Pulse propagation in a 1D array of excitable semiconductor lasers

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ABSTRACT

Nonlinear pulse propagation is a major feature in continuously extended excitable systems. The persistence of this phenomenon in coupled excitable systems is expected. Here, we investigate theoretically the propagation of nonlinear pulses in a 1D array of evanescently coupled excitable semiconductor lasers. We show that the propagation of pulses is characterized by a hopping dynamics. The average pulse speed and bifurcation diagram are characterized as a function of the coupling strength between the lasers. Several instabilities are analyzed such as the onset and disappearance of pulse propagation and a spontaneous breaking of the translation symmetry. The pulse propagation modes evidenced are specific to the discrete nature of the 1D array of excitable lasers.

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Linear oscillators coupled with springs to nearest neighbors exhibit wave propagation. This phenomenon is persistent when considering the continuous limit, i.e., when considering an elastic rope. In this limit, the wave dispersion relation is linear, unlike the discrete case of coupled systems where it is nonlinear. Here, we study the propagation of localized nonlinear waves— pulses—in coupled excitable systems. Excitable oscillators play a fundamental role in understanding the activity of neurons, cardiac tissue, and oscillatory chemical reactions. Based on a model of a 1D array of excitable semiconductor lasers, we show that pulse propagation is characterized by a hopping dynamics and that it displays a rich variety of bifurcations. Counterintuitively, we observe that pulses do not persist in the continuous limit.

I. INTRODUCTION

The propagation of excitations (spikes) in discrete excitable media plays a major role in biological systems.[1–4] It is at the heart of the conduction of information in axons and it ensures conduction delays, which are central to information processing in neural networks.[5] The possibility to process information with spikes in photonic systems has attracted recently a lot of interest because of its application potential in terms of energy consumption, parallelism, and speed.[6–9] It has been recently shown theoretically that coupled excitable semiconductor lasers can behave analogously to biological axons, allowing to transport and process information in the form of short optical spikes.[8,10]

Dissipative systems are characterized by exhibiting attractors and basins of attraction.[11–15] The boundaries of these basins of attraction are in general fractal.[11,12] The dynamics within the basin of attraction is governed by the geometry of the stable invariant manifolds associated with the respective equilibrium and the separatrix manifolds of the basin of attraction.[15,16] Manifolds are the nonlinear extension of the eigenvectors obtained in the linearized dynamics around the equilibrium. Hence, infinitesimal disturbances around the equilibrium are generally exponentially decaying to equilibrium (linear dynamical behavior). However, under certain conditions and unexpectedly, large excursions (larger than the disturbance) can be observed in the basin of attraction. This type of behavior is known as *excitability*. Excitability is a generic phenomenon encountered in many areas of science and, in particular, in biology,[17–21] chemistry,[22,23] and optics.[24,25]

From the point of view of the phase portrait geometry, excitability arises because stable invariant manifolds are folded (as a consequence of previous bifurcations) or are connected with

hyperbolic points that generate separatrix inside the basin of attraction (see Fig. 1).[20,21] Therefore, excitability is a genuine nonlinear phenomenon. The above scenario changes radically when considering excitable systems extending over space. In this context, a local perturbation above the excitability threshold is enough to excite its nearest neighbors, generating excitable pulses or waves.[19,21,26–28] These waves are known in chemical reactions,[19,27] in the cardiac muscle,[17] and in liquid crystals.[26] In lasers, their existence has been theoretically predicted in a laser with an injected signal,[28] but the experimental part lacks a convincing demonstration. In most of these works, the propagation of pulses is studied, which has a well-defined propagation speed. However, continuous models are used to characterized the propagation. It is known that discreteness can affect the propagation of wavefronts.[29,30] For example, the wavefront speed in discrete systems can present oscillations.[29–31] Even

more, discreteness can induce the propagation failure in bistable[32] or excitable[4] discrete systems.

The aim of this paper is to study theoretically the propagation of pulses in an array of excitable semiconductor lasers. Based on a one-dimensional array of coupled lasers with a saturable absorber medium, we show that the propagation of pulses is characterized by a hopping dynamics (see Fig. 1). Depending on the coupling strength between the lasers, we characterize the speed of the pulses and their bifurcation diagrams. This speed increases with the coupling strength. The propagating pulses emerge by means of a saddle-node bifurcation, as the coupling constant increases, and then the solution adapts itself and exhibits several instabilities. The observed pulses are peculiar to the discrete nature of the excitable laser coupling; that is, when the continuous limit is taken, the traveling pulses do not persist.
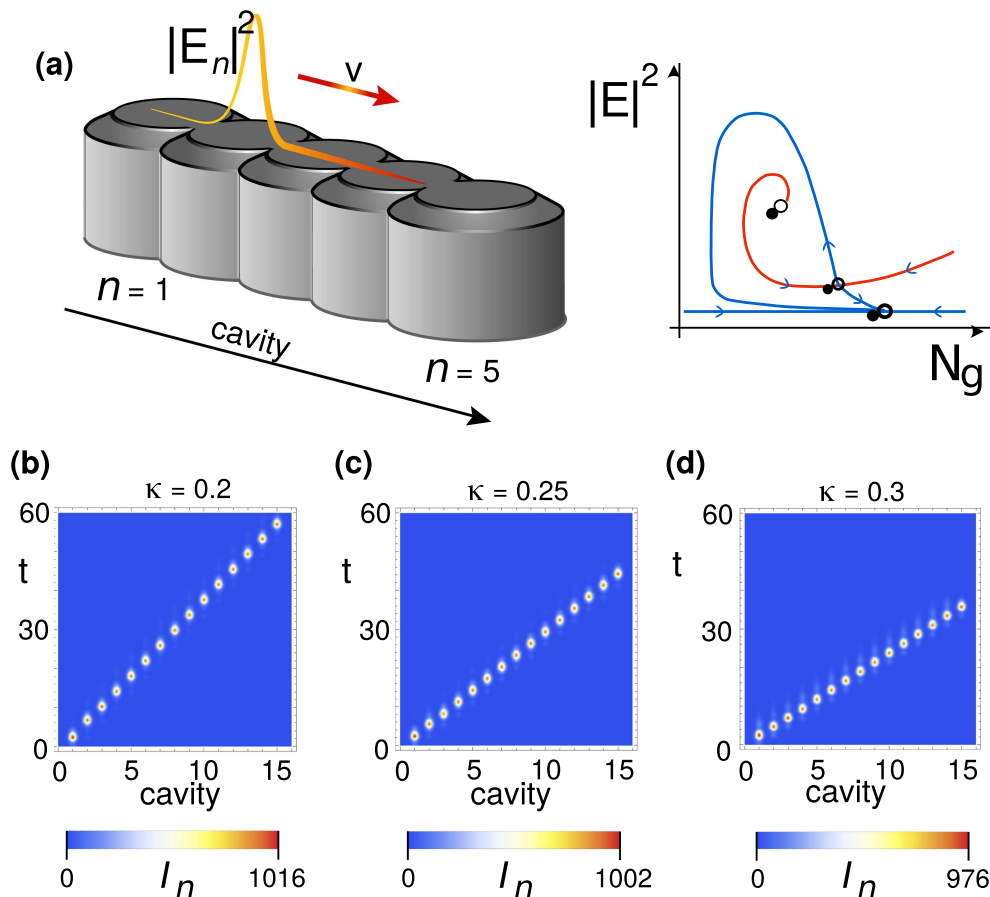

FIG. 1. Pulse propagation in an 1D array of excitable semiconductor lasers. (a) Schematic representation of an array of excitable semiconductor lasers. Lasers emit from the top and are evanescently coupled through the neighbors. The inset accounts for the typical phase portrait of a single semiconductor laser. Curves and dots account for invariant manifolds and equilibria. Spatiotemporal propagation of a pulse of an array of an excitable semiconductor laser model [Eqs. (1)–(3)] with different coupling constants $\kappa = 0.20$ (b), $\kappa = 0.25$ (c), and $\kappa = 0.30$ (d), $\alpha = 2.0$, $\beta = 0$, $A = 2.74$, $B = 2$, $b_1 = b_2 = 0.001$, and $s = 10$.

II. THEORETICAL DESCRIPTION OF AN ARRAY OF AN EXCITABLE SEMICONDUCTOR LASER

The excitable system that we consider is a micropillar laser with an integrated saturable absorber medium studied theoretically in Refs. 33 and 34 and experimentally in Refs. 35 and 36. These lasers have been shown to behave analogously to biological neurons, displaying refractory periods,[36] spike latency,[37,38] and temporal summation (coincidence detection).[39] We consider a 1D array of evanescently coupled lasers with a saturable absorber[8,10] depicted in Fig. 1(a) and described by the dimensionless set of equations,

$$\dot{E}_n = \left[(1 - i\alpha)N_{g,n} - (1 - i\beta)N_{as,n} - 1\right]E_n + i\kappa(E_{n-1} + E_{n+1}), \tag{1}$$

$$\dot{N}_{g,n} = b_1\left[A - N_{g,n}\left(1 + |E_n|^2\right)\right], \tag{2}$$

$$\dot{N}_{as,n} = b_2\left[B - N_{as,n}\left(1 + s|E_n|^2\right)\right], \tag{3}$$

where $E_n(t)$, $N_{g,n}(t)$, and $N_{as,n}(t)$ account, respectively, for the envelope of the electric field, the rescaled gain, and the rescaled absorption in the ith laser. The factors α and β are standard semiconductor parameters describing phase-amplitude coupling. κ stands for the dispersive nearest-neighbor coupling coefficient between the lasers. Non-radiative carrier recombination rates for the gain and absorber media are, respectively, b_1 and b_2. A and B account for the pump gain and non-saturable losses. The saturation parameter s in semiconductors is necessarily greater than 1. Time is rescaled to the cavity photon lifetime, which is the shortest time scale in the system (several picoseconds), and the carrier recombination time scales, which are of the order of 0.5–1 ns are, therefore, small: $b_{1,2} \ll 1$. Notice that a model similar to the set of Eqs. (1)–(3) with purely diffusive coupling has been considered to study synchronization phenomena in
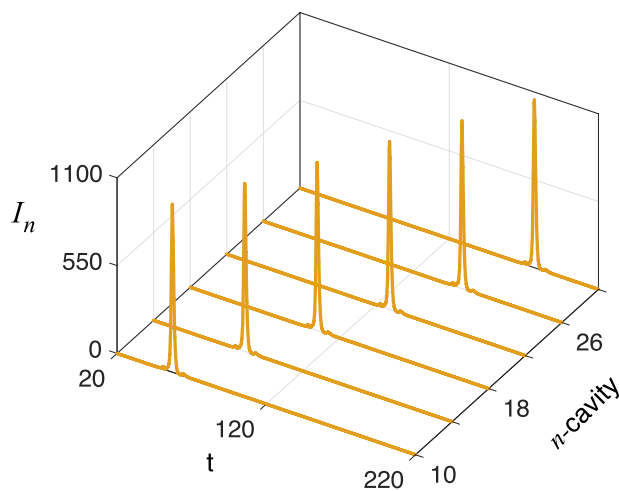

FIG. 2. Pulse propagation in an array of an excitable semiconductor laser model [Eqs. (1)–(3) with $\alpha = 2.0$, $\beta = 0$, $A = 2.74$, $B = 2$, $b_1 = b_2 = 0.001$, and $s = 10$. Spatiotemporal evolution of a perturbation above an excitable threshold of the leftmost cavity, in an array of 100 coupled micropillars with an evanescent coupling of $\kappa = 0.15$.
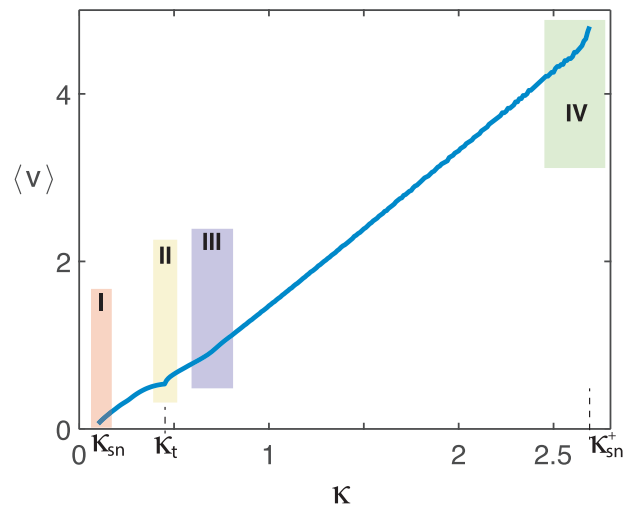

FIG. 3. Average pulse speed as a function of evanescent coupling κ of an excitable semiconductor laser model [Eqs. (1)–(3)] with $\alpha = 2.0$, $\beta = 0$, $A = 2.74$, $B = 2$, $b_1 = b_2 = 0.001$, and $s = 10$. The colored regions account for the different bifurcations observed. $\kappa_{sn} \sim 0.1$, $\kappa_t \sim 0.45$, and $\kappa_{sn}^+ \sim 2.71$ account for the critical evanescent coupling in which the pulses emerge and present a spontaneous symmetry translation-breaking, respectively.

the presence of additive noise[40] and localization phenomena when coefficients are variable (with disorder).[41] The set of Eqs. (1)–(3) under the influence of noise exhibits synchronization and with variable coefficients shows localization.[42] A single semiconductor laser with an integrated saturable absorber medium can be accurately described by rate equations for the intensity of the electric field, gain, and absorption, the *Yamada model*.[34,43] However, because of the evanescent coupling between the microlasers, one must consider the envelope of the electric field in the model written in Eqs. (1)–(3) to account for the dispersive (imaginary) coupling term.

The non-lasing solution is represented by $E_n(t) = 0$, $N_{g,n} = A$, and $N_{as,n} = B$. This state is stable for a single laser when $A - B - 1 < 0$ and corresponds to an attractor. Excitable dynamical behavior requires that[33] $s > 1 + 1/B$. Note that in semiconductor materials, this condition is fulfilled since the parameter s is a large parameter due to the gain saturation. The schematic projection of the phase portrait of a single laser in the plane $\{N_g, I = |E|^2\}$ is illustrated in Fig. 1(a). The laser threshold corresponds to a transcritical bifurcation and occurs at $A_{th} \equiv 1 + B$. In this kind of system, excitability exists near a homoclinic loop bifurcation and below the laser threshold. If the system is sent above the stable manifold of the saddle point, it makes a large excursion around the heteroclinic orbit and turns back to the stable state corresponding to the off solution of the laser: an excitable optical pulse is produced.

In Figs. 1(b)–1(d), pulse propagation in the array of semiconductor lasers is exemplified for different values of the evanescent coupling parameter, κ. This parameter can be experimentally tuned by changing the center-to-center distance between the pillars (see Ref. 44). In the regime of weak evanescent coupling, $\kappa \ll 1$, the
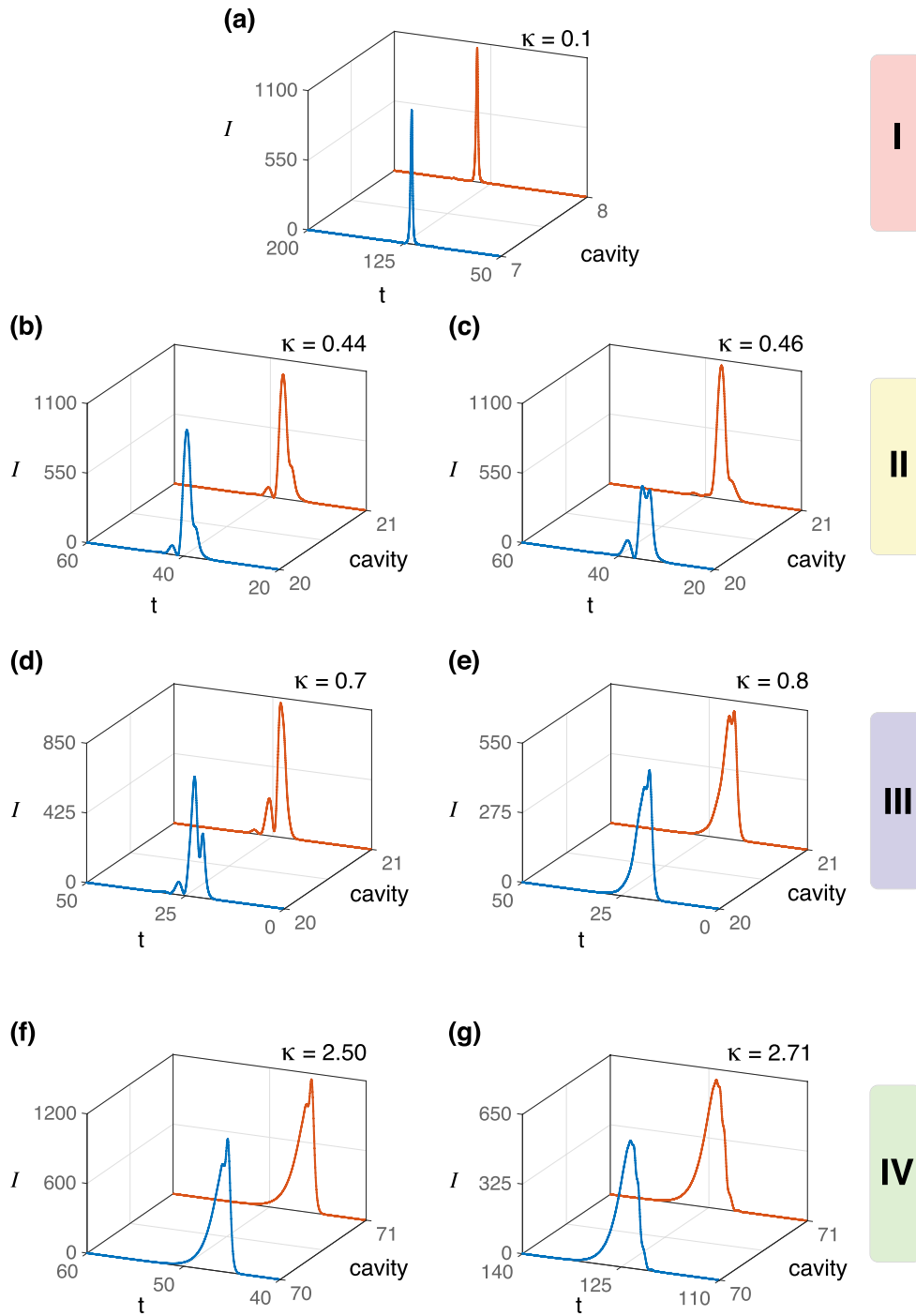
FIG. 4. Intensity of the electric field $I = |E_n|^2$ of two successive micropillars as a function of time for different evanescent coupling parameters $\kappa = 0.1$ (a), $\kappa = 0.44$ (b), $\kappa = 0.46$ (c), $\kappa = 0.7$ (d), $\kappa = 0.8$ (e), $\kappa = 2.5$ (f), and $\kappa = 2.71$ (g). Pulses spread to the right flank.

coupling time is large as compared to the photon cavity lifetime, which ensures that an excitable response can form before the energy couples to the neighboring laser. This corresponds to a saltatory propagation regime.[10] When kappa is larger, as will be shown below, the propagation mode deviates from the saltatory one. To test numerically the pulse propagation, we disturb a single laser with a perturbation amplitude value slightly above the excitable threshold (cf. Fig. 2). The first effect observed due to the discreteness of the excitable medium is that the propagation of the pulses proceeds through a hopping (or saltatory) dynamics. Namely, the lasers are turned on one by one while they emit an excitable spike, and when they go back to their off state, they excite the neighboring lasers. If the coupling strength is large enough, the leaking of energy from the initially perturbed laser to the neighboring ones can excite the neighbors and propagate the pulse. The process repeats giving rise to the observed hopping dynamics. Note that the propagation is unidirectional because of the refractory period exhibited by each excitable laser:[36,37] once the laser has fired a spike, it cannot be re-excited immediately; thus, there is a symmetry breaking of the excitable medium. This explains why the pulse that starts at one edge only propagates to a single flank.

III. PULSE SPEED AND BIFURCATION DIAGRAM CHARACTERIZATION

As we increase the value of the evanescent coupling parameter κ, the average pulse speed $\langle v \rangle$ increases. Figure 3 summarizes how the average pulse speed behaves as a function of κ. For small evanescent coupling values, we do not observe propagating pulses. The pulses appear by means of a saddle-node bifurcation from a critical coupling constant $\kappa \equiv \kappa_{sn}$. The saddle-node bifurcation is a generic mechanism of emergence of localized structures in several contexts such as nonlinear optics, plasma, and fluid.[45–49] The main features of this bifurcation are that solutions are only observed in a region of the parameter space and that a critical exponent is observed near the bifurcation for the growth rate as a function of the distance to the equilibrium. Figure 4 shows the intensity of the electric field in two successive lasers as a function of time for different evanescent coupling regions (I–IV) highlighted in Fig. 3. The pulse throughout region I is characterized by the fact that the intensity of the electric field is concentrated in a single laser [see Fig. 4(a)]. Numerically, we find for the parameters considered in Fig. 2 and close to the left edge (asymptotic limit of the speed) that the mean speed goes almost linearly with κ such that $\langle v \rangle = v_0 + v_1(\kappa - \kappa_{sn})^n$ with $v_0 = 0.061\,76$, $v_1 = 1.655$, $\kappa_{sn} = 0.1$, and $n = 0.9189$ [cf. Fig. 5(a)]. This regime corresponds to a solitary, ballistic regime.

However, close to $\kappa \equiv \kappa_t = 0.45$ (region II), we observe that there is a qualitative change in the average speed curve (see Fig. 3). Figure 5(b) shows a zoom of regions I and II. We note that the speed of the pulse varies continuously but is not differentiable at this critical point. To reveal the origin of this instability, the evolution of the electric field intensity in two successive lasers below and above the transition is shown in region II in Figs. 4(b) and 4(c). Note that below the bifurcation, the temporal profile of the pulse in the two successive lasers is identical. However, above the bifurcation, the temporal profiles in two successive lasers are not identical and they alternate. Therefore, this bifurcation corresponds
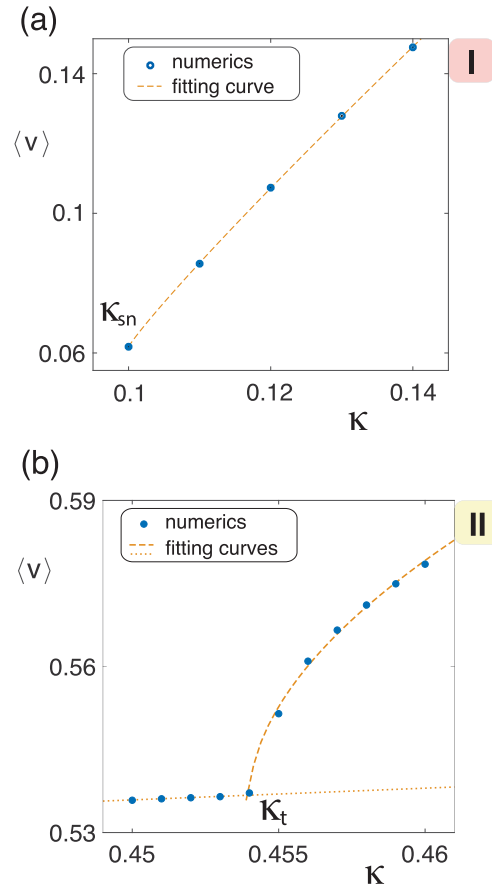

FIG. 5. Amplifications of the average pulse speed as a function of evanescent coupling κ of an excitable semiconductor laser model [Eqs. (1)–(3) with $\alpha = 2.0$, $\beta = 0$, $A = 2.74$, $B = 2$, $b_1 = b_2 = 0.001$, and $s = 10$. Points account for the pulse speed obtained numerically. (a) Emergence of hopping pulse solutions. The dashed curve is obtained by fitting $\langle v \rangle$ with $\langle v \rangle = v_0 + v_1(\kappa - \kappa_{sn})^n$ and $v_0 = 0.061\,76$, $v_1 = 1.655$, $\kappa_{sn} = 0.1$, and $n = 0.9189$. (b) Spontaneous translational symmetry-breaking instability. The dashed curve is obtained by fitting $\langle v \rangle$ with $\langle v \rangle = a_0 + a_1(\kappa - \kappa_t)^n$ and $a_0 = 0.5363$, $a_1 = 0.6659$, $\kappa_t = 0.454$, and $n = 0.5367$.

to a spontaneous translational symmetry-breaking. This transition occurs for both pulses that propagate to the right or left flank, and then one expects this bifurcation to be of the pitchfork type.[11–15] To characterize the pitchfork bifurcation, we fit in Fig. 5(b) the average speed with $\langle v \rangle = a_0 + a_1(\kappa - \kappa_t)^n$ with $a_0 = 0.5363$, $a_1 = 0.6659$, $\kappa_t = 0.454$, and $n = 0.5367$. The dependence is thus compatible with the expected square root law.

Likewise, to characterize the spontaneous translational symmetry-breaking bifurcation, we introduce the total intensity in the nth laser as an order parameter,

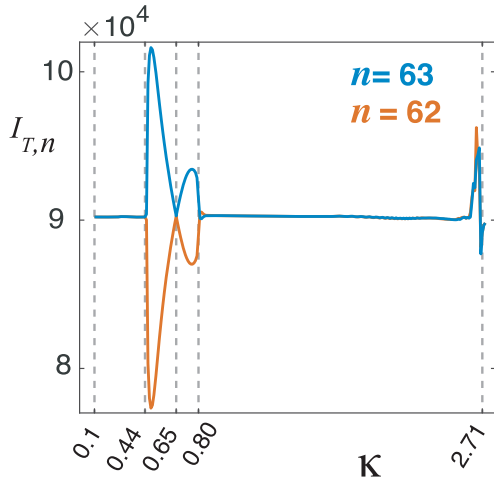$$I_{T,n} = \int dt |E_n|^2(t). \tag{4}$$

FIG. 6. Total intensity of the electric field $I_{T,n}$ in the n-micropillar as a function of evanescent coupling for a model [Eqs. (1)–(3)] with $\alpha = 2.0$, $\beta = 0$, $A = 2.74$, $B = 2$, $b_1 = b_2 = 0.001$, and $s = 10$.

Figure 6 shows the total intensity for two successive lasers as a function of the evanescent coupling parameter. Below the bifurcation, the total intensity between two successive lasers is identical. However, above the bifurcation, the total intensity is dissimilar, and

there is a concentration of energy in one of the lasers. The pulse intensities in successive lasers become almost identical at $\kappa = 0.65$; otherwise, there is a translational symmetry breaking for $0.44 < \kappa < 0.80$. From $\kappa = 0.80$ onward, the system recovers the spatial translation invariance (cf. Fig. 6). Increasing the value of the coupling κ, the propagative pulses persist for rather large values until they disappear by a saddle-node bifurcation for $\kappa \equiv \kappa_{sn}^+ = 2.71$. For κ greater than κ_{sn}^+, no pulses are observed (see Fig. 3). Physically, this is expected since the energy will flow to the neighboring cavities before reaching the excitable threshold. Therefore, propagation is not possible anymore.

It is worth noting that the continuous limit of the set of Eqs. (1)–(3) is obtained considering an infinitely large coupling constant ($\kappa \to \infty$). Therefore, we conclude from the previous observations that in the continuous limit the model [Eqs. (1)–(3)] has no propagating pulses. In order to support this conjecture, we analyze in the parameter space (κ, A) the region where the pulses with hopping dynamics are observed. The results are shown on Fig. 7. As the gain pump parameter A decreases, the window of coupling constants where propagating pulses are observed shrinks. Below $A = 2$, no propagating pulses are observed since the system is not excitable anymore. Similar bifurcation diagrams are observed for the average pulse speed for different evanescent coupling constants and pump gain as shown in Fig. 7(b). Interestingly, the mean speed can also be controlled through the pump[8] as can be seen in Fig. 7(c). This is a very important feature from an experimental point of view since the coupling constant is often fixed by fabrication. The yellow shadowed
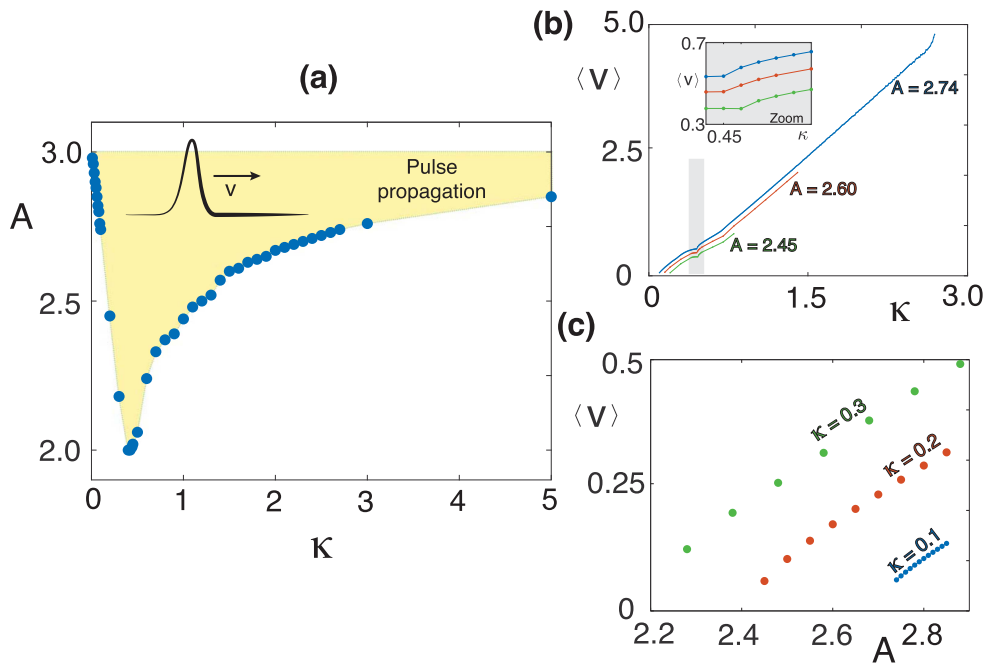

FIG. 7. Phase diagram of the pulse obtained for the model [Eqs. (1)–(3)] with $\alpha = 2.0$, $\beta = 0$, $B = 2$, $b_1 = b_2 = 0.001$, and $s = 10$. (a) Pulse phase diagram in the A–κ space. Points account for the limits of pulse obtained numerically. The average pulse speed as a function of evanescent coupling κ (b) and pump gain A (c).

region shows the region where the propagating pulses are observed. Note that for a finite pump gain in the excitable region ($2 < A < 3$) and in the continuous limit, no propagating pulses are expected.

The continuous limit of the set of Eqs. (1)–(3) plus diffusive coupling was studied in Refs. 50 and 51. Propagation of pulses is shown for non-radiative carrier recombination rates of the same order as the one for the electric field ($b_1 \sim b_2 \sim 1$). For semiconductor micropillar lasers, these rates differ from several orders of magnitude since the electric field decay time is much smaller than the carrier decay times. Hence, for the typical parameters of micropillar lasers, it is not possible to observe pulses in the continuous limit.

IV. CONCLUSION

Continuous spatially extended excitable systems can sustain the propagation of pulses. This phenomenon is intuitively based on the fact that the system without spatial coupling is excitable; that is, if an equilibrium suffers a sufficiently large disturbance, the dynamical system exhibits large excursions in the basin of attraction. When considering the spatially extended system, one expects that when a region is perturbed, it will excite the surrounding areas generating the emission of pulses or waves. The persistence of this phenomenon for coupled (discrete) excitable systems is expected. However, the phenomenon of pulse propagation in discrete, coupled excitable systems is not obvious and depends on the coupling. We show that pulse propagation in discrete, coupled excitable lasers is characterized by a hopping dynamics and that it presents a rich bifurcation structure. We also show that the observed pulses do not necessarily persist in the continuous limit. These results pave the way for the experimental study of such hopping dynamics in optics with a potential impact on neuromimetic systems and information processing.[8]

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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