer. It has been shown\(^3\) that the above adaptive laws assure global stability of the adaptive system and that \(\lim_{t \to \infty} e_c(t) = 0\).

(b) Indirect control. In Figure 1(b) an identifier is used to estimate the parameters of the plant. The adaptive laws for adjusting the parameter estimates are normally given by

\[
\begin{align*}
\dot{\hat{\theta}}_p(t) &= \alpha_{i1} e_i(t) r_p(t) \\
\dot{\hat{k}}_c(t) &= -\alpha_{i2} e_i(t) u(t)
\end{align*}
\] 

(1)

When indirect control is used, no explicit reference model is needed but \(\theta(t)\) and \(k_c(t)\) are adjusted in such a manner that the transfer function of the plant together with the controller (for constant values of the parameters) approaches that of the reference model. This is accomplished by defining closed-loop estimation errors as follows:

\[
\begin{align*}
-a_p + k_p \theta^* + a_m &= 0 \\
k_p k_c^* - k_m &= 0
\end{align*}
\]

(2)

The control parameters are then adjusted to decrease the errors \(\epsilon_\theta(t)\) and \(\epsilon_{k_c}(t)\) as shown below:

\[
\begin{align*}
\dot{\hat{\theta}}_p(t) &= -\text{sgn}(k_p) \beta_{c1} \epsilon_{\hat{\theta}}(t) \\
\dot{\hat{k}}_c(t) &= -\text{sgn}(k_p) \beta_{c2} \epsilon_{k_c}(t)
\end{align*}
\]

Figure 1(b). Indirect MRAC scheme for first-order plants
This in turn entails the following change in the identification adaptive laws given by equations (1):

\[ \dot{a}_p(t) = \alpha_{11} e_i(t) r_p(t) + \beta_{11} e_\theta(t) \]
\[ \dot{k}_p(t) = -\alpha_{12} e_i(t) u(t) - \beta_{12} \left[ \theta(t) e_\theta(t) + k_c(t) e_{k_c}(t) \right] \]

In Reference 4 it is shown that this results in \( e_i(t) \to 0 \) as \( t \to \infty \) as well as \( \lim_{t \to \infty} (e_\theta(t), e_{k_c}(t)) = 0 \). This in turn can be used to show that \( \lim_{t \to \infty} e_c(t) = 0 \) or that the plant output follows the output of the reference model asymptotically.

Comment 1. The indirect method suggested here involves the dynamic adjustment of control parameters and avoids the use of algebraic equations as is commonly done.

Comment 2. In the indirect approach it is seen that the identification error \( e_i(t) \) and the closed-loop estimation errors \( e_\theta(t) \) and \( e_{k_c}(t) \) feature in the adaptive laws but \( e_c(t) \) does not.

(c) Direct–indirect control. In this case the control error \( e_c(t) \) as well as the identification error \( e_i(t) \) play a part in the adjustment of the control parameters \( \theta(t) \) and \( k_c(t) \) (see Comment 4). Figure 1(c) shows the overall system when the combined approach is used.

The output \( \hat{r}_p(t) \) of the identification model, the output \( r_p(t) \) of the plant and the output \( r_m(t) \) of the reference model are generated and yield the errors \( e_c(t) \) and \( e_i(t) \). Once again, if the closed-loop estimation errors \( e_\theta(t) \) and \( e_{k_c}(t) \) are generated as shown in equations (2), the

![Diagram of the Combined MRAC approach for a first-order plant](image-url)
adaptive laws for adjusting $\hat{\alpha}(t)$, $\hat{k}(t)$, $\theta(t)$ and $k_c(t)$ can be chosen as

\[
\begin{align*}
\dot{\hat{\alpha}}(t) &= \alpha_{11} e(t) r_p(t) + \beta_{11} e_{\theta}(t) \\
\dot{\hat{k}}(t) &= -\alpha_{12} e(t) u(t) - \beta_{12} [\theta(t) e_{\theta}(t) + k_c(t) e_k(t)] \\
\dot{\theta}(t) &= -\text{sgn}(k_p) [\alpha_{21} e_c(t) r_p(t) + \beta_{21} e_{\theta}(t)] \\
\dot{k}_c(t) &= -\text{sgn}(k_p) [\alpha_{22} e_c(t) e_c(t) + \beta_{22} e_k(t)]
\end{align*}
\]  

(3)

where $\alpha_{ij}$, $\beta_{ij}$, $\alpha_{cj}$ and $\beta_{cj}$ for $j = 1, 2$ are constant positive adaptive gains.

In Reference 1 this combined method is shown to result in a quadratic Lyapunov function in all the error variables, whose time derivative along any trajectory is negative semidefinite. Hence the overall system is globally uniformly stable and $\lim_{t \to \infty} \{ e_c(t), e(t), e_{\theta}(t), e_k(t) \} = 0$.

Comment 3. As in the direct and indirect methods, the adaptive laws in the combined method also have adaptive gains which can be adjusted to improve performance.

Figure 2(a). Combined direct–indirect MRAC of a general plant
Comment 4. In the combined approach as given above, $e_\varepsilon(t)$ and $e_\psi(t)$ as well as $e_\psi(t)$ and $e_\psi(t)$ feature in the adaptive laws. The use of all four signals is not necessary but merely sufficient for stable adaptation. In Reference 1 it is shown that stability can also be obtained in some cases where only some of these error signals are used in the adaptive laws.

2.2. Second-order plant

The adaptive control of a second-order plant using a combined approach proceeds along the same lines as for a first-order plant. The block diagram representation of the adaptive system using the combined method for a general plant of relative degree $n^* \geq 2$ is shown in Figure 2(a).

Comment 5. Since $n^* = 2$ is a special case, stable adaptive laws can be determined without using augmented errors. This method, however, cannot be directly extended to systems with $n^* \geq 3$. The controller in this particular case is of the form shown in Figure 2(b).

3. CONTROLLER DESIGN AND IMPLEMENTATION FOR THE EXAMPLE PROBLEMS

The basic adaptive approach has been described in Section 2 when no external disturbances are present. The implementation of the controller therefore proceeds along the same lines as in Section 2.
3.1. First-order plant

(i) Set up a reference model with input \( c(t) \).
(ii) Set up the identification model as described in Section 2.
(iii) Set up a controller with two adjustable parameters.
(iv) Simulate direct control with nominal values for the plant and adjust the adaptive gains so that the speed of response is sufficiently high.
(v) Simulate the response of the system with the given disturbance but fine-tune the adaptive gains.
(vi) Check whether these adaptive gains are also adequate for other values of the plant parameters.
(vii) Repeat the procedure for indirect control alone. A comparison of the best that the two methods can offer separately given an indication as to how they should be combined in the new approach.
(viii) Simulate the adaptive system using the combined approach.

3.2. Second-order plant

While the complexity of the controller increases with the order of the system, the general philosophy in designing the controller is the same in all cases. However, owing to the number of signals that are used in generating the control input, a considerable amount of trial and error in the choice of the adaptive gains is inevitable.

In view of the greater complexity of the system in this case, as well as the fact that the various equations describing the reference model, identifier and controller was not given in the previous section, we describe the procedure in greater detail here. Only the combined approach is described since it subsumes the other two methods.

**Comment 6.** It is well known that certain unknown disturbances can be cancelled exactly using adaptation by overparametrizing the adaptive controller. This was also carried out in the simulation studies and, whenever relevant, further details needed to understand the design procedure are provided.

We now analyse the combined direct–indirect adaptive approach applied to the case of a second-order plant with relative degree \( n^* = 2 \), i.e. \( W_p(s) = k_p Z_p(s)/R_p(s) \) and \( W_m(s) = k_m Z_m(s)/R_m(s) \) with \( Z_p(s) = 1, Z_m(s) = 1, R_p(s) = s^2 + a_2 s + a_1 \) and \( R_m(s) = s^2 + a_2^m s + a_1^m \).

The plant to be controlled, the corresponding reference model chosen and the structure of the controller are shown in Figures 2(a) and 2(b).

The vectors of the plant and model parameters are denoted as \( a = [a_1, a_2]^{\top} \), \( a_m = [a_1^m, a_2^m]^{\top} \) respectively and the plant estimates as \( \hat{a}(t) = [\hat{a}_1(t), \hat{a}_2(t)]^{\top} \). In this particular case the vectors \( b = [1, 0]^{\top} \) and \( b_m = [1, 0]^{\top} \) are completely known. The control parameter vector is given by \( \theta(t) = [k_c(t), \theta_1(t), \theta_0(t), \theta_2(t)]^{\top} \) and the control sensitivity vector by \( \omega(t) = [c(t), \omega_1(t), r_p(t), \omega_2(t)]^{\top} \), the filtered version of which is denoted as \( \hat{\omega}(t) = [\hat{c}(t), \hat{\omega}_1(t), \hat{r}_p(t), \hat{\omega}_2(t)]^{\top} \).

The desired values \( \theta^* \) of the control parameter vector \( \theta(t) \), as shown in Reference 4, satisfy relationships that allow us to write the closed-loop estimation errors as follows:

\begin{align*}
\epsilon_{\theta_1}(t) & = \hat{\theta}_0(t) + \theta_1(t) \\
\epsilon_{\theta_2}(t) & = k_m \hat{\theta}_1(t) + \hat{k}_p(t) \theta_0(t) \\
\epsilon_{\theta_0}(t) & = \hat{k}_p(t) \theta_2(t) + k_m \hat{\theta}_0(t) \\
\epsilon_{k_c}(t) & = \hat{k}_p(t) k_c(t) - k_m
\end{align*}

(4)
where \( \hat{\beta}_0(t) \), \( \hat{\alpha}_0(t) \) and \( \hat{\alpha}_1(t) \) are the estimates of the parameters \( \beta_0 \), \( \alpha_0 \) and \( \alpha_1 \) defined as

\[
\beta_0 = \alpha_2^m - a_2
\]

\[
\alpha_1 = \frac{1}{k_m} \left[ a_1^m + \lambda a_2^m - a_1 - a_2(\lambda + \beta_0) \right]
\]

\[
\alpha_0 = \frac{1}{k_m} \left[ \lambda a_1^m - \lambda k_m \alpha_1 - a_1(\lambda + \beta_0) \right]
\]

If no explicit identification model is used, the adaptive laws for the combined approach are

\[
\dot{k}_c(t) = -\alpha_c \text{ sgn}(k_p) e_c(t) c(t) / k_m - \beta_c \text{ sgn}(k_p) e_i(t)
\]

\[
\dot{\theta}_1(t) = -\alpha_c \text{ sgn}(k_p) e_c(t) \tilde{c}(t) / k_m - \beta_c e_i(t)
\]

\[
\dot{\theta}_0(t) = -\alpha_c \text{ sgn}(k_p) e_c(t) \tilde{r}_p(t) / k_p - \beta_c \text{ sgn}(k_p) e_i(t)
\]

\[
\dot{\theta}_2(t) = -\alpha_c \text{ sgn}(k_p) e_c(t) \tilde{r}_p(t) / k_m - \beta_c \text{ sgn}(k_p) e_i(t)
\]

\[
\dot{k}_p(t) = -\beta_i [\theta_0(t) e_{ii}(t) + \theta_2(t) e_{ii}(t) + k_c(t) e_i(t)]
\]

\[
\dot{k}_i(t) = -\beta_i e_i(t)
\]

\[
\dot{\alpha}_1(t) = -\beta_i k_m e_{ii}(t)
\]

\[
\dot{\alpha}_0(t) = -\beta_i k_m e_{ii}(t)
\]

where \( \alpha_c \), \( \beta_c \) and \( \beta_i \) are constant positive adaptive gains.

4. SIMULATION RESULTS

4.1. Example problem number 1

The performance of a first-order plant together with an adaptive controller using the combined approach is shown in Figure 3. Since there are two identification parameters and two control parameters to be adjusted (i.e. eight adaptive gains), there is considerable freedom in the design. However, since the ultimate objective is to design a single adaptive controller which will be satisfactory for all values of the plant parameters, many simulations have to be performed and adaptive gains chosen by a trial and error procedure. The final adaptive laws chosen were those given by equation (3) with \( \alpha_{c1} = \alpha_{c2}, \beta_{c1} = \beta_{c2}, \alpha_{i1} = \alpha_{i2}, \beta_{i1} = \beta_{i2} \), all fixed at 40, and \( a_i = a_m = k_m = 1 \).

The plant has a transfer function \( W_p(s) = k_p / (s + a_p) \) where \( k_p \) and \( a_p \) lie in the intervals \( 0.5 \leq k_p \leq 3 \) and \( -1 \leq a_p \leq 3 \). The reference input \( c(t) \) (solid line) and the disturbance \( d(t) \) (dashed line) are defined as shown in Figure 3. The response of the plant as well as the control input when \( k_p = 1 \) (the nominal value) and \( a_p \) takes on three values, which include the extreme values 3 and -1 as well as the nominal value 1, are shown in Figure 3(b). Similar responses for \( k_p = 0.5 \) and \( k_p = 3 \) are shown in Figures 3(a) and 3(c) respectively.

The adaptive controller is seen to provide tight control of the output for all nine cases. The control input is seen to be minimum in all cases when \( a_p = -1 \) (dashed line) and maximum when \( a_p = 3 \) (solid line) over most of the time interval.

The fact that precise control of the output is possible with a large disturbance \( d(t) \) and parameter uncertainties shows the advantage of using adaptive control over conventional control.
4.2. Example problem number 2

In this case the plant transfer function \( W_p(s) \) is given by \( W_p(s) = k_p/(s^2 + a_2 s + a_1) \) and \( k_p, a_2 \) and \( a_1 \) lie in the intervals \( 0.5 \leq k_p \leq 3, \ -0.6 \leq a_2 \leq 3.4 \) and \( -2 \leq a_1 \leq 4 \).

For the nominal value \( k_p = 1 \) and for the extreme values \( k_p = 0.5 \) and \( k_p = 3 \), the output of the plant for five different cases as well as the control input computed for the nominal values of the parameters \( a_2 = 1.4 \) and \( a_1 = 1 \) are shown in Figures 4(b), 4(a) and 4(c) respectively.

![Figure 3. Simulation results for first-order plant](image-url)
These five combinations of the parameters $a_2$ and $a_1$ are

(i) $a_1 = 4$, $a_2 = 1.4$
(ii) $a_1 = -2$, $a_2 = 1.4$
(iii) $a_1 = 1$, $a_2 = 1.4$
(iv) $a_1 = 1$, $a_2 = 3.4$
(v) $a_1 = 1$, $a_2 = -0.6$

Figure 4. Simulation results for second-order plant
Owing to the large number of possible combinations of the three parameters which can occur, the choice of suitable adaptive gains for them is no longer simple. Different values of adaptive gains are found to be satisfactory for different values of the plant parameters. The values $\alpha_c = \beta_c = \beta_l = 200$, $\lambda = 2$ and $\alpha = 0.7$ used in the simulations are those which yield satisfactory response for the maximum range of plant parameter values. No single value of adaptive gain could be determined for which all the outputs were close to the output of the reference model during the entire observed interval. Oscillatory response generally occurs when the plant is unstable and the corresponding control input is also found to be large and oscillatory in nature.

5. SUMMARY AND CONCLUSIONS

In this paper a new method recently proposed by the authors is used to adaptively control a linear time-invariant plant with unknown parameters. The method combines the well known direct and indirect control approaches for the adjustment of the control parameters. The key to the success of the method lies in the fact that the parametrizations of the plant and the controller are such that the true parameters and the desired control parameters are linearly related. If the instantaneous values of the control parameters and the estimates of the plant are known, closed-loop estimation errors which can be measured are linearly related to the parameter errors. By judiciously incorporating the closed-loop estimation errors in both the identification and control laws, the overall system can be made globally uniformly stable. Since the method is not currently well known, all the relevant details for first-order and second-order systems are given in the paper.

The simulation results presented indicate that the combined method performs satisfactorily even in the presence of large external disturbances. For first-order systems the amplitude of the input is less than 7 in all cases and the tracking error is found to be small. For second-order systems the input is found to be quite oscillatory and has a large amplitude in the interval $2 \leq t \leq 8$. Surprisingly the amplitude is relatively very small for $t > 8$. In all cases the error at the terminal time $t = 20$ is found to be close to zero.

The advantage of the combined method lies in the fact that it uses both the control and identification errors as well as plant parameter estimates while adjusting the control parameters. Its full potential is realized when the adaptation is observed over a long period of time. In the present context, where only the interval $[0, 20]$ is of interest, the principal advantage of using the combined approach is that a less oscillatory and more accurate response than direct control is achieved.

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