Localized states beyond asymptotic parametrically driven amplitude equation,

Departamento de Fisica.
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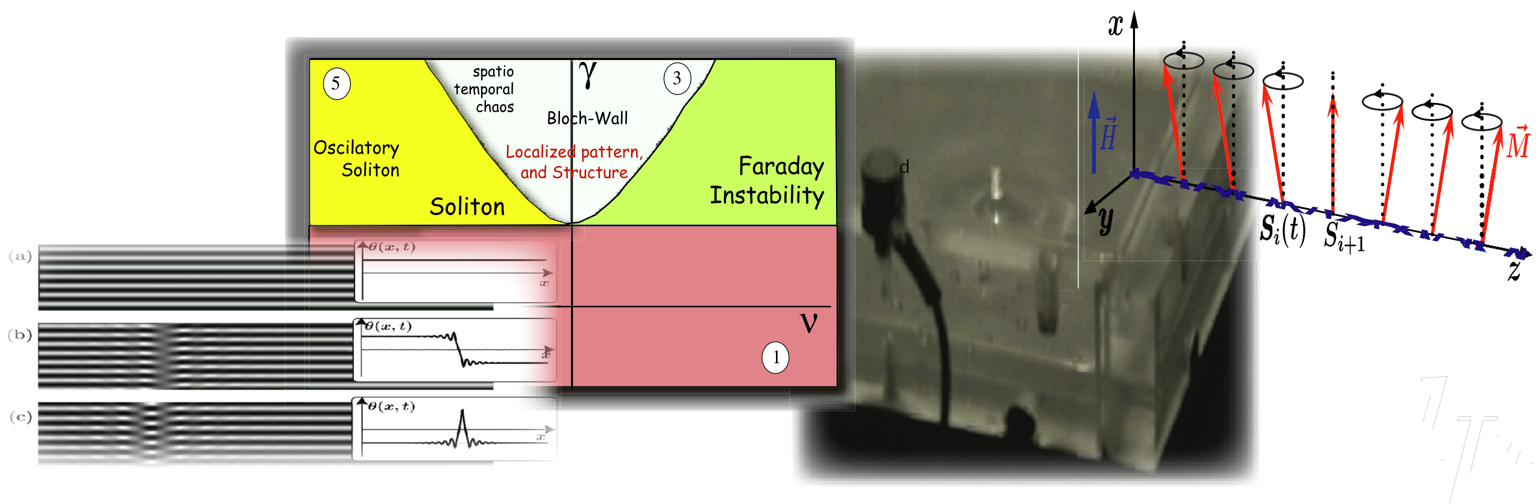

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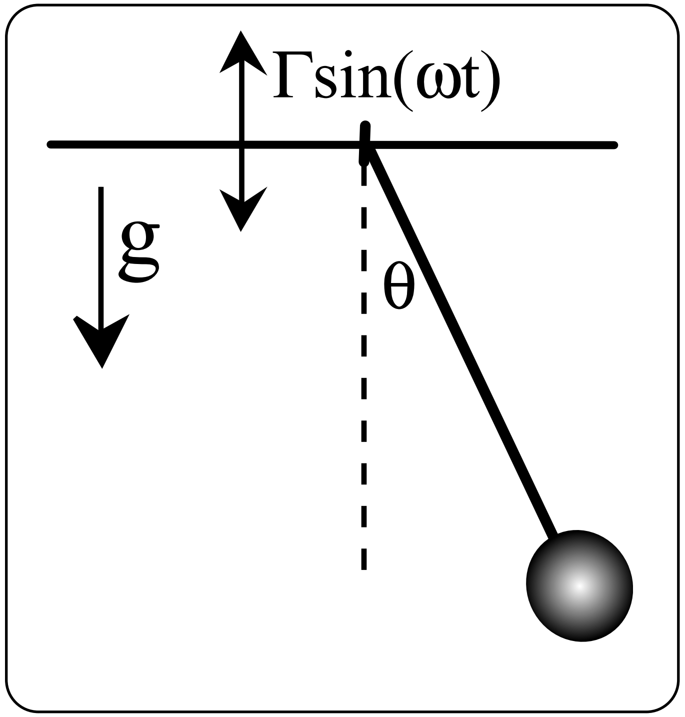
Outline

- Parametrically driven systems,
- Parametrically driven pendula chain and particle type solutions,
- Parametrically driven damped non-linear Schrodringer equation,
- Amended amplitud equation,
- Aplication: Magnetic wire,
- Conclusions

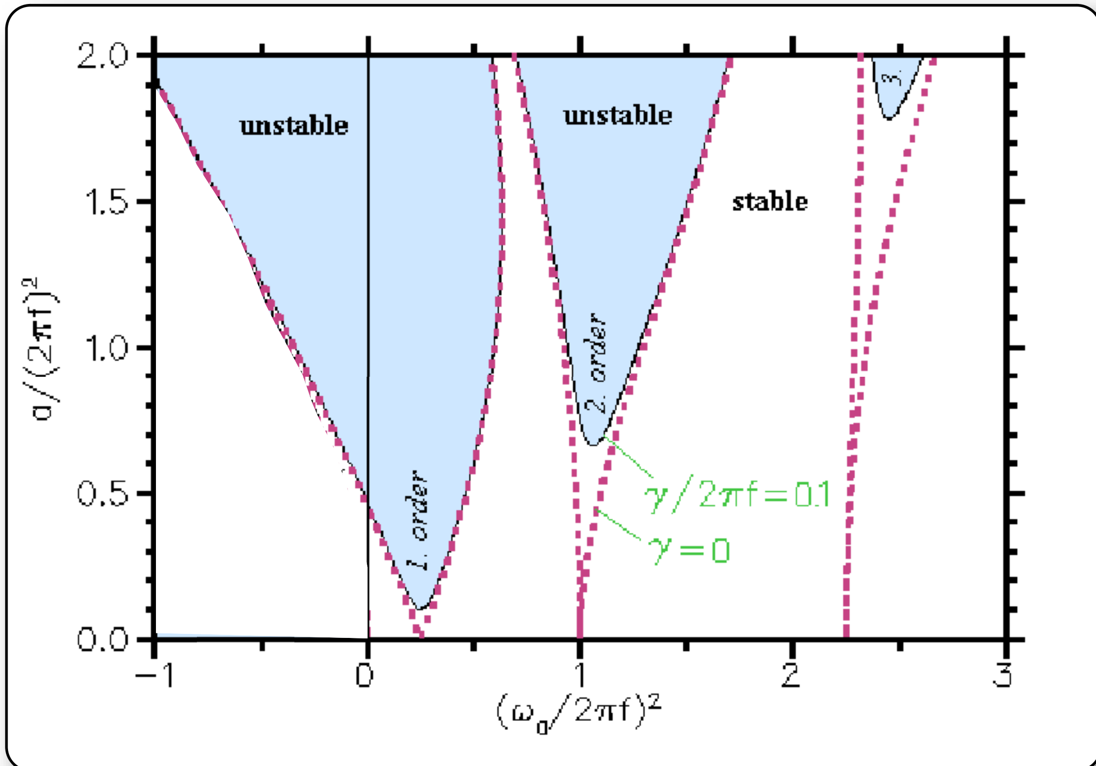
Example of parametrical instability: vertically driven pendulum



- The pendulum is described by

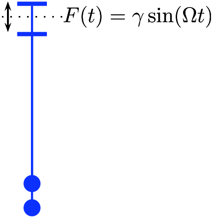
$$\ddot{\theta}(x,t) = -\left(\omega_o^2 + \gamma \sin(\omega t)\right) \sin(\theta) - \mu\dot{\theta}$$

where $\omega_o = \sqrt{\frac{g}{l}}$



- The vertical solution θ=0 is unstable (Arnold's tongue)

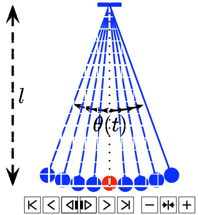
- Pendulo simple con forzaje vertical



- Pendulo simple con forzaje vertical

Ecuación de Mathieu:

$$\ddot{\theta} = -\left(\frac{g}{l} + \gamma \sin(\Omega t)\right)\sin\theta - \mu\dot{\theta} \quad (1)$$



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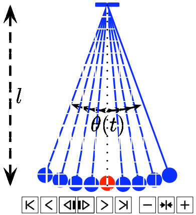
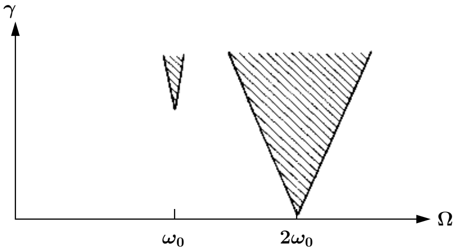
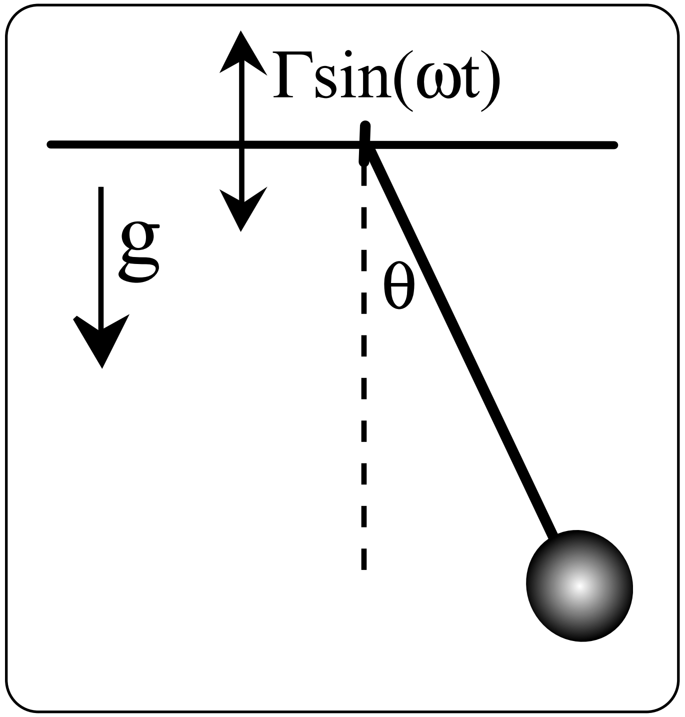




Inestabilidades paramétricas: Lengua de Arnold

Example of parametrical instability: vertically driven pendulum



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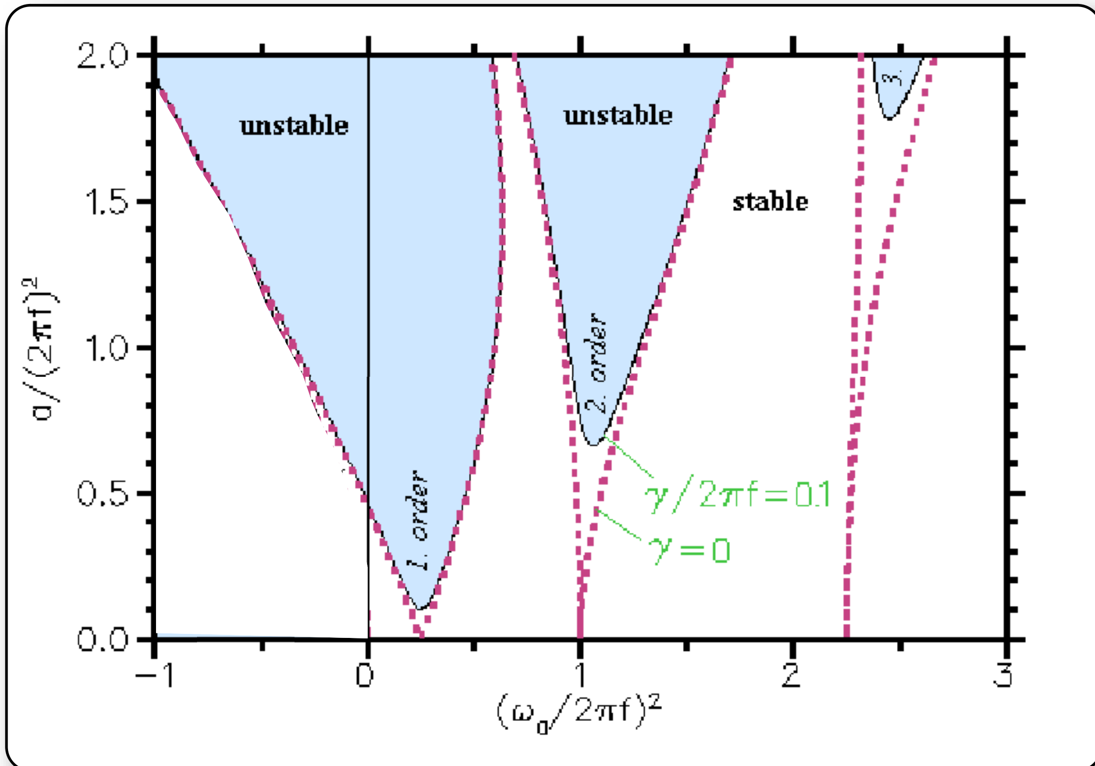

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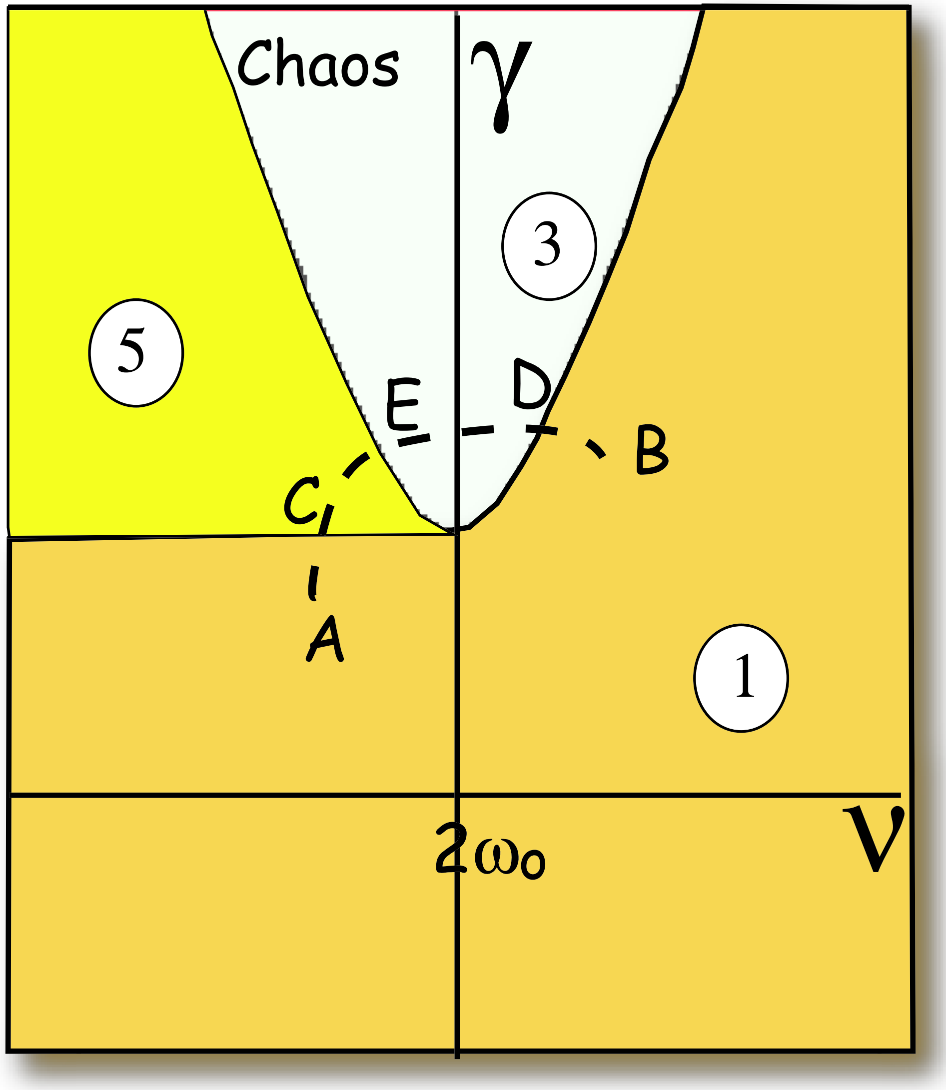
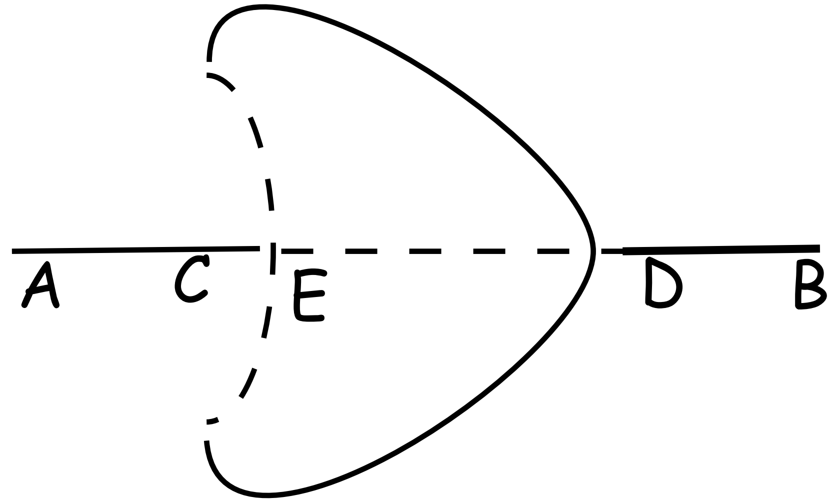
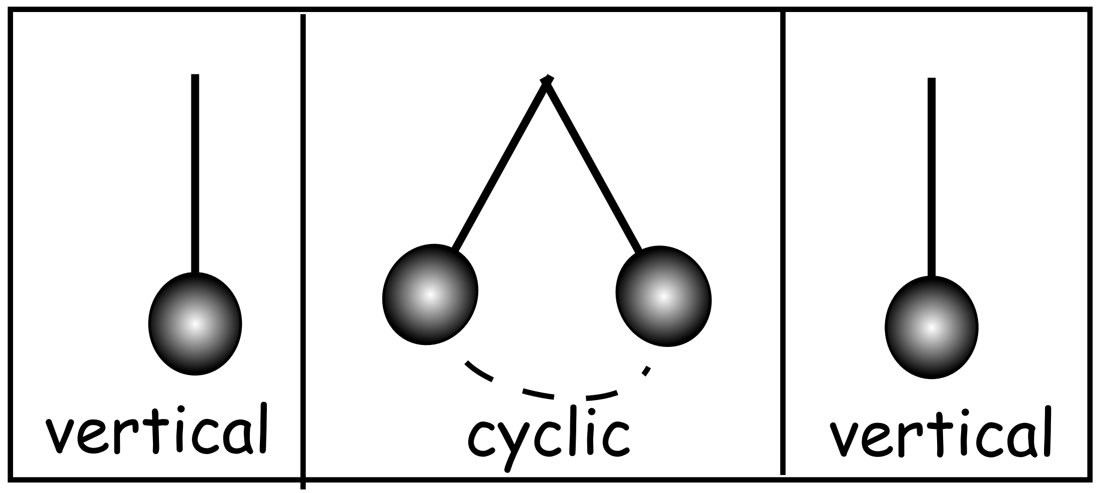
Bifurcation diagram close to subharmonic instability



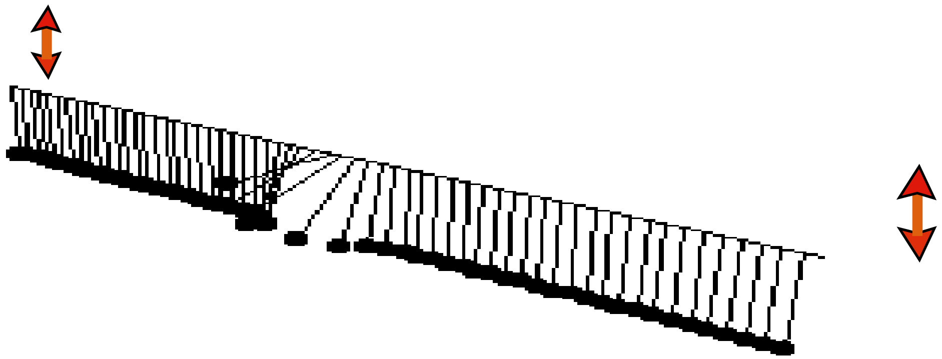
Stationary solutions

- Bifurcation diagram at the curve AB





Parametrically driven damped pendula chain



In the continuos limit this system is decribe by

$$\ddot{\theta}(x,t) = -\left(\omega_o^2 + \gamma\sin(\omega t)\right)\sin(\theta) - \mu\dot{\theta} + k\partial_{xx}\theta,$$

where ω_0 is the natural frequency, γ and ω are the amplitude and frequency of forcing, μ is the damped and k is coupled constant.

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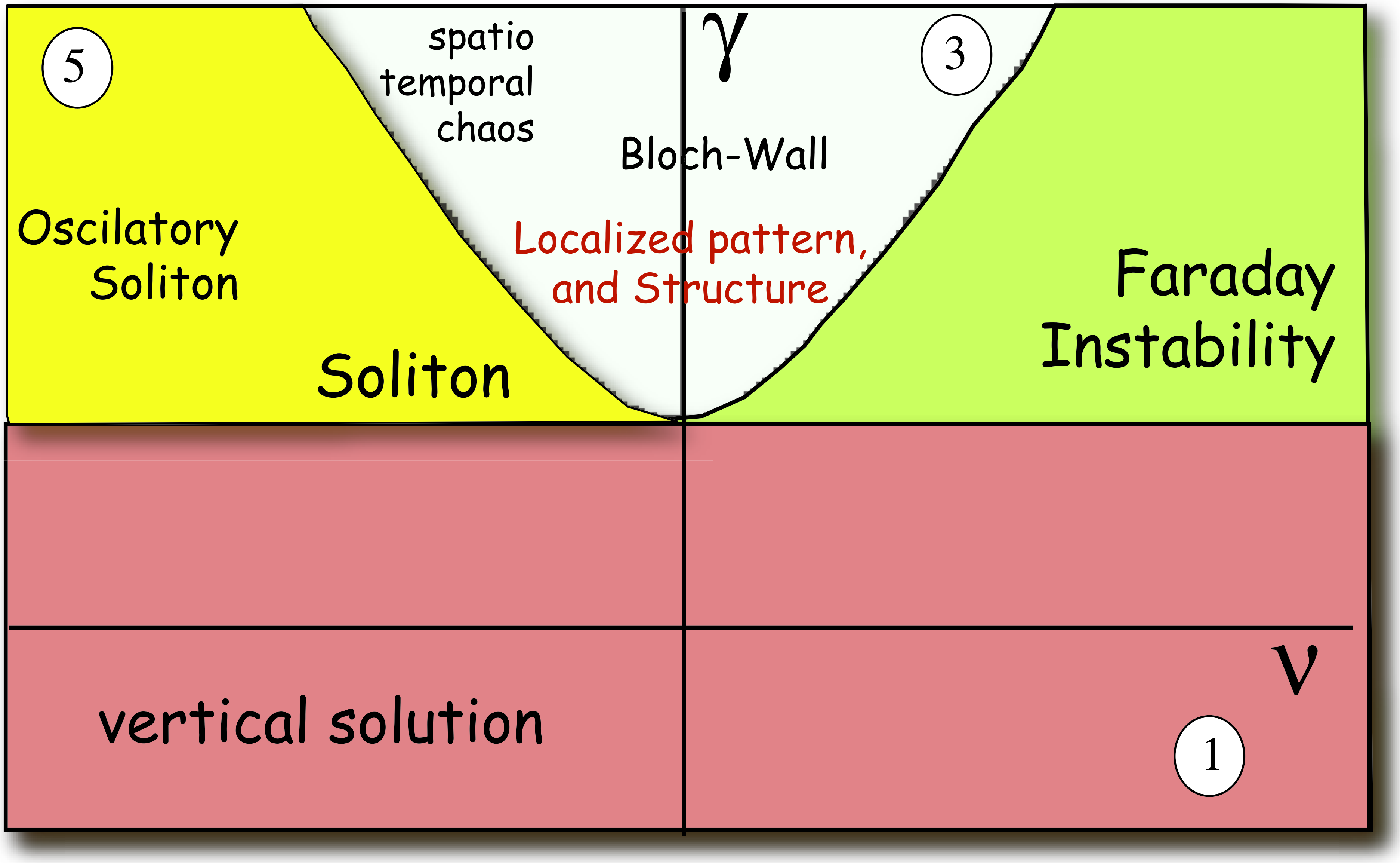
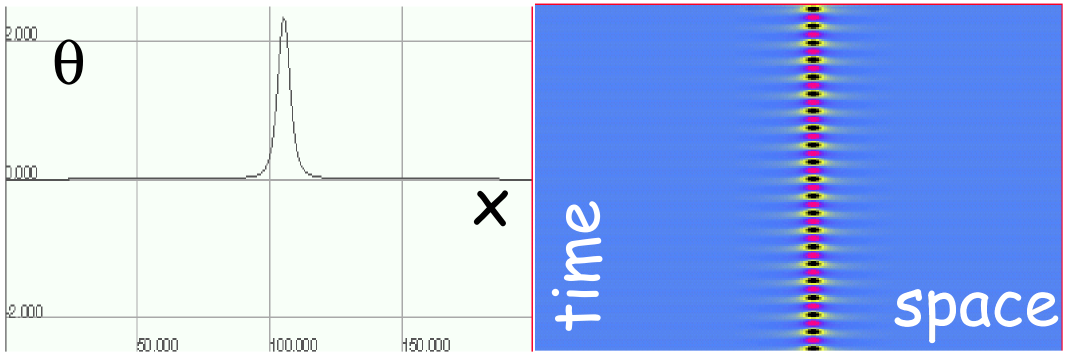
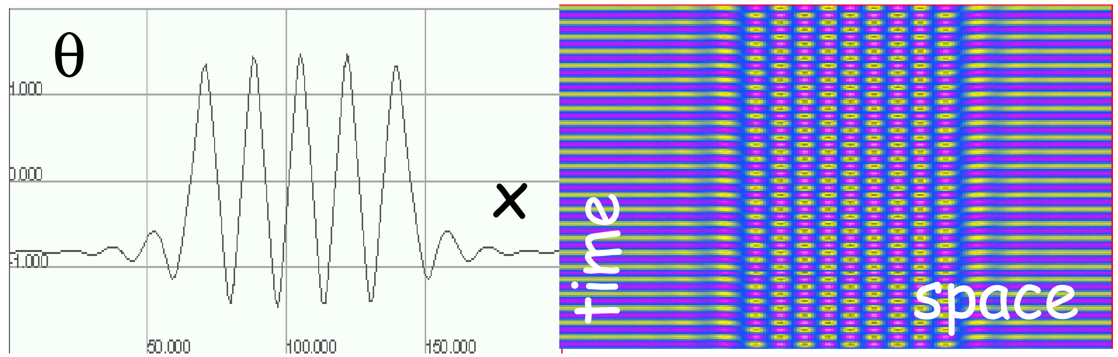


Particle type solutions

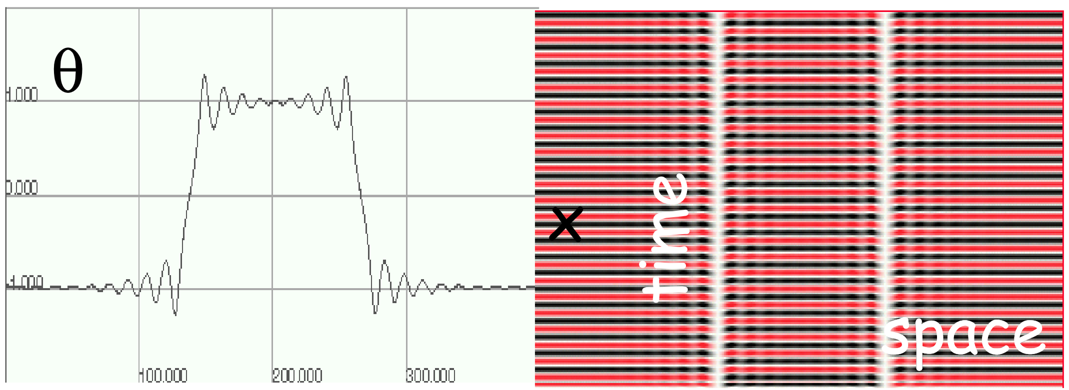
Soliton



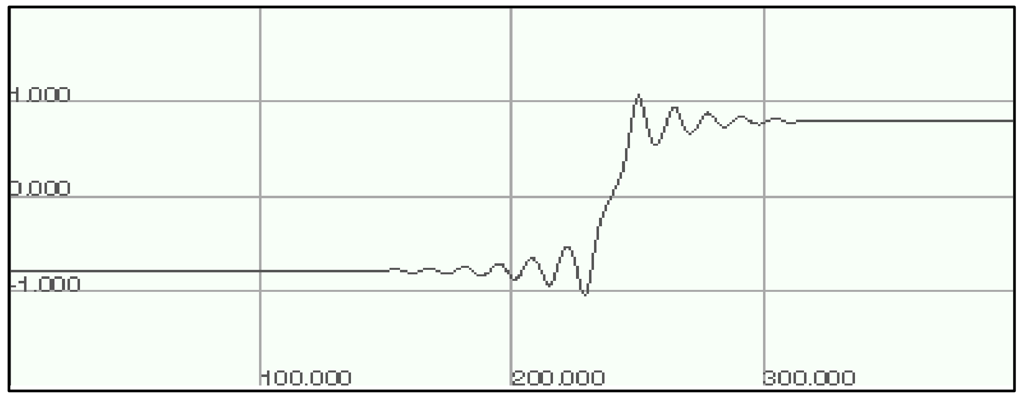
Localized patterns



Localized Structure



Kink or wall



Amplitude equation

The above model can be simplified if we restrict ourselves to the small amplitude solutions, whose main harmonic frequency is close to ω_0. Intoducing the ansatz

$$\theta(x,t) = A(T,X)e^{i(1+\nu)t} - \frac{A^3(T,X)}{48}e^{i3(1+\nu)t} + \frac{|A(T,X)|^2 A}{8}e^{i3(1+\nu)t} + c.c + h.o.t$$

where $\omega_o = 1,\ \omega = 2(1+\nu),\ T = \nu t,$ and $X = \sqrt{\nu}x$ The amplitude satisfies the parametrically driven and damped non-linear Schrodiger equation

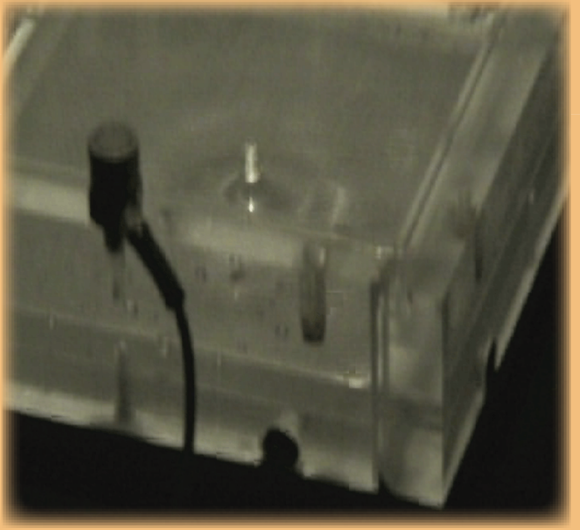
$$\underbrace{\partial_t A = -i\nu A - i\frac{|A|^2}{4}A - \frac{i}{2}\partial_{xx}A}_{\text{Nonlinear Schrodinger equation}} - \mu A - \frac{\gamma}{4}\bar{A}$$

Nonlinear Schrodinger equation

The parametrically driven and damped nonlinear schrodinger equation is a model used to describe pattern and soliton in various media:

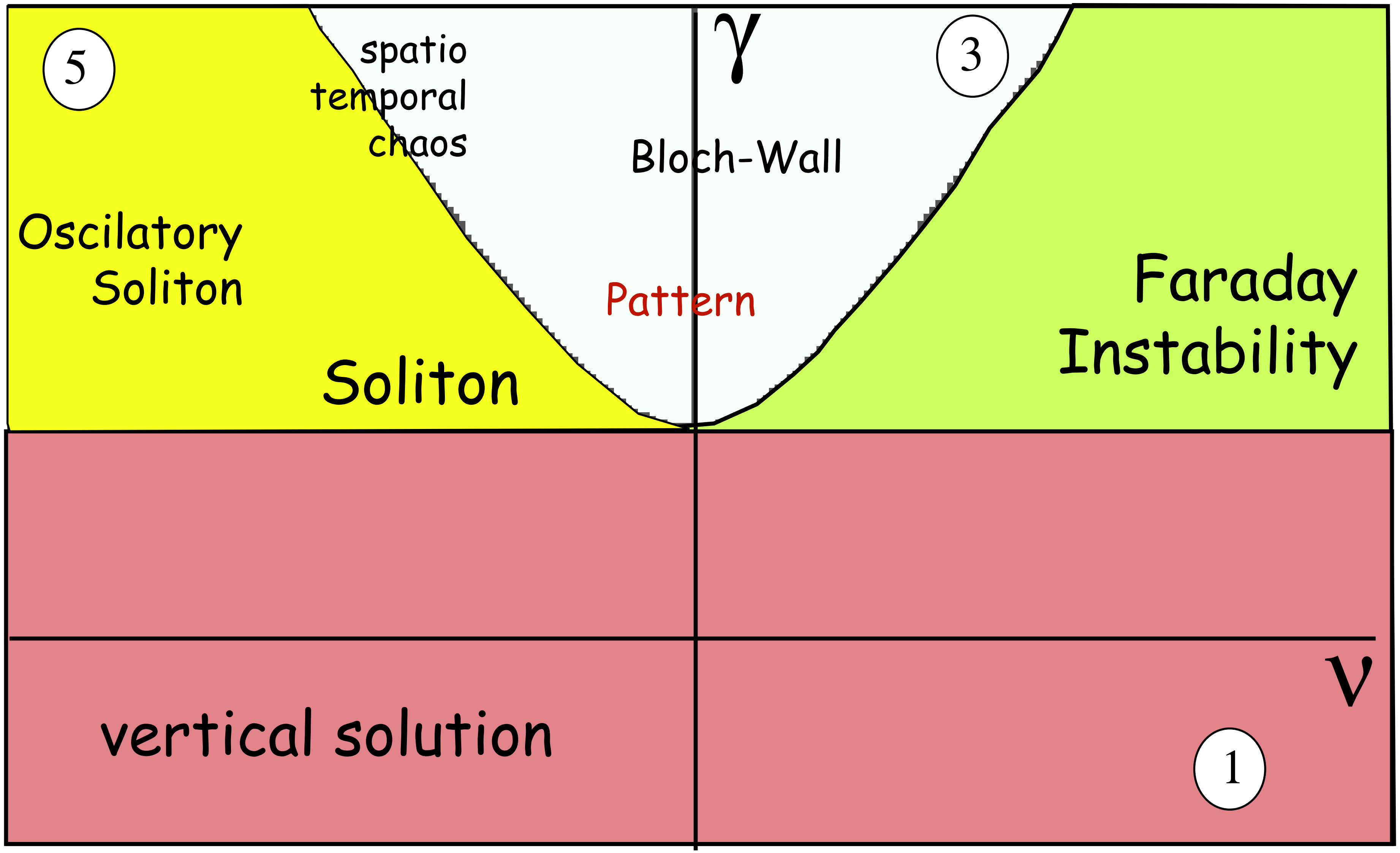
- Vertically oscillating layers of water.

- Light pulses in optical fibers.
- Kerr-type optical parametric oscillator.
- Magnetization soliton in easy-plane ferromagnets exposed to oscillatory magnetic field.
- Coupled Josephson junction.
- Parametrically driven chain of pendulums.



Courtesy Clerc, Residori & Falcon

Bifurcation diagram of the amplitude equation close to subharmonic instability



Stability of uniform steady states inside Arnold's Tongue

The parametrically driven damped Non-linear Schrodinger equation,

$$\partial_\tau A = -i\nu A - i|A|^2 A - i\partial_z^2 A - \mu A + \gamma \bar{A}$$

has the uniform solution

$$\begin{aligned} A_\pm &= \pm R_0 e^{i\theta_0} = \pm\sqrt{-\nu + \sqrt{\gamma^2 - \mu^2}}\, e^{i\theta_0} \\ \cos 2\theta_0 &= \frac{\mu}{\gamma} \end{aligned}$$

which is unstable and marginal only for zero detuning.

Variacion del espectro

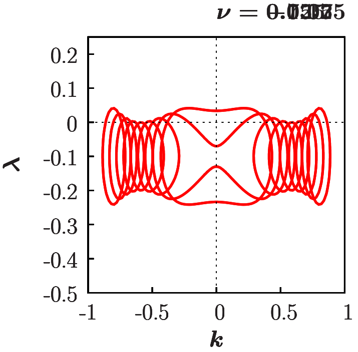
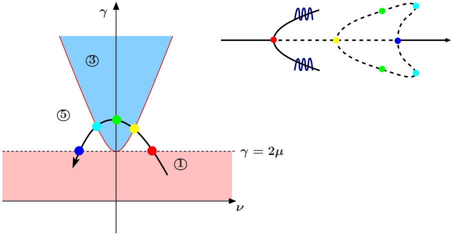


Diagrama de bifurcación de la PDNLS

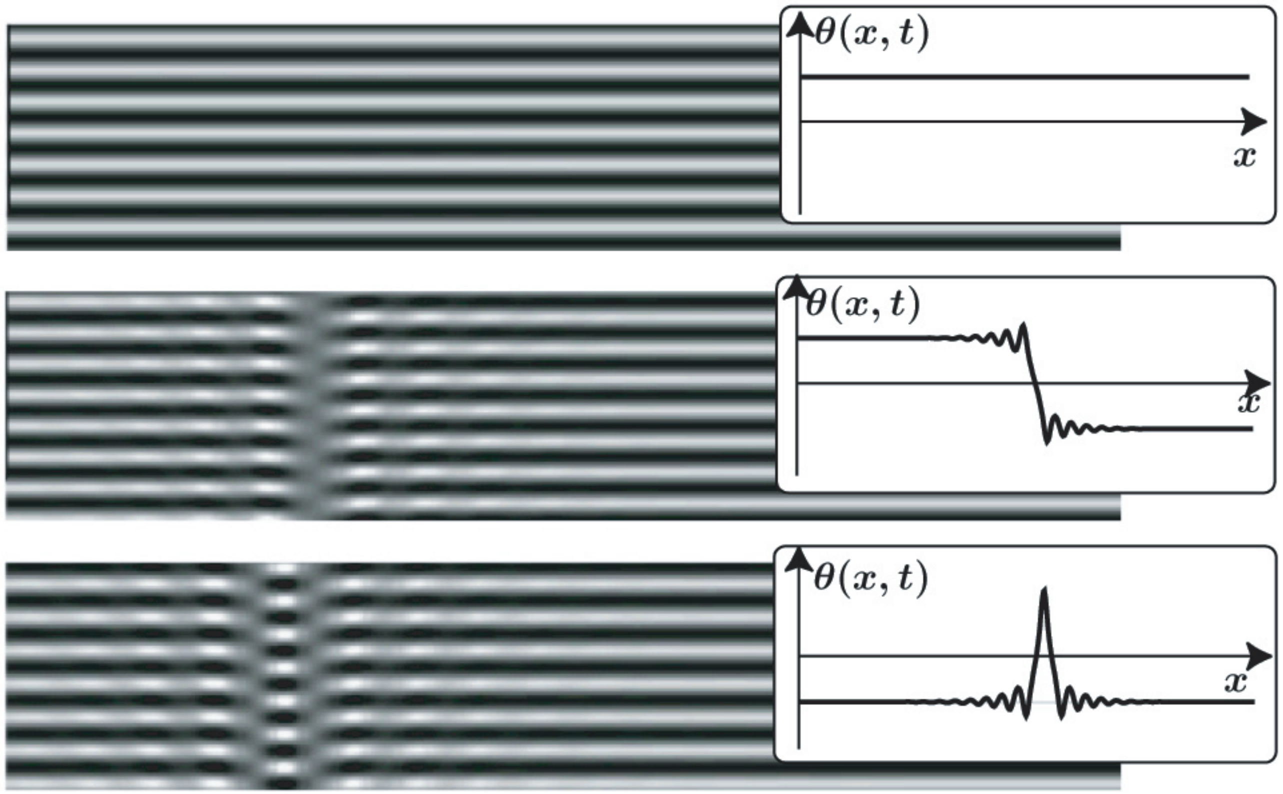


Entonces segun la ecuación de shcrödinger paremetrica no lineale la solución estacionaria homogénia es inestable dentro de la lengua de Arnold.

Parametrically driven nonlinear schrodinger description

- The conventional approach to these systems, the parametrically driven damped nonlinear Schrodingerequation, does not account for localized states observed in horizontally driven pendula chain.

(a)
(b)
(c)

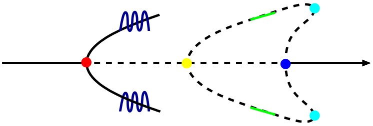


- Uniform oscillatory states.
- Kink states: solutions that links two uniform oscillations.
- Family of localized states: As consequence of kink and anti-kink interactions which alternates between attractive andrepulsive.

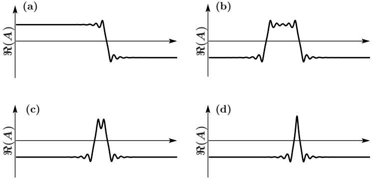
Derivando la ecuación de amplitude con los nuevos terminos pertinantes de la expansión de Taylor, encontramos la nueva ecuación amplitud:

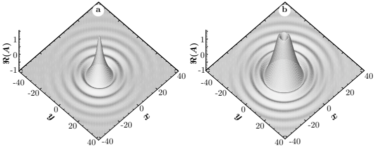
PDNLS Enmendado

$$\partial_\tau A = -i\nu A - i\left|A\right|^2 A - i\partial_x^2 A - \mu A$$
$$+ia\left|A\right|^4 A + \gamma\left(b\left|A\right|^2 \bar{A} + cA^3\right)$$



LS de la PDNLS Enmendado

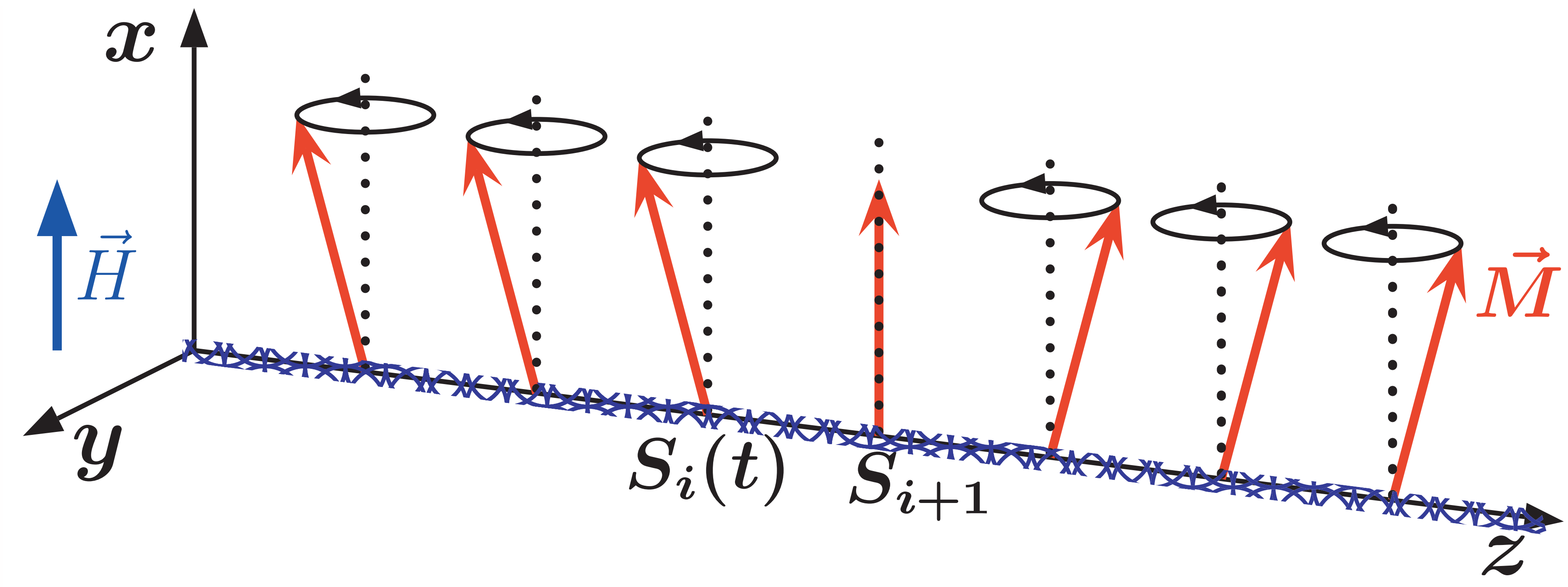




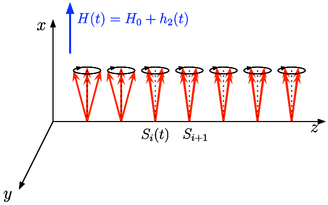
Entonces recuperamos el comportamiento del sistema inicial.

Aplication

- In a magnetic wire--with an easy-plane ferromagnetic--forced with a transversal oscillatory magnetic field.



El modelo



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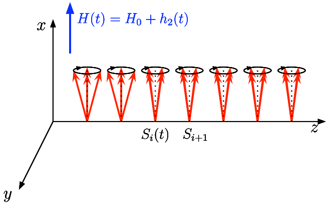


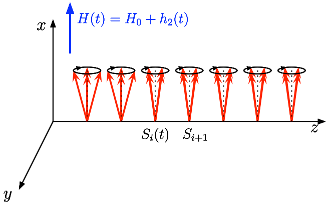
Hamiltoniano y ecuación de movimiento

- El Hamiltoniano

$$\mathcal{H} = \sum_{i}^{N} \left[-J\vec{S}_i\vec{S}_{i+1} + 2D(S_i^z)^2 - g\mu(S_i^x)H_x \right] \qquad (9)$$

  - Interacción dentro spines vecinos
  - Anisotropía: permite la existencia de fácil-plano
  - Campo magnético exterior

El modelo



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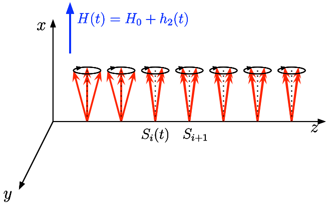

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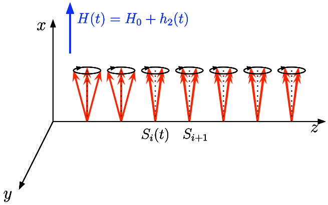

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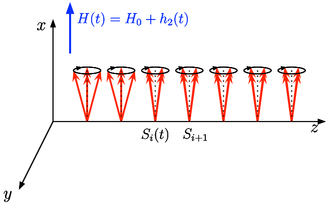

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- La ecuación de movimiento

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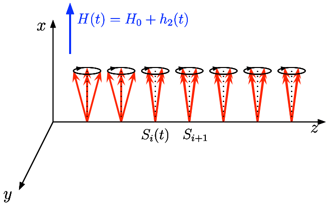

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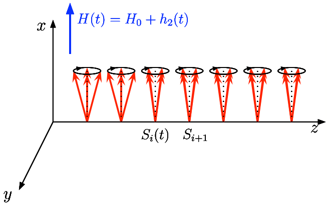
- La ecuación de movimiento

$$\hbar\dot{\vec{S}}_i = -\vec{S}_i \times \frac{\partial \mathcal{H}}{\partial \vec{S}_i} \qquad (10)$$

Hamiltoniano y ecuación de movimiento

El modelo



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- La ecuación de movimiento

$$\frac{\partial \mathcal{H}}{\partial \vec{S}_i} = -J\vec{S}_{i+1} + 2J\vec{S}_i - J\vec{S}_{i-1} + 4DS_i^z\vec{e}_z - g\mu H_x\vec{e}_x - 2J\vec{S}_i \qquad (10)$$

- límite continua: $\vec{S}_i(t) \longrightarrow \vec{S}(x,t)$

$$\Longrightarrow \frac{Jdx}{\hbar}\left(\frac{-\vec{S}_{i+1} - 2\vec{S}_i + \vec{S}_{i-1}}{dx}\right) \rightarrow l_{ex}\partial_x^2\vec{S}(x,t)$$

l_{ex}:longitud de interaccion
Luego, despues algunos normalizaciones llegamos à:

Ecuación de Landau-Lifshitz-Gilbert

$$\mathbf{M}_t = \mathbf{M}\times\mathbf{M}_{xx} - \beta\left(\mathbf{M}\cdot\vec{e}_z\right)\left(\mathbf{M}\times\vec{e}_z\right) + \mathbf{M}\times\mathbf{H} - \alpha\mathbf{M}\times\mathbf{M}_t \qquad (11)$$

Atenuación de Gilbert≡ Disipación de Rayleigh

- $\alpha = 0$:sistema reversible $(t \rightarrow -t)$ con simetría de reflexión $(M_x, M_y, M_z) \rightarrow (M_x, -M_y, -M_z)$
- Aproximación cuasi-reservible:$\alpha \ll 1, h_2 \ll 1, \partial_x\mathbf{M} \ll 1 \Longrightarrow M_x = \sqrt{1-(M_y^2+M_z^2)} \approx 1 - \frac{M_y^2+M_z^2}{2} + \cdots$

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Después de largos cálculos obtenemos:

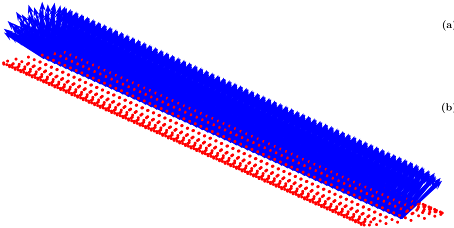
$$\begin{aligned}\ddot{M}_z &= -H_0\left(\beta+H_0\right)M_z+\frac{\beta H_0}{2}M_z^3+\left(\beta+2H_0\right)\partial_x^2 M_z \\ &\quad -\alpha\left(\beta+2H_0\right)\dot{M}_z-\beta_2\sin(\Omega t)M_z \qquad (12a)\\ M_y &\approx \frac{\alpha}{H(t)}\left(\beta+H(t)\right)M_z \qquad (12b)\end{aligned}$$

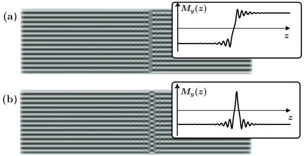
Entonces si introducimos el mismo ansatz que en la cadena de péndulo llegamos a la misma ecuación de amplitud enmendado con:

$$\begin{aligned}\omega_0 &= \sqrt{H_0(\beta+H_0)} \qquad (13a)\\ \mu &= \frac{\alpha}{2}(\beta+2H_0) \qquad (13b)\\ \gamma &= \frac{h_2(\beta+2H_0)}{4\omega_0} \qquad (13c)\end{aligned}$$

Simulaciones numericas

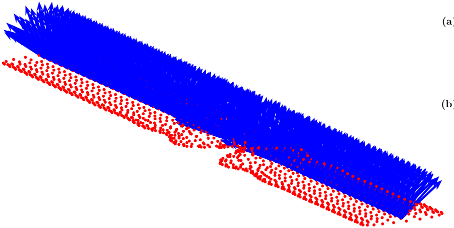
- Solución homogénea

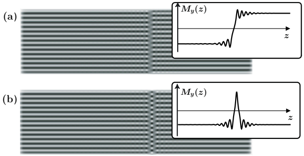




Simulaciones numericas

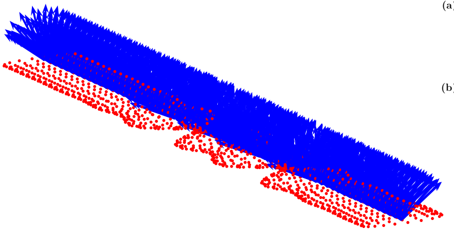
- Kink

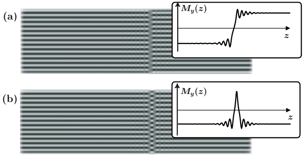




Simulaciones numericas

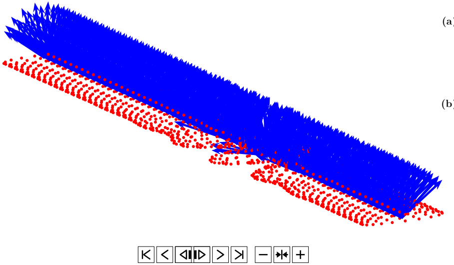
- Estructura Localizada de tipo horne

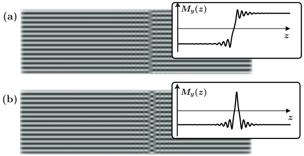




Simulaciones numericas

- Estructura Localizada

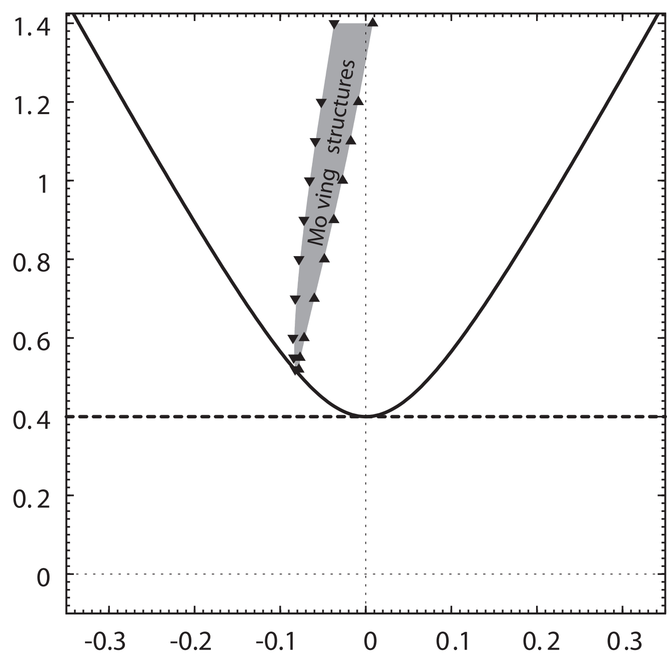




Conclusions

- A novel type of localized states which link asymptotically homogeneous precession states in a magnetic wire parametrically driven with a magnetic field is studied.
- The conventional approach of this system, PDNSL in one-dimension has been a successful model to explain pattern and localized states which connect uniform states in parametrically driven quasi-reversible systems. However, this model lacks of this novel family of localized states, which connect asymptotically a uniform oscillatory state with itself.
- The improvement of this model by the consideration of higher order terms allow us to recover and to account for this localized state. Due to the unified description that we have considered, the same family of localized states is observed in parametrically drivendamped pendula chain.

Outlook



- Non variational dynamics