Strong nonlocal interaction stabilizes cavity solitons with a varying size plateau

Cristian Fernandez-Oto[a], Mustapha Tlidi[a], Daniel Escaff[b], Marcel Clerc[c] and Pascal Kockaert[d]

[a]Faculté des Sciences, Université Libre de Bruxelles (U.L.B.), C.P. 231, Campus Plaine, B-1050 Bruxelles, Belgium;
[b]Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes, Av. San Carlos de Apoquindo 2200, Santiago, Chile;
[c]Departamento de Física, Universidad de Chile, Blanco Encalada 2008, Santiago, Chile;
[d]OPERA-photonique, Université Libre de Bruxelles (U.L.B.), CP 194/5, 50, Av. F.D. Roosevelt, B-1050 Bruxelles, Belgium.

ABSTRACT

Cavity solitons are localized light peaks in the transverse section of nonlinear resonators. These structures are usually formed under a coexistence condition between a homogeneous background of radiation and a self-organized patterns resulting from a Turing type of instabilities. In this issue, most of studies have been realized ignoring the nonlocal effects. Non-local effects can play an important role in the formation of cavity solitons in optics, population dynamics and plant ecology. Depending on the choice of the nonlocal interaction function, the nonlocal coupling can be strong or weak. When the nonlocal coupling is strong, the interaction between fronts is controlled by the whole non-local interaction function. Recently it has shown that this type of nonlocal coupling strongly affects the dynamics of fronts connecting two homogeneous steady states and leads to the stabilization of cavity solitons with a varying size plateau. Here, we consider a ring passive cavity filled with a Kerr medium like a liquid crystal or left-handed materials and driven by a coherent injected beam. We show that cavity solitons resulting for strong front interaction are stable in one and two-dimensional setting out of any type of Turing instability. Their spatial profile is characterized by a varying size plateau. Our results can apply to large class of spatially extended systems with strong nonlocal coupling.

Keywords: Cavity solitons, localized structures, nonlinear optics, nonlocal coupling, optical bistability, metamaterials, nonlinear dynamics

1. INTRODUCTION

The emergence of spatial-temporal dissipative structures far from equilibrium is a well-documented issue since the seminal works of Turing[1] and Prigogine and Lefever.[2] Dissipative structures have been observed in numerous nonlinear optical systems.[3] Localized structures (LSs) of light in nonlinear laser systems are called cavity solitons belong to this class of structures. They appear on a homogeneous background and require multistable regime, i.e. coexistence of a homogeneous steady state and periodic structures such as hexagons and stripes.[4] When such LSs are brought in proximity of one another they start to interact via their oscillating, exponentially decaying tails resulting even in clustering phenomena.[5,6]

However, self-organization phenomenon are not limited to the local coupling, also occurs in different dissipative systems with nonlocal coupling like chemical reactions,[7] population dynamics,[8] nonlinear optics[9] and vegetation patterns.[10] Notice that these works are focus on weak nonlocal coupling, i.e. a coupling that decays faster than an exponential. In the other hand, couplings that decay slower than an exponential have received scant attention –this kind of coupling is called strong nonlocal coupling.[11] In a recent work has been shown that localized structures may be stable in systems with a strong nonlocal coupling.[12] The mechanism of stabilization

Further author information: (Send correspondence to C.F.-O.)
C.F.-O.: E-mail: criferna@ulb.ac.be, Telephone: 32 483 585170

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of a single LS due to strong nonlocal coupling in Scalar Nagumo-Model has been theoretically predicted. Our preliminary studies on this model in the context of population dynamics showed that the nature of the interaction between fronts is affected by the choice of the nonlocal kernel.[12] Studying such LSs induced by strong nonlocal coupling in Left-handed materials has not been carried out.

Strong nonlocal response has been measured in nematic liquid crystals cells.[13] This experimental work showed that the nonlocal variation of the refractive index cells filled with the E7 LC is well fitted with a Lorentzian. Also strong nonlocal coupling has been observed in photorefractive materials experiments.[14] In this case, the source of the strong nonlocal coupling is the thermal medium effect.

In this work we investigate numerically the effect of the strong nonlocal interaction in an optical ring cavity and its importance to stabilize the localized structures with a varying size plateau. We show how in a one dimensional system two fronts interact to create this kind of structures, and the relevance to the distance to the Maxwell point in the size of the localized structure. We also present a stable localized structure in a two dimensional system.

2. MODEL

We consider a system formed by an optical ring cavity filled with two adjacent layers, a slab of a right-handed material and another slab of a left-handed material. The cavity is driven by an external coherent laser beam E_i. It has been shown that the electric field in this system can be described by the well-know Lugiato-Lefever model when the varying of the envelop is slow, i.e. in the mean field approximation.[15] This kind of systems have an inherent nonlocality that comes from the metamaterial and, when we consider that, the equation which describes the system is[16]

$$\partial_t E = \quad E_i - (1+i\theta)E + i|E|^2 E + iD\nabla^2 E \ - i\gamma \int_{-\infty}^{\infty} E\left(\mathbf{r}+\mathbf{r}', t\right) K\left(\mathbf{r}'\right) d\mathbf{r}', \tag{1}$$

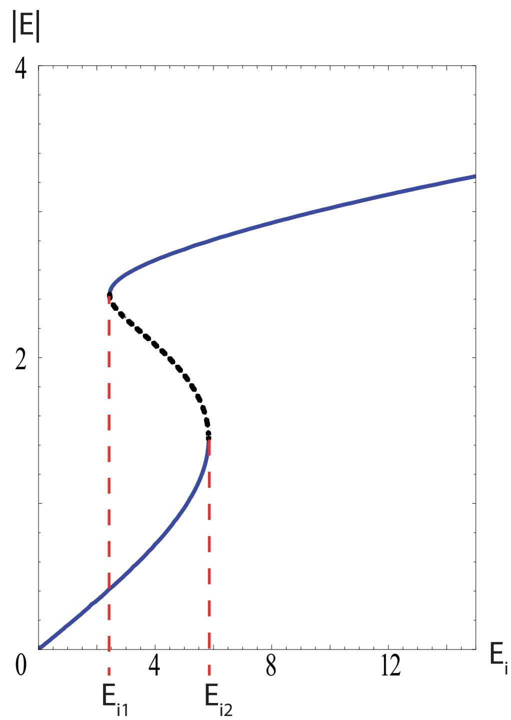

Figure 1. Stationary states versus E_i. The curves are the analytical results of the stationary homogeneous states in Eq. 1.The bistable region is between E_1 and E_2. The value of the detuning $\theta = 6.0$. Dashed black line correspond to the unstable state.

where t is time, E is the normalized complex enveloped of the electric field, θ represent the detuning parameter, D correspond to the diffraction coefficient, ∇^2 is the laplacian operator, and the injection E_i is a real parameter. The last term on the right size of equation (1) describes the linear nonlocal respond of the left-handed material that couples the electric field in the whole space. The intensity of the nonlocal interaction is γ. The nonlocal coupling is describe by the kernel function $K(\mathbf{r}) = \delta(\mathbf{r}) - f_\sigma(\mathbf{r})$, where $\delta(\mathbf{r})$ is the delta function and

$$f_\sigma(\mathbf{r}) = \frac{N_n}{\left(1 + (|\mathbf{r}|/\sigma)^2\right)^n} \tag{2}$$

is a Lorentzian which accounts for the nonlocal interaction of the field, σ is the characteristic length of the non-local interaction, n is the power of the Lorentzian that describes how the nonlocal interaction decays with the distance, and N_n is a normalization constant.

3. BISTABILITY AND FRONTS

The homogeneous steady states (HSS) of Eq. (1) exhibit a bistable behavior when $\theta > \sqrt{3}$. It is well known that in the absence of nonlocal coupling, and if $\Delta > 2$, the HSS exhibits a patten forming instability leading to the formation of periodic structures that occupy the whole space available in the transverse plane.[15] In this work, we focus on the regime far from any kind of pattern forming instability and we consider only a bistable regime as shown in Fig. 1. In the range $E_{i1} < E_i < E_{i2}$, there is a coexistence between two different homogeneous stable states. It is then possible to create fronts connecting the two stable HSS. Mathematically, this fronts are called heteroclinic solution of Eq. (1) that connect both states. In one dimension systems the front has a core, which could be defined as the middle point in the interface connecting both stable states. At the maxwell point where the two HSS are equally stable,[17] the velocity of the fronts vanishes. As we can see in Fig. 2, when one state (as example the lower one) is more stable than the other (the upper state), the most stable state invades all the space with a well defined velocity. Two opposite fronts move with an opposite velocity for the same values of parameter as shown in Fig. 2. The model Eq. (1) admits fronts with damping oscillations around only one state as illustrated in Fig. 2. It is well know that in the case of local or weak nonlocal coupling the profile of the front decays to the stable solution according to exponentially law with the distance to the core. However, when using a Lorentzian as nonlocal coupling the front's the front decays to the stable solution according to rather a power law.[12]

4. CAVITY SOLITONS WITH DIFFERENT SIZE PLATEAU

In one dimensional setting, two opposite fronts can create stable cavity solitons like structure as illustrated in Fig. 3. The interaction between two front stabilizes then the CS. Without this interaction a single front will moves

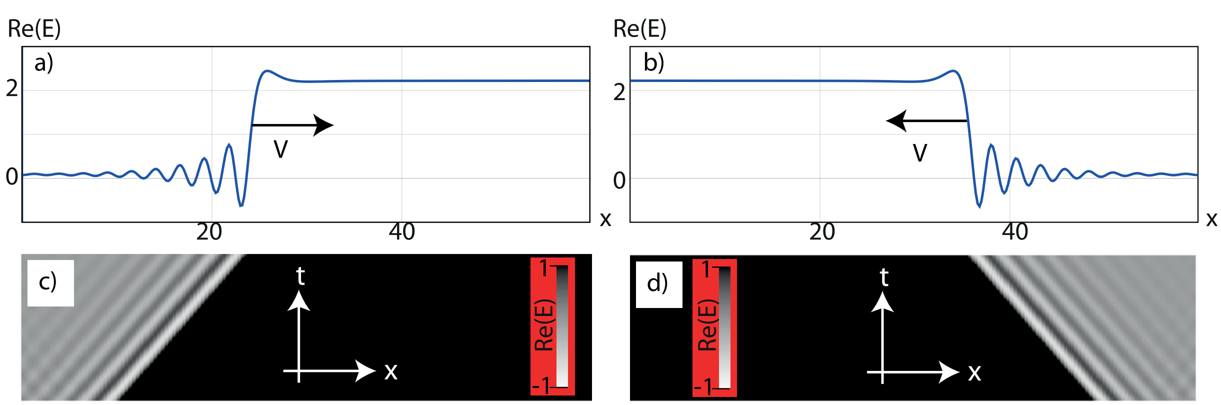

Figure 2. The real part of the field of two opposite fronts profiles are illustrated in a) and b). The results have been obtained numerically from Eq. 1. Their spatial-temporal evolution are in c) and d) respectively. The parameters are $\theta = 6$, $E_i = 2.96$, $D = -1$, $\gamma = 1$, $\sigma = 0.7$ and $n = 1$

as already shown in Figure 2. Note that both figures have been obtained for the same values of all parameters and with the same boundary conditions. The localized structure does not have oscillations in the plateau (upper state), it has oscillations only in the lower state (cf. Fig. 3 a). The space-time diagram of Fig. 3 c) illustrates how an initial condition (cf. Fig. 3 b) evolves towards a stable localized structure (cf. Fig. 3 a).

It has been shown in1D that weak nonlocal coupling with damping oscillations could stabilize localized structures.[18,19] However, our localized solutions are different from the one describes in[18,19] in two different ways:

(i)The plateau is devoid from any oscillations. The stabilization without damping oscillations is attributed to the strong nonlocal coupling.

(ii) possess a plateau with a varying size as shown in Fig. 4. The size vary strongly with the change of the

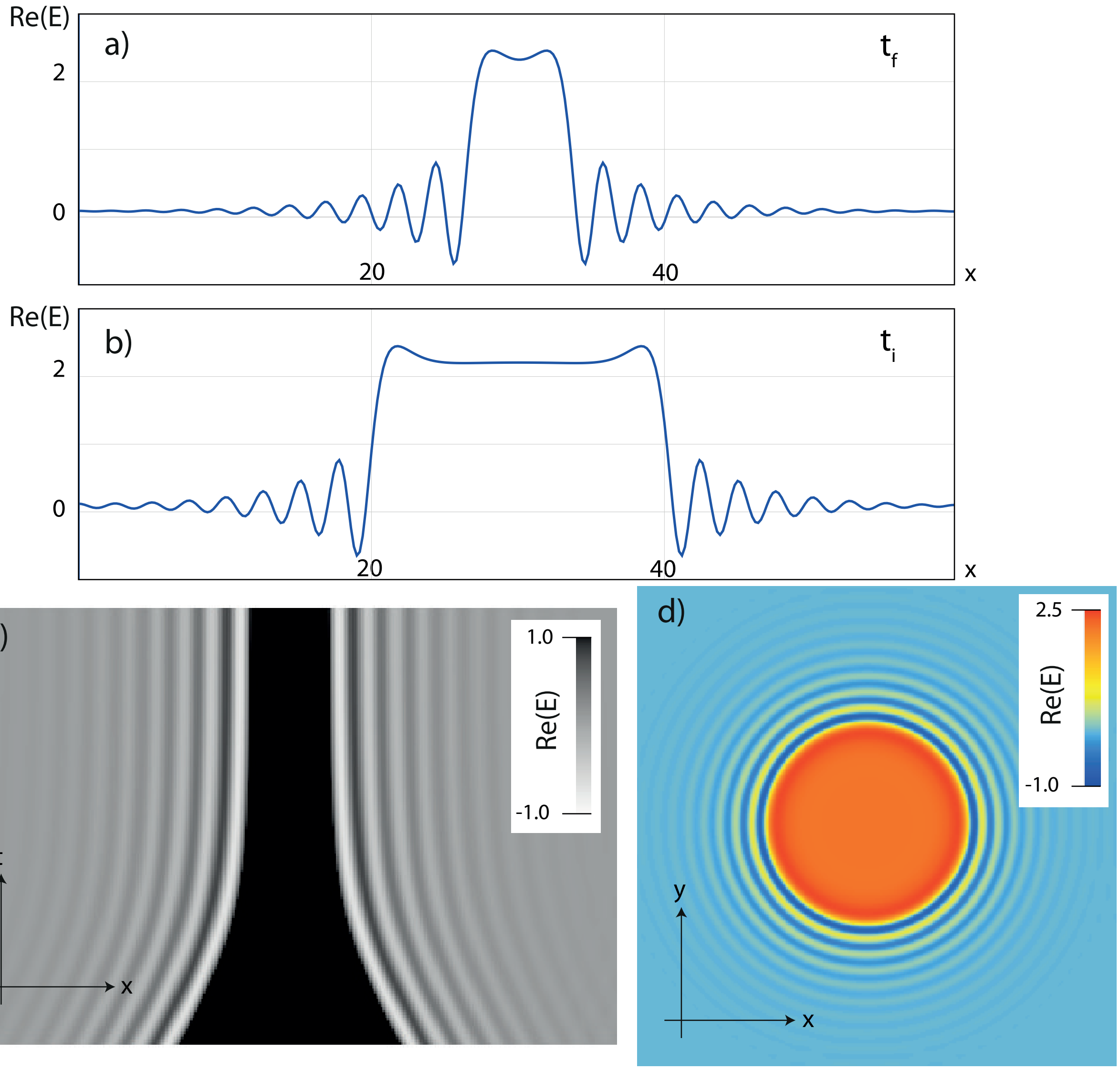

Figure 3. (Color online) The real part of the field of stable localized structures are illustrated. The results have been obtained numerically from Eq. 1. A stable equilibrium is illustrated in a), its initial condition in b) and the spatial-temporal diagram between both states in c). In a), b) and c) the parameters are $\theta = 6$, $E_i = 2.96$, $D = -1$, $\gamma = 1$, $\sigma = 0.7$ and $n = 1$. In d) the parameters are $\theta = 5.93$, $E_i = 3.0$, $D = -1$, $\gamma = 1$, $\sigma = 0.4$ and $n = 1.1$

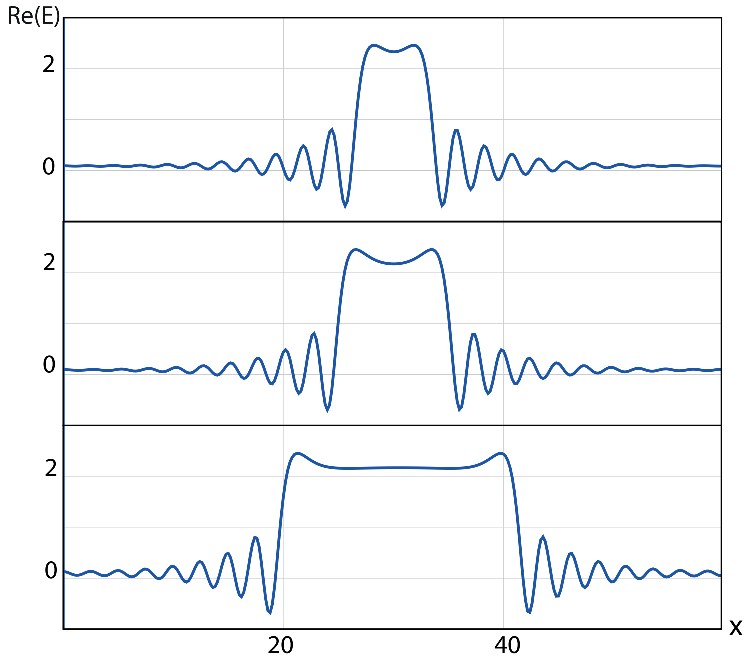

Figure 4. (Color online) Different size plateau of cavity solitons. The results have been obtained numerically from Eq. 1. It is shown the real part of the field for different injections. The parameters are $\theta = 6$, $E_i = 2.96$, $D = -1$, $\gamma = 1$, $\sigma = 0.7$ and $n = 1$

injected field inside the hysteresis loop involving two HSS. The size of localized structure dependents on the value of the control parameter E_i and becomes infinite when the system reaches to the Maxwell point.

Strong nonlocal coupling stabilizes the 2D localized solutions as shown in Fig. 3 d). In the case of local or a weak nonlocal coupling, the 2D structures are instable in the absence damping oscillations.

5. CONCLUSION

In this work we show strong nonlocal coupling in an optical ring cavity filled with a left-handed material and a Kerr type of material allows to stabilize a new type of localized structures or cavity solitons. This device is described by the well known mean field model, i.e., the LL model. The stabilization mechanism is attributed to strong nonlocal coupling mediated by a Lorentzian like function. These solutions possess a varying plateau. The width of the plateau dramatically changes with the intensity of the injected field and becomes infinite at the Maxwell point. This mechanism is robust in one and two dimensions. However, this solutions are possible only when the system operates in the bistable regime of parameters. Here has been shown numerically the stability of this kind of structures and how their width change dramatically when the systems is close to the Maxwell point. This results are quite general and can be applied to a broad kind of systems –that exhibit bistability– with strong nonlocal interaction.

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