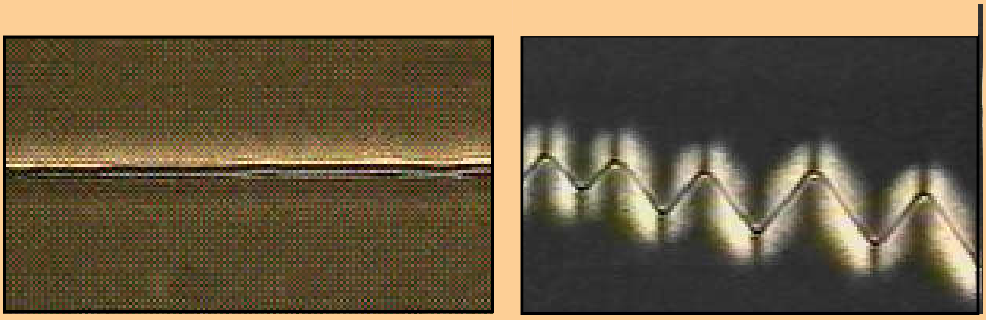
Zig-zag Instability of an Ising Wall in Liquid Crystals



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Experimental Set-up

Nematic liquid crystal cell

5 CB

d ~100mm

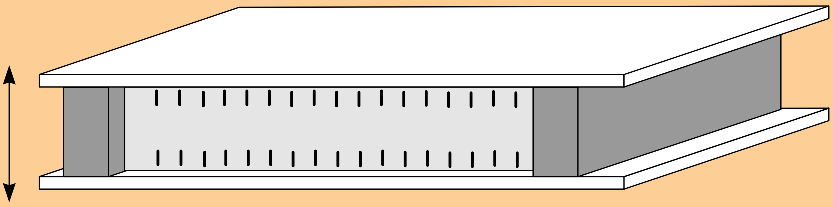
c_a =1.142

e_a =11.3

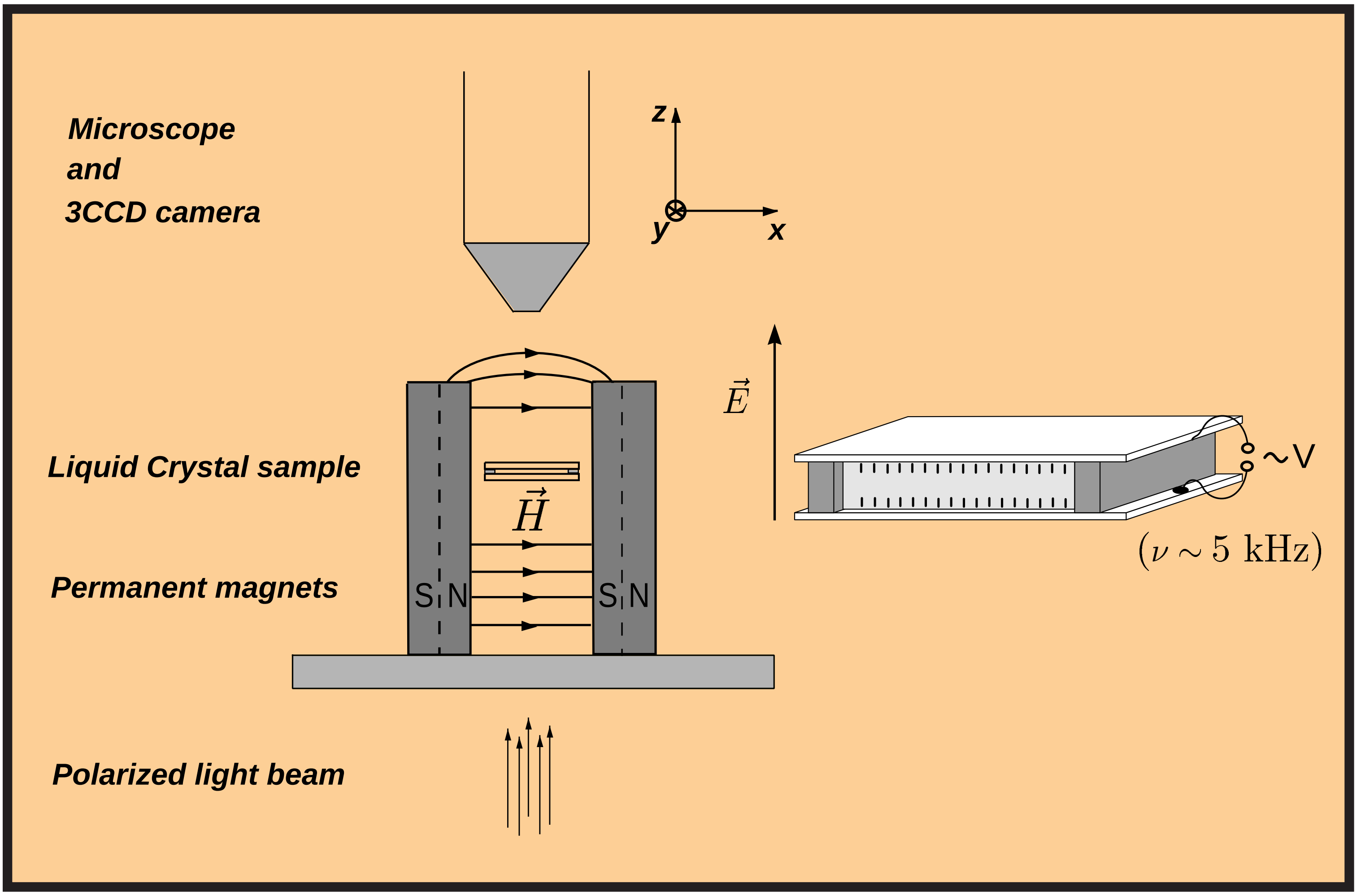
$K_1 = 6\ 3$

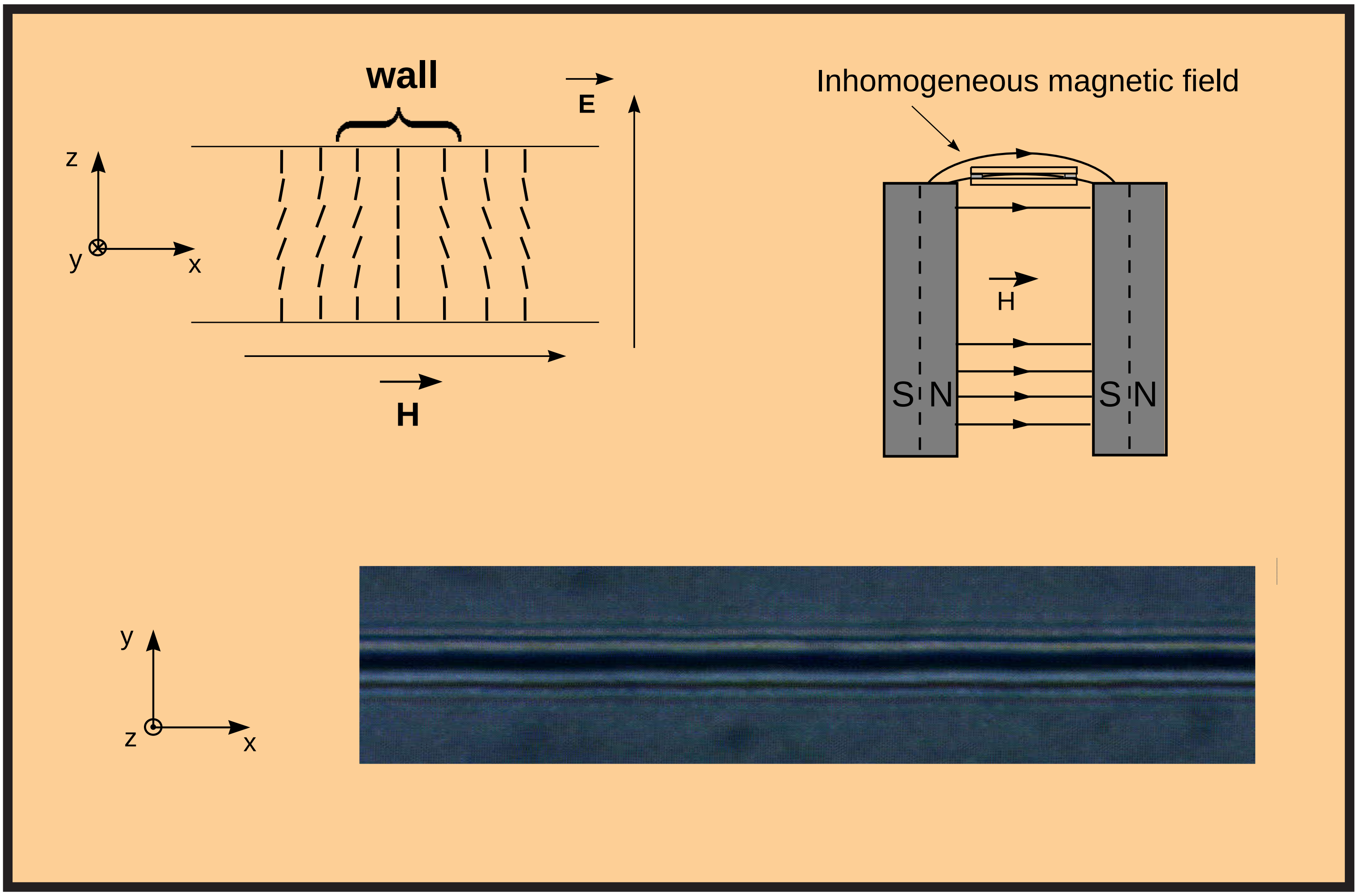
$K_2 = 4\ 1 \quad (10^{-12}\mathrm{N})$

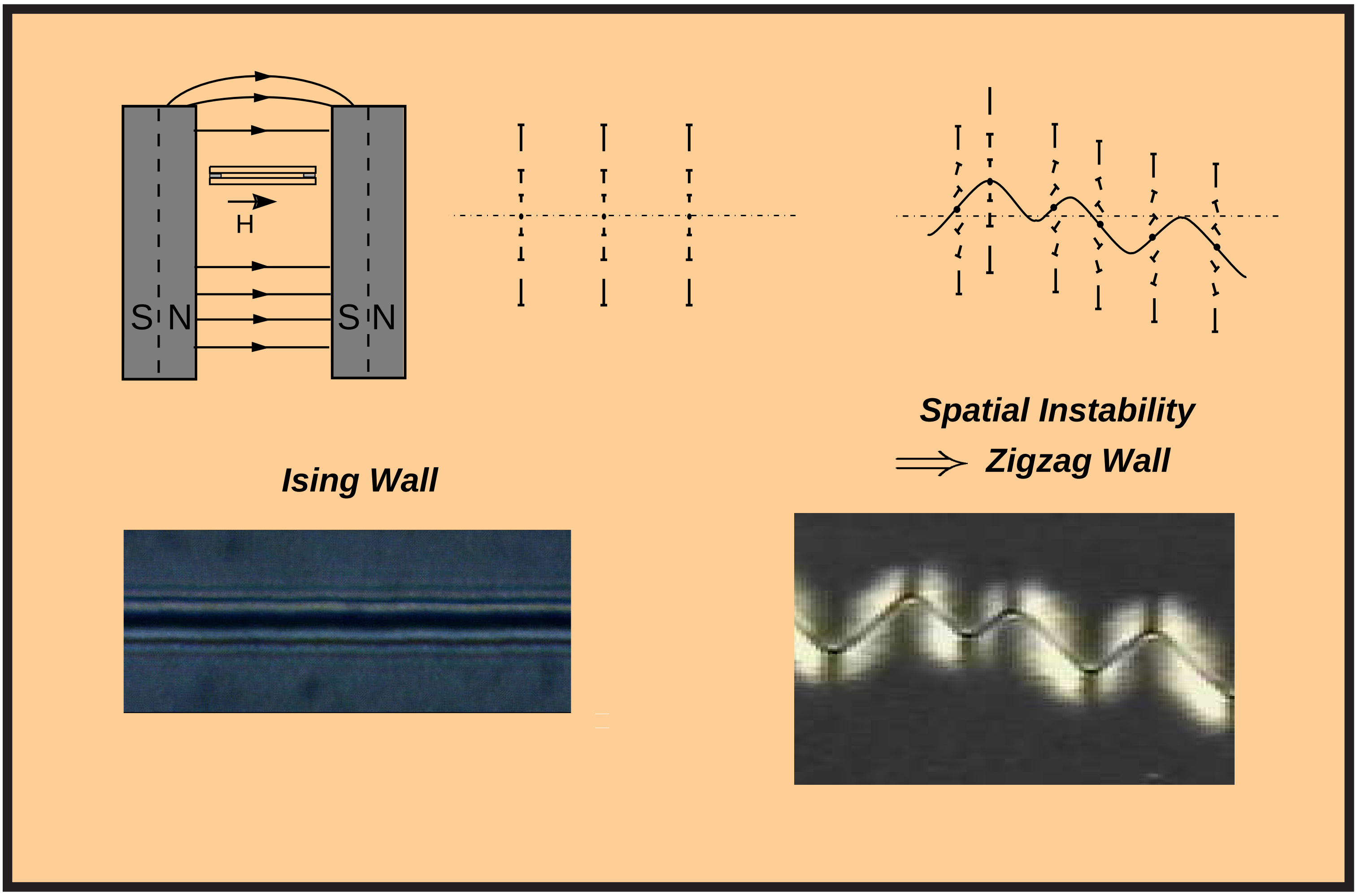
$K_3 = 8\ 4$

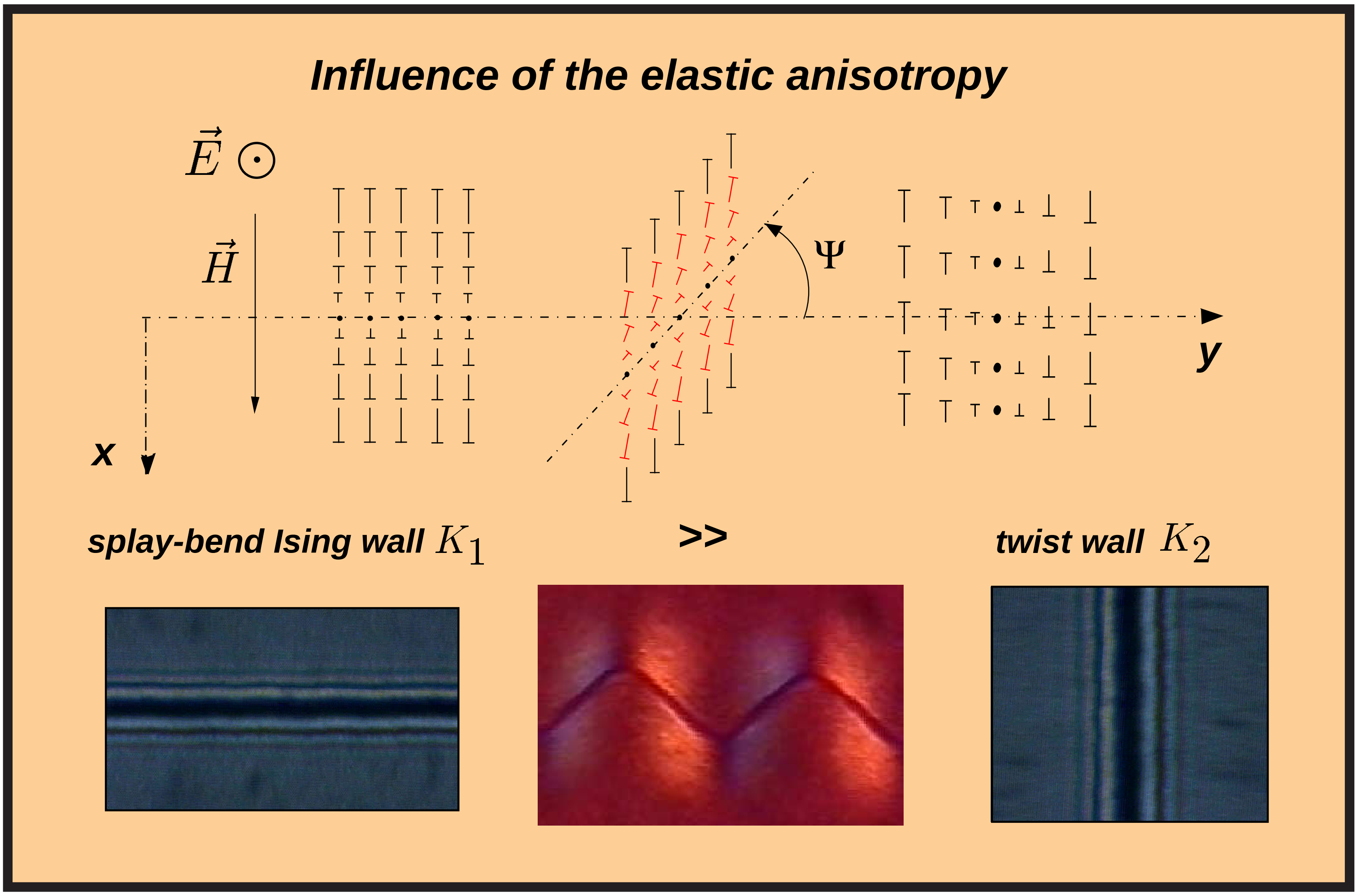


Homeotropic anchoring


Microscope
and
3CCD camera
z
y
x
$\vec{E}$
~V
Liquid Crystal sample
$\vec{H}$
$(\nu \sim 5$ kHz$)$
Permanent magnets
S N
S N
Polarized light beam


wall
E
Inhomogeneous magnetic field
z
y
x
H
S N
S N
y
z
x


H
S N
S N
Ising Wall
Spatial Instability
⟹ Zigzag Wall


Influence of the elastic anisotropy
$\vec{E}$ ⊙
$\vec{H}$
x
y
Ψ
splay-bend Ising wall K_1
>>
twist wall K_2

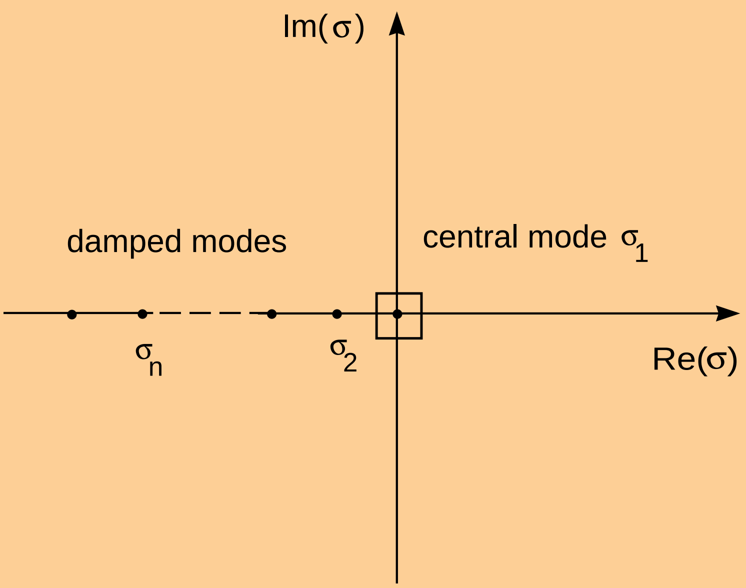
Derivation from the Frank free energy

$$\frac{\partial \vec{n}}{\partial t} = -\frac{\delta \mathcal{F}}{\delta \vec{n}}\,, \qquad \vec{n}\cdot\vec{n} = 1\,,$$

where

$$\mathcal{F} = \int \frac{1}{2}\Big[K_1(\vec{\nabla}\vec{n})^2 + K_2(\vec{n}.(\vec{\nabla}\times\vec{n}))^2 + K_3(\vec{n}\times(\vec{\nabla}\times\vec{n}))^2 - \epsilon_a(\vec{n}.\vec{E})^2 - \chi_a(\vec{n}.\vec{H})^2\Big]\, d\tau$$

For not too high elastic anisotropy, the magnetically-induced Fréedericksz transition is characterized by a unique marginal mode which is the first Fourier mode along the z-axis.



$$\begin{cases} n_x = X\cos\left(\frac{\pi z}{d}\right) \\ n_y = Y\cos\left(\frac{\pi z}{d}\right) \\ n_z = 1 - \frac{X^2+Y^2}{2}\cos\left(\frac{\pi z}{d}\right)^2 \end{cases}$$

The order parameter to be considered is a scalar parameter $Z(x,y,t)$ ***.***

Using the following change of variable

$$\begin{pmatrix} X \\ Y \end{pmatrix} = Z \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{K_1 - K_2}{2\gamma_c} \partial_{xy} Z \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{K_1 - K_2}{4\gamma_c^2} \left(K_2 \partial_x^2 + K_1 \partial_y^2 \right) \partial_{xy} Z \begin{pmatrix} 0 \\ 1 \end{pmatrix} + h.o.t.$$

one get the dynamical equation for the scalar order parameter $Z(x, y, t)$

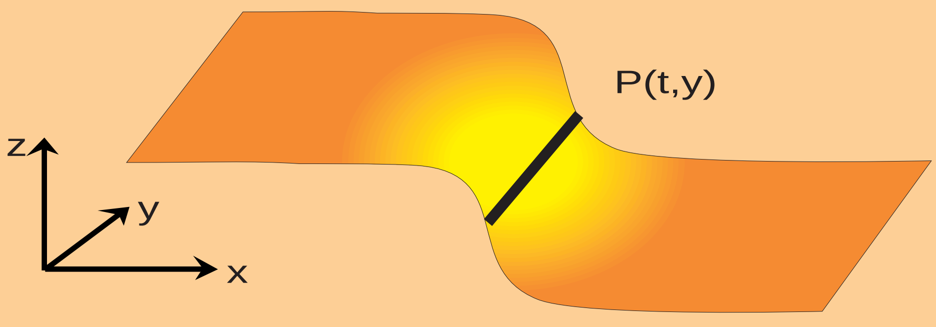
near the Fréedericksz transition threshold (Landau Eq.) :

$$\gamma_1 \partial_t Z = \epsilon^2 Z - aZ^3 + (K_1 \partial_x^2 + K_2 \partial_y^2) Z$$

where

$$\epsilon^2 = \chi_a H^2 - \epsilon_a E^2 - K_3 \frac{\pi^2}{d^2} \;, \qquad a = \frac{1}{2}(K_1 - \frac{3}{2} K_3) \frac{\pi^2}{d^2} - \frac{3}{4} \epsilon_a E^2 + \frac{3}{4} \chi_a H^2$$

Interface dynamics

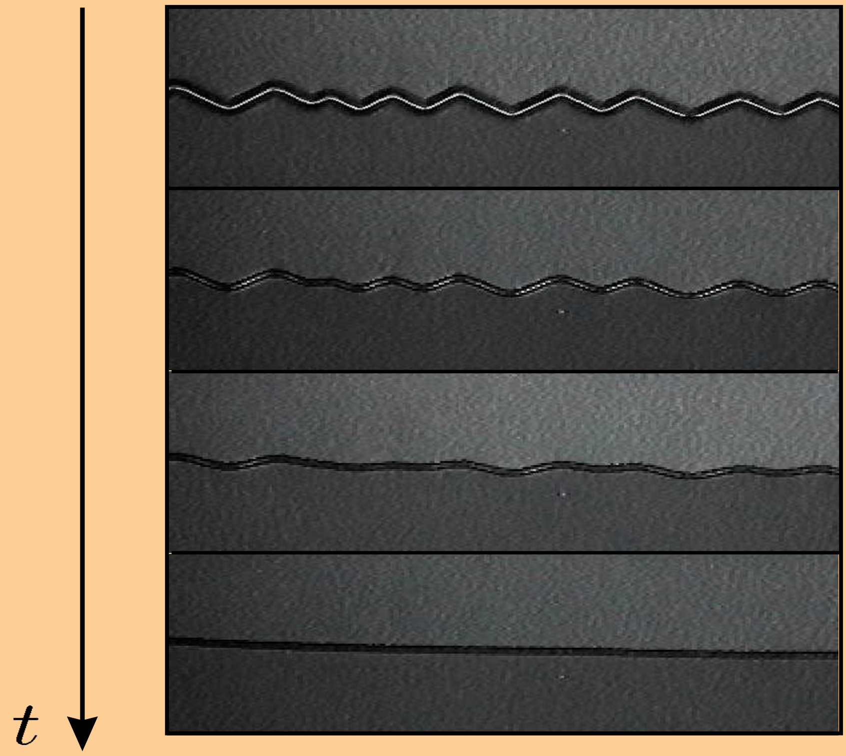


$Z_o = \frac{\epsilon}{\sqrt{a}} \tanh\left(\frac{\epsilon}{\sqrt{2}} (x - P) \right)$ ***is a solution of the previous equation that accounts for the splay-bend Ising wall.***

In order to investigate the interface dynamics, one has to introduce the following expression :

$$Z(x, y, t) = \frac{\epsilon}{\sqrt{a}} \tanh\left(\frac{\epsilon}{\sqrt{2}} (x - P(y, t)) \right) + \eta(x - P, P)$$

where $\eta(x-P,P)$ *is a wall perturbation. This can be a solution if* P *checks the next equation*



$$\partial_t P = k_2 P_{yy}$$

- **In the anysotropic case** $(K_2 << K_1)$

$$\partial_t Z = \epsilon Z - a Z^3 + K_1 \partial_x^2 Z + K_2 \partial_y^2 Z + \frac{3}{4}\left(Z (\partial_y Z)^2 - \frac{Z^2}{2} \partial_{y^2} Z \right)$$
$$+ \frac{(K_1 - K_2)^2}{2\gamma_c} \partial_{x^2 y^2} Z + \frac{(K_1 - K_2)^2}{4\gamma_c^2} K_1 \partial_{x^2 y^4} Z$$

where $\gamma_c = \frac{1}{2} \chi_a H_c^2 \quad (\epsilon = 0 \text{ for } H = H_c)$

K_1, K_2, K_3 **are the splay, twist and bend elastic constants**

γ_1 **is the rotational viscosity.**

Interface dynamics

$$Z_o = \frac{\epsilon}{\sqrt{a}} \tanh\left(\frac{\epsilon}{\sqrt{2}}(x-P)\right)$$

$$\partial_t P = D_2 P_{yy} + D_3 P_y^2 P_{yy} - D_4 P_{4y}$$

where

$$\begin{cases} D_2 &= K_2 - \dfrac{2 K_1 \epsilon}{5\ \gamma_c} + \dfrac{3 K_3 \epsilon}{40b} \\ D_3 &= \dfrac{48\epsilon}{7{\gamma_c}^2} {K_1}^2 \\ D_4 &= \dfrac{2\ \epsilon}{5\ \gamma_c^2} K_1^2 \end{cases}$$

- $\partial_t P = -\dfrac{\delta \mathcal{F}[P]}{\delta P}$ **with** $\mathcal{F}[P] = \int \left[\frac{\epsilon}{2} P_y^2 + \frac{1}{12} P_y^4 + \frac{1}{2} P_{yy}^2\right] dy$

  $\Rightarrow$ ***relaxational dynamics***

- ***The preceding equation is a continuity equation :***

$$\partial_t P = \partial_y(\epsilon P_y + \frac{1}{3} P_y^3 - P_{3y})$$

In a infinite medium, this corresponds to the conservation of the global quantity

$$\int P\ dy$$

- ***With the new variable*** $\Lambda = P_y$ ***, this reads***

$$\partial_t \Lambda = \partial_{yy}(\epsilon\Lambda + \frac{1}{3}\Lambda^3 - \Lambda_{yy}) = \partial_{yy}\frac{\delta\mathcal{F}[\Lambda]}{\delta\Lambda}$$

where $$\mathcal{F}[\Lambda] = \int \left[\frac{\epsilon}{2}\Lambda^2 + \frac{1}{12}\Lambda^4 + \frac{1}{2}\Lambda_y^2\right] dy$$

This is a Cahn-Hilliard equation for a one-dimensional system.

- ***Variational problem under constraint***

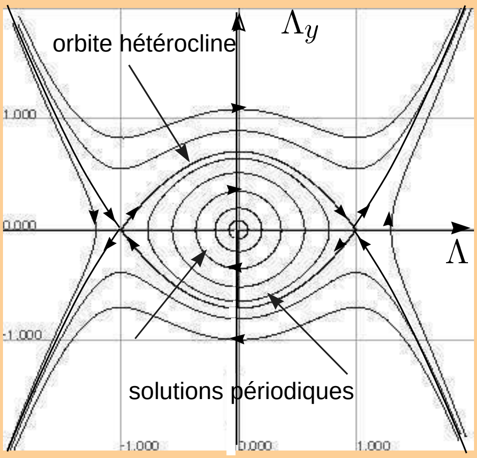
$$\begin{cases} \partial_t\Lambda = \partial_{yy}\left[\frac{\delta\mathcal{F}}{\delta\Lambda}\right] \\ \mathcal{G}[\Lambda] = \int \Lambda \; dy = M \end{cases} \Leftrightarrow \begin{cases} \frac{\delta\mathcal{F}[\Lambda]}{\delta\Lambda} + \lambda\frac{\delta\mathcal{G}[\Lambda]}{\delta\Lambda} = 0 \\ \int \Lambda \; dy = M \end{cases}$$

with λ *Lagrangian multiplier*

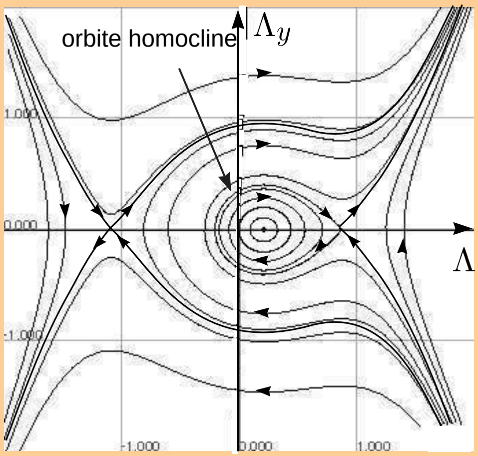
- ***Stationary solutions***

$$\partial_t P = 0 \Rightarrow -P_y + {P_y}^3 - P_{3y} = K$$
$$\Rightarrow -\Lambda + \Lambda^3 - \Lambda_{yy} = K$$

$K = 0$

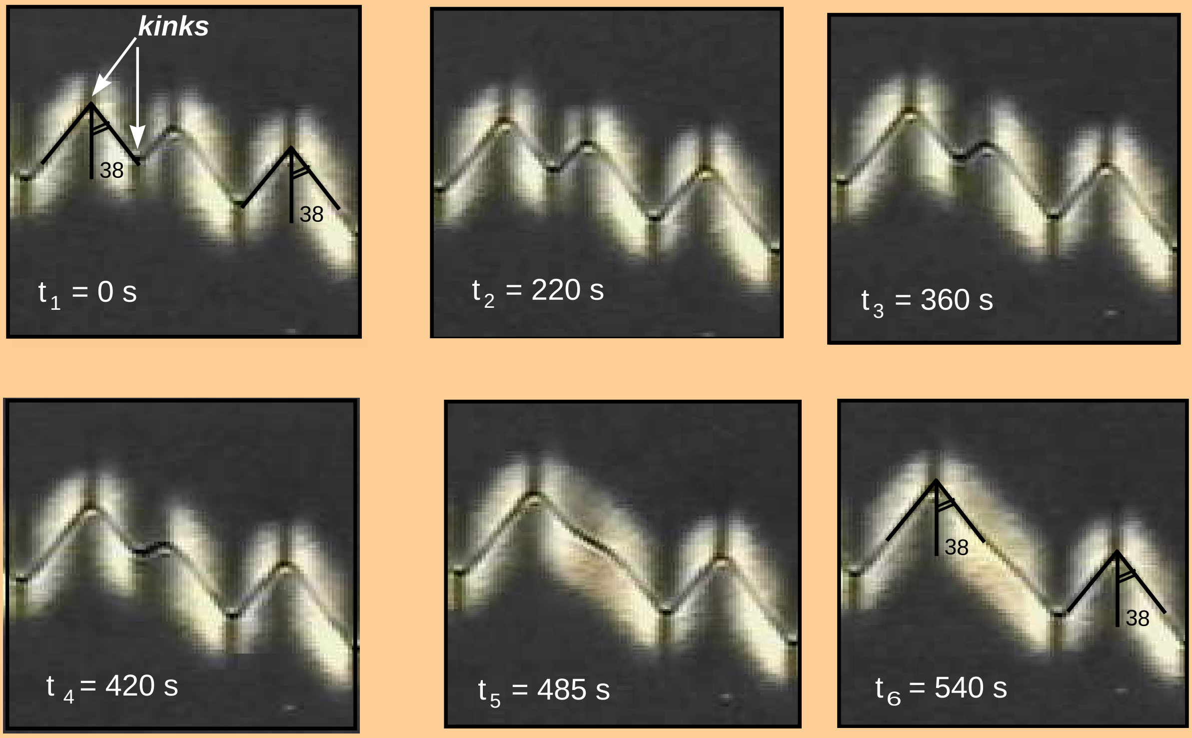


$K \neq 0$



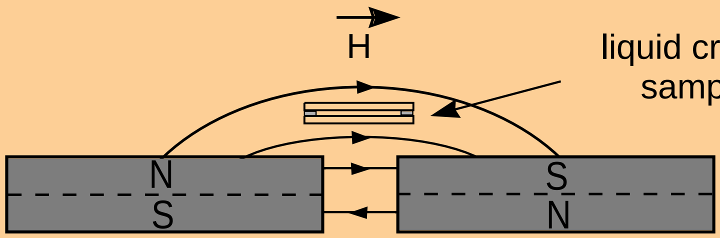
- Domain dynamics

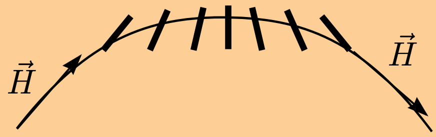
$$\partial_t P = \epsilon P_{yy} + P_y^2 P_{yy} - P_{4y}$$

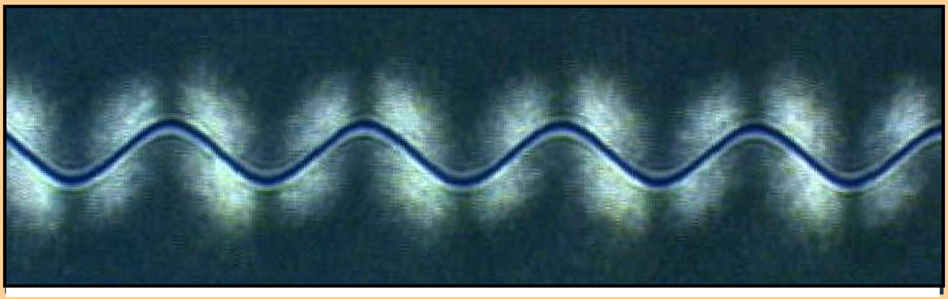


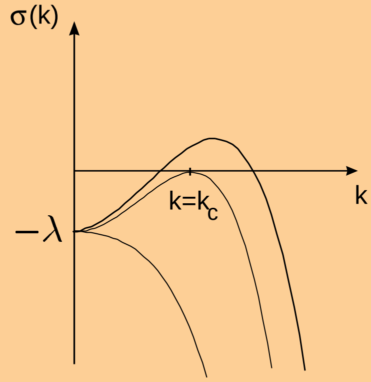
- ***Numerical simulations***

Loss of the translational invariance $(x \leftrightarrow x + x_o)$







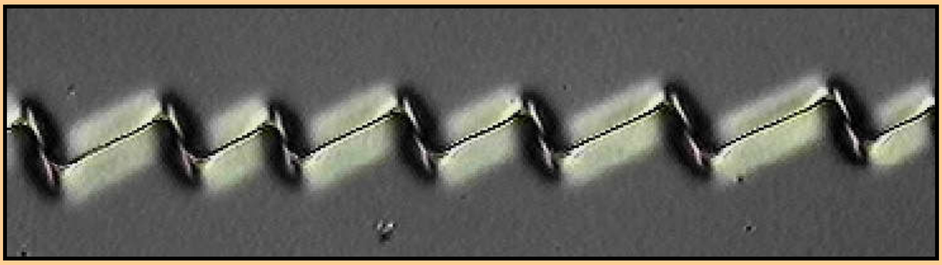


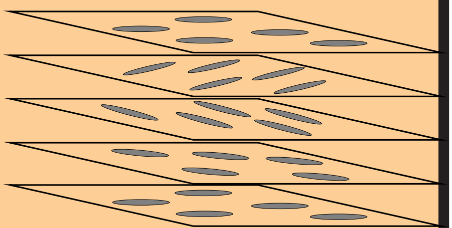
$$\partial_t P = -\lambda P + \epsilon P_{yy} + P_y^2 P_{yy} - P_{4y}$$

$$\lambda \propto \frac{\Delta H_z}{L}$$

Loss of the reflection symmetry $(y \leftrightarrow -y)$

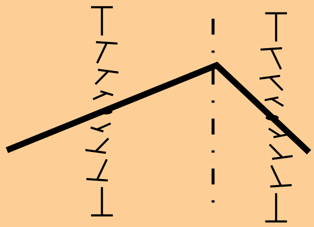
Experiment with slightly cholesteric material





5 CB doped with ZLI-811

$$\partial_t P = \varepsilon' P_{yy} - \beta P_y P_{yy} + P_y^2 P_{yy} - P_{yyyy}$$



where β depend on the chirality χ of the cholesteric material