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Noise induced rolls propagation

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Abstract. Interfaces in two-dimensional systems exhibit unexpected complex dynamical behaviors. We present a robust effect of noise in two dimensional extended systems: the motion of a static front connecting a stripe pattern with uniform state due to fluctuations. Numerical simulations of a prototype model show that noise induces rolls propagation. To give a unified description of this robust effect we place ourselves at the onset of the spatial bifurcation and we derive an stochastic normal form, which exhibits the same dynamics. Close to the Maxwell point, an interface equation is obtained, the over damping forced Sine-Gordon model, which allows us to explain origin of this stochastic phenomenon.

1 Introduction

The description of macroscopic matter, that is, matter composed by a large number of microscopic constituents, is usually done using a small number of coarse-grained or macroscopic variables. When spatial inhomogeneities are considered these variables are spatio-temporal fields whose evolution is determined by deterministic partial differential equations (PDE). This reduction is possible due to a separation of time and space scales, which allows a description in terms of the slowly varying macroscopic variables, which are in fact fluctuating variables due to the elimination of a large number of fast variables whose effect can be modelized including suitable stochastic terms, *noise*, in the PDE. The influence of noise in nonlinear systems has been the subject of intense experimental and theoretical investigations in the last decades [1–13]. Far from being merely a perturbation to the idealized deterministic evolution or an undesirable source of randomness and disorganization, noise can induce specific and even counterintuitive dynamical behavior. The most well-known examples in zero dimensional systems are noise induced transitions [1,2] and stochastic resonance (see the review [3] and references therein). More recently, examples in spatially extended system were found, such as, noise induced phase transitions [4–7], noise-induced patterns [8–10], stochastic spatio-temporal intermittency [11], noise-induced travelling waves [12] and noise induced front propagation in one space dimension [13]. Here, we present a new robust effect of noise in two dimensional extended systems: *the motion of a static front connecting a stripe pattern with a uniform state due to additive noise.* A first preliminary discussion of this effect in one space dimension was done [13–15] and the aim of this article is to study and characterize the universal mechanism which is at the origin of the front motion in the presence of noise.

The concept of front propagation emerged in the field of populations dynamics [16], and the interest in this type of problems has been growing steadily in Chemistry, Physics and Mathematics. In Physics, front propagation plays a central role in a large variety of situations, ranging from reaction diffusion models, to general pattern forming systems (see the review [17] and references therein). A front solution is a solution which links spatially two extended states. One of the most studied front solutions in one-extended systems is the front connecting a stable uniform state with an unstable one: *the FKPP-front* [18], the stable state invades the unstable one. The speed of propagation of this type of front is not unique and it is fixed by the initial conditions [19]. Another well-known type of front, the *normal front*, connects two stable uniform states [20]. The speed of this kind of front is unique and for a variational system it is proportional to the difference of free energy between the two uniform states. In Fig. 1 the dashed curve represents the typical behavior of the speed of a normal front as a function of an arbitrary parameter. Note that the speed of a front is zero only at the Maxwell point where both states have the same energy. This picture is modified when one considers a front connecting an spatially periodic state with a uniform one, which we call P-front in one dimensional extended systems. In this case the speed is zero not only in one point but in a whole interval of variation of the relevant parameter, *the pinning range* [21], and additive noise will induce front motion [13–15]. In Fig. 1 the continuous line represents the typical speed of these fronts and the interval [η_a,η_b] represents the pinning range. The effect of additive noise on the speed of a normal front is just a random fluctuation of its speed. On the other hand the influence of multiplicative noise in a front, which will not occupy us in this paper, has been extensively studied in the literature, particularly concerning the issue of velocity selection [22].

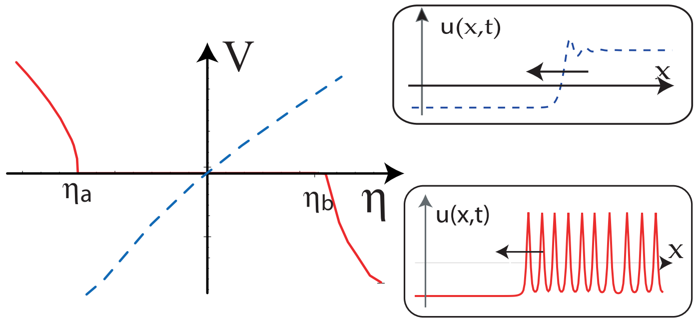

Fig. 1. Speed of the front as function of one parameter. The dashed curve depicts the typical behavior of the speed of a normal front as function of arbitrary parameter and the continue curve represents the speed of a front that link a spatial periodic state and uniform one. For the sake of simplicity the origin represents the Maxwell point. The pinning range is depicted by the interval between η_a and η_b. The inset figures represent a normal (upper) and P-front(lower), respectively.

In two spatial dimensions (2d), the systems exhibit a rich variety of extended states like uniform states, patterns (rolls, hexagons, squares and so forth), waves, chaos, oscillatory, spatio-temporal chaos and so on. Hence, the variety of interfaces or front solutions is vast. However, few experimental and theoretical studies in two-extended dimensions have been carried out on fronts connecting patterns and uniform states [23–26]. Recently, numerical simulations of a prototype model, the isotropic Swift-Hohenberg equation, has shown that a flat interface connecting a stripe pattern with a uniform state is transversely unstable and do not exhibit the locking phenomena or pinning effect (depinning effect [24, 27]). This unstable behavior is characterized by the appearance of an initial wave number which is subsequently replaced by zigzag dynamics, which presents a complex coarsening. By means of amplitude equation, we show here that an isotropic and anisotropic system exhibits depinning and pinning effect for an interface connecting a rolls pattern and a uniform state, respectively [27].

The aim of this manuscript is then to study the effect of noise in a static front connecting a stripe patterns with a uniform state in an anisotropic system. From a prototype model, we show that noise induces rolls propagation. To give a universal description of this phenomenon at the onset of a spatial bifurcation, we derive an stochastic normal form [28–34], which exhibits the same dynamical behaviors. Close to the Maxwell point, we deduce an equation for the interface, the over damped forced Sine-Gordon model, which allows us to explain the origin of this stochastic phenomenon. Numerical simulations of the interface equation are in good agreement with the stochastic normal form and prototype model.

2 Stable interface connecting a stripe pattern with a uniform state

Isotropic systems which exhibit coexistence between an stripe pattern and a uniform state do not have stable interface connecting these state (stable flat interface) [27]. This statement is understood by means of the Newell-Whitehead-Segel amplitude equation, where the flat interface suffers a transversal instability. We shall see that an anisotropic system can exhibit stable static or movable front between an stripe pattern and a uniform state. A simple anisotropic model that exhibits coexistence between a uniform state and a stripe pattern is

$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - \left(\partial_{xx} + q^2\right)^2 u + D\partial_{yy} u, \tag{2.1}$$

where $u(x,t)$ is an order parameter, ε is the bifurcation parameter, q is the wave number of the stripe pattern, ν the control parameter of the type of bifurcation (supercritical or subcritical), and D diffusion parameter. The model (2.1) describes the confluence of an stationary and an spatial subcritical bifurcations with reflection symmetry ($u \to -u$) for an anisotropic system, when the parameters scale as $u \sim \varepsilon^{1/4}$, $\nu \sim \varepsilon^{1/2}$, $q \sim \varepsilon^{1/4}$, $\partial_t \sim \varepsilon$, $\partial_x \sim \varepsilon^{1/4}$, and $\partial_y \sim \varepsilon^{1/2}$ ($\varepsilon \ll 1$). This bifurcation is a codimension three-point, that is, physically one needs to fix three parameters to observe the above dynamics. However, in order to observe a front connecting an stripe pattern with a uniform state one only needs to fix one parameter (codimension one: coexistence between different extended states). For small and negative ε and $-9\nu^2/40 \equiv \varepsilon_{sn} < \varepsilon < 0$, the system exhibits coexistence between a uniform state $u(x,y,t) = 0$ and a stripe pattern $u(x,y,t) = \sqrt{\nu}\left(\sqrt{2(1+\sqrt{1+40\varepsilon/9\nu})}\cos(qx)\right) + o(\nu^{5/2})$. For $\varepsilon = \varepsilon_{sn}$ the model has a saddle-node bifurcation that give rises to stable and unstable pattern and for $\varepsilon = 0$ the uniform state becomes unstable. When one considers the above model in one spatial dimension ($u(x,y,t) = u(x,t)$), it is well-known that in the coexistence region ($\varepsilon_{sn} < \varepsilon < 0$) the model exhibits a front connecting a spatially periodic solution and uniform state. For ε close to 0 (ε_{sn}), the pattern (uniform) state invades the uniform (pattern) state, i.e. the system displays front propagation. The front is motionless at the interval $\varepsilon_- < \varepsilon < \varepsilon_+$, the pinning range [21]. Note that in this region, the state which is energetically more favorable does not invade the less favorable one.

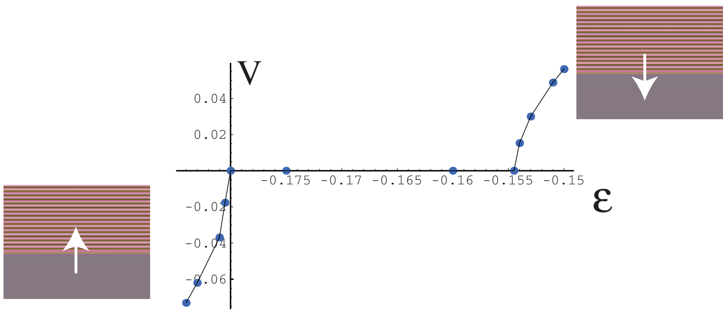

Fig. 2. Speed of the front connecting a stripe pattern with uniform state of model (2.1) as function of bifurcation parameter ϵ with $q = 0.7$, $\nu = 1.0$, and $D = 1$.

In two spatial dimension the above scenarios continue working due to the anisotropy and the pinning range is renormalized, that is, the locking phenomenon between an stripe pattern and a uniform state persists in 2D. In Fig. 1, we show the pinning range of model (2.1). Inside the pinning range, one expects that the flat interface is transversally stable, that is, a perturbation in the y-direction disappears and the interface converges to the flat state. In Fig. 3, we show the diffusive evolution of this interface.

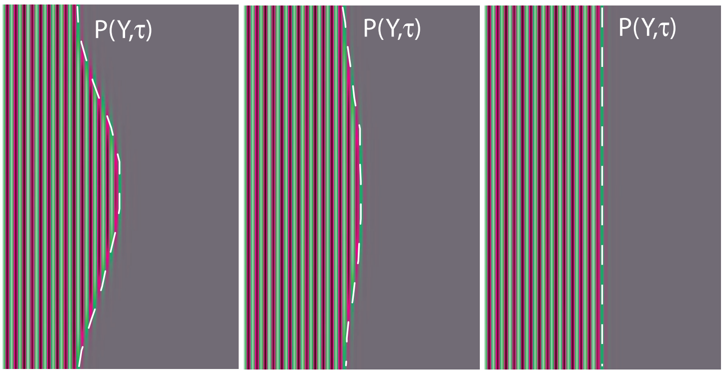

Fig. 3. Pinning effect: Numerical simulation of model (2.1) with $\epsilon = -0.165$, $\nu = 1.0$, $D = 1.0$ and $q = 0.7$, demonstrating that the flat interface is stable. The time increases in the frames from left to right. The dashed curve represents the interface between the different extended states.

2.1 Noise induced front propagation

In order to understand the mechanism through which noise modifies the dynamics of the flat interfaces, we consider the model (2.1) with an additive noise

$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - \left(\partial_{xx} + q^2\right)^2 u + D\partial_{yy} u + \sqrt{\eta}\zeta(x, y; t), \tag{2.2}$$

where $\zeta(x, y; t)$ is a gaussian white noise with zero mean value and correlation $\langle \zeta(x, y; t)\, \zeta(x', y'; t')\rangle = \delta(x - x')\,\delta(y - y')\,\delta(t - t')$, and η represents the intensity of the noise. When additive white noise is taken into account, one may expect random fluctuations of the interface between the two states. However, numerical simulations of above model show that the front propagates from one state to the other with a stochastic velocity, as it is depicted in Fig. 4. The numerical method used in the simulation is the Runge-Kutta algorithm with time step equal 0.01, and spatial mesh $dx = 1/400$ and $dy = 1/200$. Depending on the region of parameters, the front can propagate to the periodic spatial state or to the homogeneous one. Hence, noise induces the most favorable state to invade the unfavorable one and the conversion of random fluctuations into direct motion of the interface is responsible of the propagation; noise prefers to create or remove a stripe, because the necessary perturbations to nucleate or destroy a stripe are different.

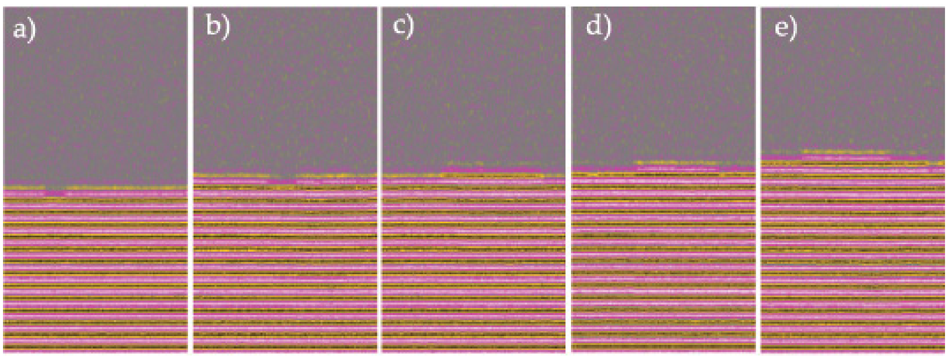

Fig. 4. Noise induces rolls propagation: Numerical simulation of model (2.2) with $\epsilon = -0.165$, $\nu = 1.0$, $D = 1.0$, $q = 0.7$, and $\eta =$. The time increases in the frames from left to right.

3 Stochastic normal form

In order to give a unified description of the noise induced interface propagation, we consider the system close to a spatial bifurcation with weak nonlinear response, one has that for small ν and $-9\nu^2/40 < \varepsilon < 0$, the system exhibits coexistence between a stable homogenous state $u = 0$ and an stripe pattern. In this parameters region, one finds a front between these two states. A front between a homogeneous and a spatial oscillatory state can be described by the ansatz

$$u = \left\{ A\left(X = \frac{x}{l_1}, Y = \frac{y}{l_2}, \tau = \frac{9\widetilde{\nu}^2\left|\varepsilon\right|}{10}t \right) e^{iqx} + c.c. + w_1\left(x,y,\tau\right) \right\}, \tag{3.1}$$

here $\nu = \widetilde{\nu}\sqrt{|\epsilon|}$, $l_1 = 2q\sqrt{10}/3\nu$, $l_2 = \sqrt{10D}/3\nu$ and $A(X,Y,\tau)$ is the envelope of the pattern that describes the front solution (when the envelope is uniform and not null the initial model (2.2) has an stripe pattern), $w_1\left(x,y,\tau\right)$ is a small correction, and $\{X,Y,\tau\}$ are slow variables. In this ansatz (3.1), we consider that q is of order one or larger than the other parameters. The ansatz (2.2) is the starting point leading the stochastic normal form as explain in [28–34]. This normal form is the stochastic unfolding of the usual spatial normal form and it reads

$$\partial_\tau B = \sigma B + \left|B\right|^2 B - \left|B\right|^4 B + \vec{\nabla}^2 B + \frac{\sqrt{\eta}\beta}{\left|\varepsilon\right|^{9/4}} e^{-iqX/(\alpha\sqrt{|\varepsilon|})}\zeta\left(X,Y,\tau\right), \tag{3.2}$$

where $\sigma \equiv 10\epsilon/9\nu^2, B(X,Y,\tau) \equiv \sqrt{10/3\nu}A(X,Y,\tau), \alpha \equiv 3\widetilde{\nu}/2q\sqrt{10}, b \equiv 10^{5/2}\sqrt{2q\sqrt{D}11}/81$ $\sqrt{3\widetilde{\nu}^9}$ and $\vec{\nabla}^2 \equiv \partial_{XX} + \partial_{YY}$.

At this point we comment the role of non resonant terms which one could add to the normal form (3.2). The dominant non-resonant terms, which will induce in the equation of the interface a contribution of the same small exponential order as the noise term is (a_0 and b_0 are O(1) as $|\epsilon| \to 0$)

$$\left(\frac{a_0}{3}B^3 - \frac{b_0}{2}|B|^2 B^3\right) e^{\frac{2iqX}{\alpha\sqrt{|\epsilon|}}},$$

will give in the equation for the interface an additional term which can be neglected in comparison with the drift terms induced by the noise.

3.1 Noise induces interface propagation

For $-0.25 \leq \epsilon \leq 0$ the deterministic part of model (3.2), exhibits coexistence of two uniform states. Due to the spatial forcing in (3.1) the non null uniform state becomes a stripe pattern. Close to this region one can find a front connecting these states. In fig. 6a, it is illustrated the typical front solution observed in this model inside the pinning range. When we consider the stochastic term in model (3.2) in the pinning range, noise induces propagation of the most favorable state as it is depicted in Fig. 6. Due to the universal nature of the normal for [17], for any anisotropic system which exhibits a static interface connecting a stripe pattern and a uniform state, noise will generically induce interface propagation.

4 Interface equation

When one considers only the deterministic normal form in (3.2), it is straightforward to show that the system exhibits a front solution between two homogeneous states, 0 and $\sqrt{(1+\sqrt{1+4\sigma})/2}$, when $\sigma < 0$. This front propagates from the stable state (lowest free

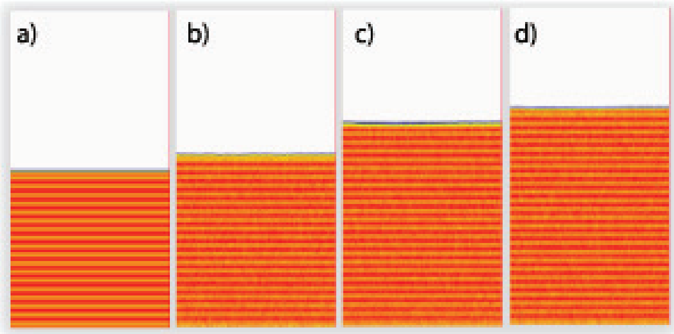

Fig. 5. Noise induces interface propagation: Numerical simulation of model stochastic normal form (3.2) with $\sigma = -0.178$, $q = 8.177$, and $\eta = 0.2$. The time increases in the frames from left to right.

energy) to the metastable one, and it is static when the Maxwell point is reached $\sigma_M = -3/16$, and it has the form

$$B\left(X - P, Y, \tau\right)_{\pm} = \sqrt{\frac{3/4}{1 + e^{\pm\sqrt{3/4}(X-P)}}}e^{i\theta}, \tag{4.1}$$

where P is the position of flat interface, and θ is an arbitrary phase. The front solution B_+ (B_-) represents the interface that links stripe pattern (uniform state) with uniform state (stripe pattern). It is important to note that the flat interface solution (4.1) is parameterized by a symmetry group characterized by two quantities the position and phase of the interface (P, θ). To describe the dynamics exhibited by (2.1, 3.2) and the locking phenomena, we must consider the stochastic normal form (3.2), which include the noise term with its implicit rapidly varying spatial oscillations due to the factor exponential factor. We consider the noise term as a perturbation and we promote the position of the interface to a field $P(Y, \tau)$, which will describe the dynamical behaviors of this interface. Close to the Maxwell point ($\sigma_M = -3/16$), we use the ansatz (We shall work with from B_-)

$$B\left(y, \tau\right) = \left[R_0\left(X - P(Y, \tau), Y\right) + \widetilde{\epsilon}\rho\left(X, P(Y, \tau)\right)\right] e^{i\theta + \widetilde{\epsilon}\Theta(X, P(Y, \tau))}$$

where $R_0 = |B_-|$, $\widetilde{\epsilon}$ is a small parameter of order $\Delta\sigma = (\sigma - \sigma_M)$, and $\{\rho, \Theta\}$ are correction functions which depend of time only through $P(Y, \tau)$. Introducing the above ansatz in equation (3.2) and linearizing in $\{\rho, \Theta\}$ we obtain the following solvability condition for the position of the interface

$$\partial_\tau P = \frac{3a}{\sqrt{8}}\Delta\sigma + \partial_{YY}P + |J|\cos\left(\frac{2qP}{\alpha\sqrt{|\epsilon|}} + \varphi\right) + \frac{\sqrt{\widetilde{\eta}}}{\varepsilon^{9/4}}\xi\left(Y, \tau\right)$$

where $\xi\left(Y, \tau\right)$ is a δ-correlated gaussian white noise with mean value zero and correlation $\langle \xi\left(Y, \tau\right)\xi\left(Y', \tau'\right)\rangle = \delta\left(\tau - \tau'\right)\delta\left(Y - Y'\right)$, $\widetilde{\eta} = \beta^2\eta a/8$, $a = \int dx(\partial_x R_o)^2 = 3/4$. J is a complex number given by $J = -(a^2\beta^2\eta/16\Delta|\epsilon|^{9/2})\int_{-\infty}^{\infty} dx e^{i\frac{2qx}{\alpha\sqrt{|\varepsilon|}}}\partial_x R_0(x)\partial_{xx}R_0(x)$. In the previous formula Δ is the discretization length in the y-axis which appears there due to the δ-correlation in space of $\xi(Y, \tau)$ (if this correlation is $\langle \xi\left(Y, \tau\right)\xi\left(Y', \tau'\right)\rangle = C\left(Y - Y'\right)\delta\left(\tau - \tau'\right)$ one would have $C(0)$ instead of $1/\Delta$ in formula for J) and $\tan(\varphi) = \Im(J)/\Re(J)$ ($\Re$ and $\Im$ are the real and imaginary part, respectively). Putting $\tau = \tau'/|J|$, $Y = Y'/\sqrt{|J|}$, $P = \alpha\sqrt{|\epsilon|}/2q\left(\Lambda + \pi/2 - \varphi\right)$, and $\xi' = |J|^{3/4}\xi$, one obtains (over-damped forced Sine-Gordon model)

$$\partial_{\tau'}\Lambda = \gamma - \sin\left(\Lambda\right) + \partial_{Y'Y'}\Lambda + \sqrt{\mu}\xi'\left(Y', \tau'\right), \tag{4.2}$$

with the forcing $\gamma = 6aq\Delta\sigma/8|J|\alpha\sqrt{|\epsilon|}$, and the effective noise intensity $\eta' = 4q^2\widetilde{\eta}/\alpha^2|J|^2|\epsilon|^{7/2}$, and correlation $\langle \xi'\left(Y_1', \tau_1'\right)\xi'\left(Y_2', \tau_2'\right)\rangle = \delta\left(Y_1' - Y_2'\right)\delta\left(\tau_1' - \tau_2'\right)$. The field Λ describes the evolution of the interface connecting a stripe pattern with a uniform state. The first term of right hand size of model (4.2), the constant forcing, is associated to the different of energy between the extended states. The periodic term describes the interaction of the large scale envelope variation with the small scale of the pattern state, that is, it is consequence of the spatial for cing modulating the noise in eq. (3.2). The third term, diffusive type, describes the transversal coupling of the interface. In Fig. 3, we show this typical diffusive character. The last term contains the fluctuations, to which is submitted the interface. It is worth to note that the effective stochastic term in the interface equation is a simple extended white noise. In the derivation of the equation (4.2) for the interface, the periodic term in the drift comes from the multiplicative noise term interpreted in the stratonovich sense and we have used the techniques developed in [35] for its derivation. The calculations is very similar to what we have done in [13, 14].

4.1 Dynamics of deterministic interface equation

The dynamics of deterministic over damped Sine-Gordon model ($\mu = 0$) is characterized by the parameter γ. If this parameter $\gamma < -1$, i.e. the uniform state is most favorable than the stripe pattern, then the uniform state invades the stripe pattern and the flat interface is stable, that is, a perturbation in the interface spreads when the interface propagates. As result of the periodic term the speed of the interface is not uniform, it is oscillatory.

When $\gamma = \gamma_- \equiv -1$, the system exhibits simultaneously infinite equilibriums points at $\Delta_0 = 0, \pm 3\pi/2, \pm 5\pi/2, \ldots$, that is, the system presents a simultaneous saddle-node bifurcation. For $|\gamma| < 1$ the model exhibits infinite equilibria, which satisfies $\sin \Delta_0 = -\gamma$. Hence, the system exhibits equilibriums state for $|\gamma| < 1$, which appear and disappear by saddle-node bifurcation in $\gamma = \gamma_-$ and $\gamma = \gamma_+ \equiv 1$, respectively. This region of parameter represents the pining range of the interface connecting a stripe pattern with a uniform one. Note that the different equilibria has different energy, for negative (positive) γ the most stable state is negative (positive) infinite, as result of the interaction of the large-scale envelope variation with the small scale underlying in the pattern state the there are thresholds that separate consecutive stable position of the flat interfaces. The threshold that separates a more energetically position from a lower one has the form of a bump [41], when the perturbation is smaller than this threshold the system evolves to the initial flat state. In the case that the perturbation is larger than the threshold the interface starts to propagate from the perturbation as two counter-propagative fronts and finally the system arrives to a new flat interface with lower global energy, that is, for negative (positive) γ the position of the interface decreases (increases). When $\gamma = 0$, the system reaches the Maxwell point, i.e. all equilibria are energetically equivalent. For $\gamma > 1$, i.e. the stripe pattern is the most favorable state, then this state invades the uniform one and the flat interface is stable and it propagates to the right size.

4.2 Dynamics stochastic interface equation

The above pictures is modified when the stochastic term of model (4.2) is taken into account. Outside the pinning range the noise in average does not modify the deterministic behavior and in the pinning range the noise induces interface propagation. For negative (positive) γ the interface moves to the left (right). In Fig. 6 this propagation is shown. One can figure out

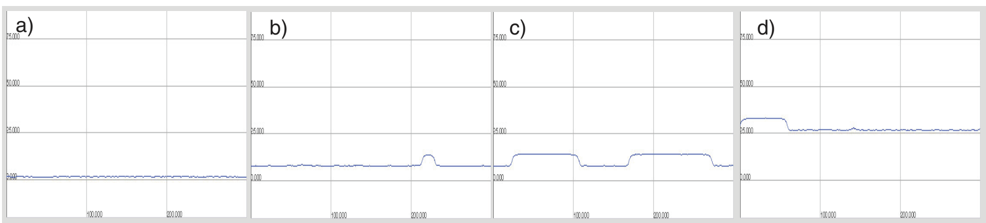

Fig. 6. Numerical simulation of model (4.2) with $\gamma = 0.95$, and $\mu = 1.0$. The time increases in the frames from left to right.. The time increases in the frames from left to right.

the mechanism of this motion: the flat interface is stable, then the small fluctuations spread in the flat interface, however, a large fluctuation can cross the threshold that separates this equilibrium from the a lower energetically one and the interface propagates to the flat interface with lower energy. Hence, we have easily identify the mechanism trough which noise induces rolls or uniform state propagation.

5 Conclusion

We have presented a new robust effect of noise in two dimensional extended systems: the motion of a static front connecting a stripe patterns with a uniform state due to additive noise. This phenomenon initially can be conjecture from previous results 1D [13,15], however in isotropic system the effect disappears [24].

Numerical simulations of a prototype model show that noise induces rolls propagation. In order to give a unified description of this effect at the onset of an spatial bifurcation, we derive an stochastic normal form, which exhibits the same phenomenon. Close to the Maxwell point, the equation for the interface is the over damp forced Sine-Gordon model, which allows us to explain origin of this stochastic effect. All these models exhibit qualitative the same dynamical behaviors. The speed of the from can in principle be calculated using similar argument to the ones in [13,14] and will be published elsewhere.

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