PATTERNS AND LOCALIZED STRUCTURES IN NONLINEAR NONLOCAL SYSEM

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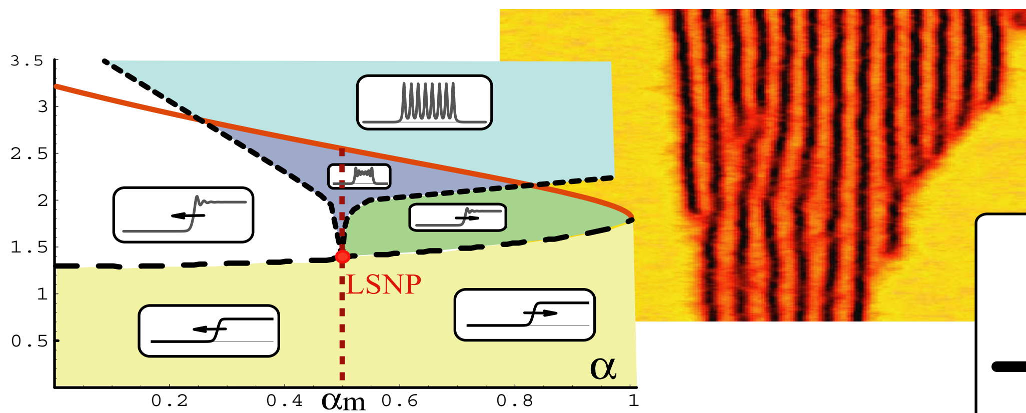
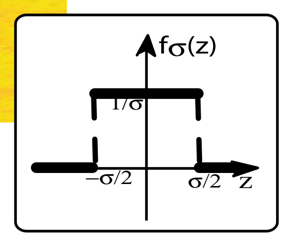


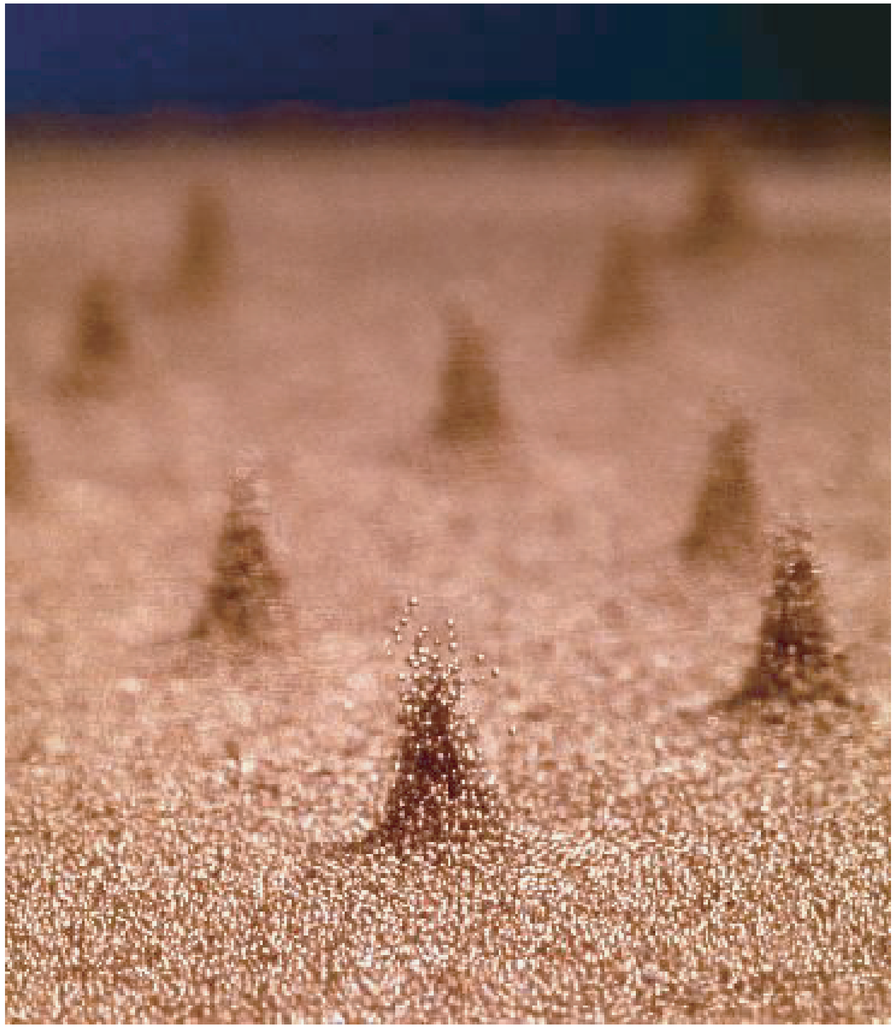


Outline

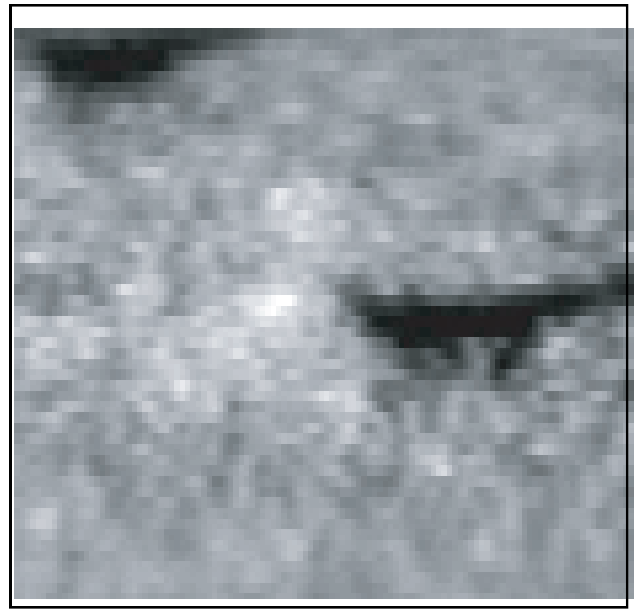
- Localized and Front solutions in experiments.
- Main ingredients of localized structures.
- Patterns in Non-local Fisher model.
- Non-local Nagumo model.
- Bifurcation diagram of non-local Nagumo model.
- Remarks of particle-type solutions in non-local Nagumo model.
- Conclusions.

Localized and Front solutions in experiments

- Fluidized granular matter

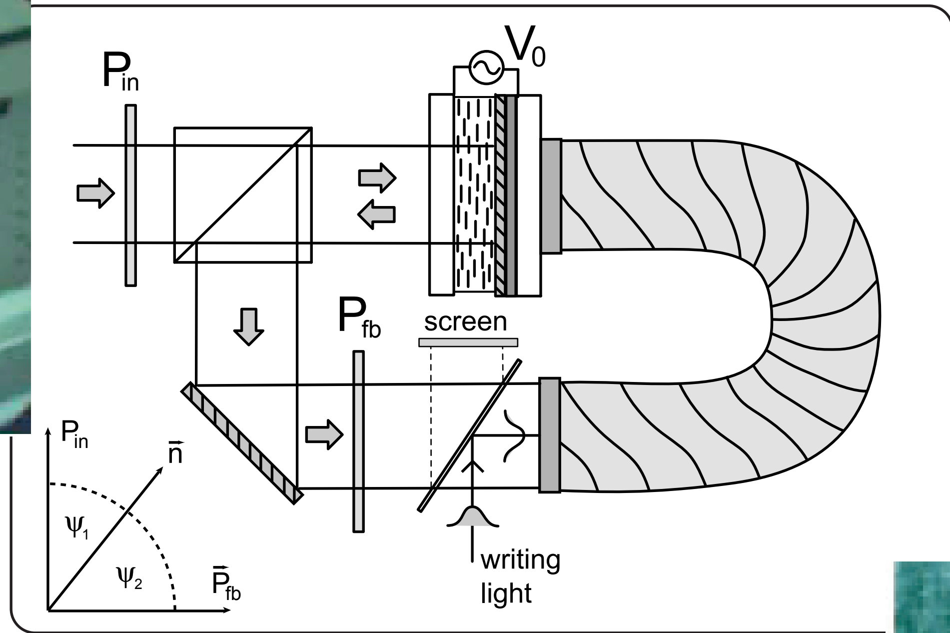


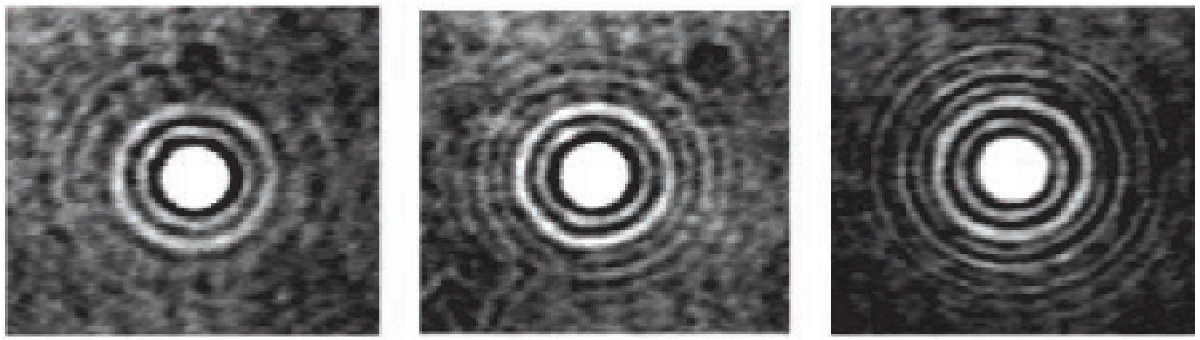
Localized excitations in a vibrating layer of sand (Oscillons)

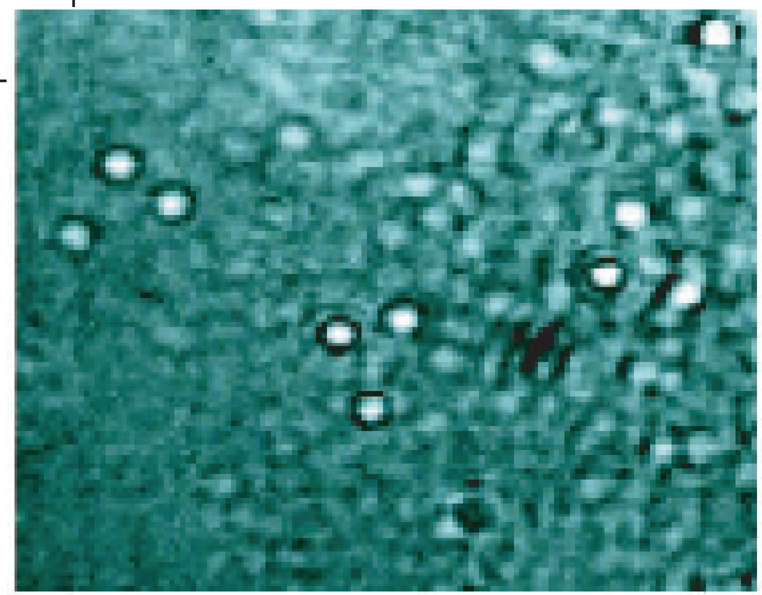


P. Umbanhowar, F. Melo and H. Swinney, Nature, 382, 793 (1996)

- liquid crystal light valve with optical feedback

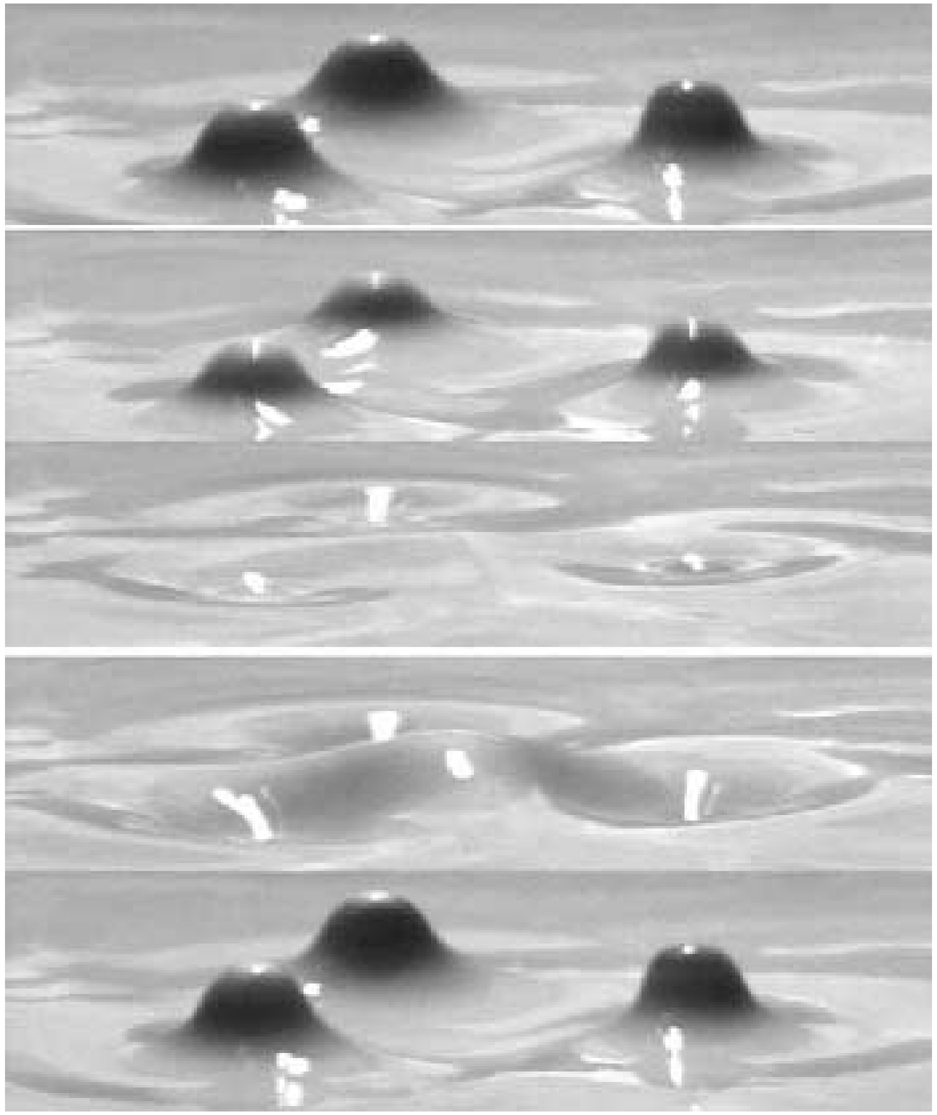






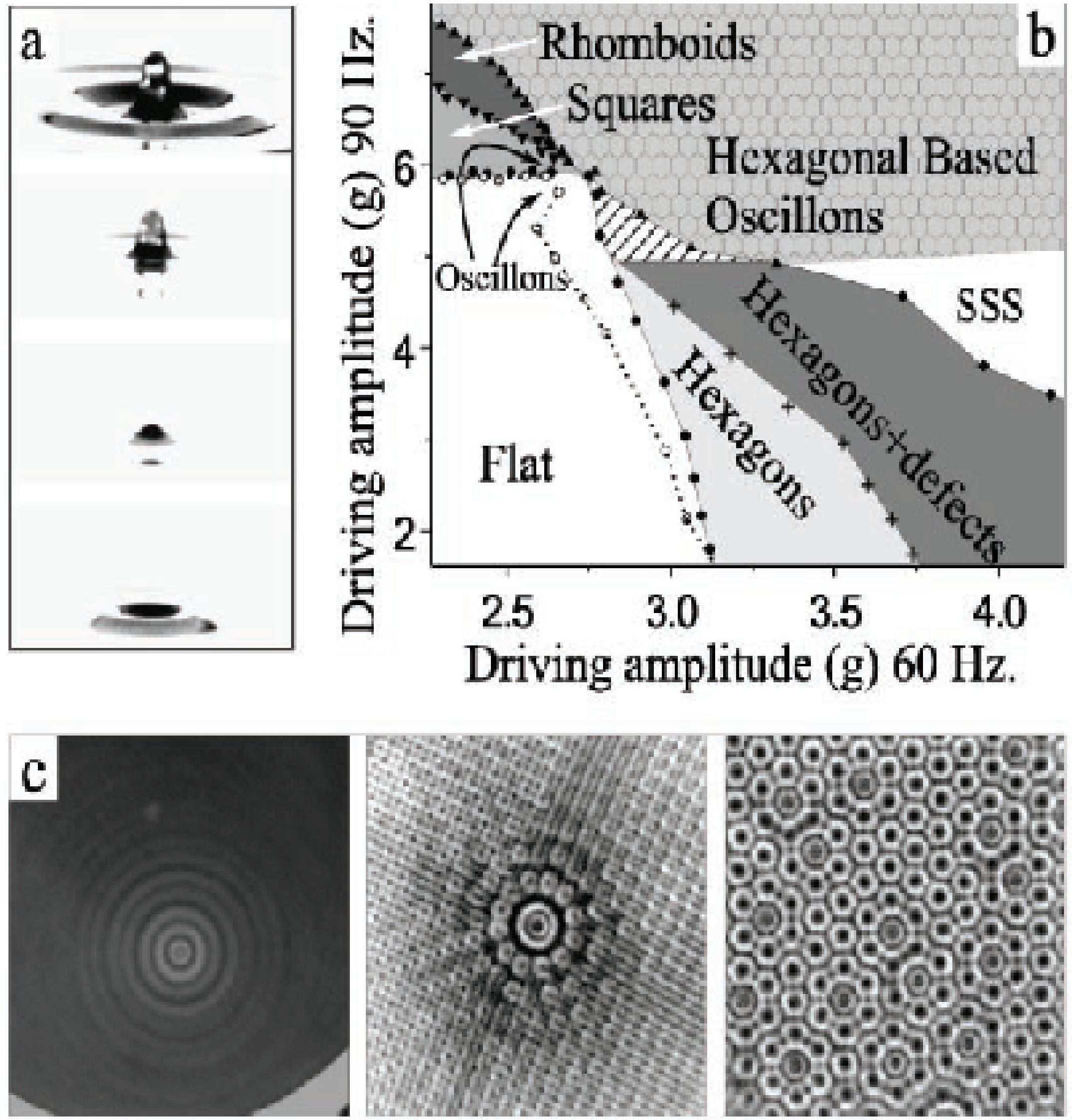
PRA 52, 791 (95), Phys. Rep. 318 (99)

- Vertically vibrated colloidal Suspension



PRL, 83, 3190 (1999).

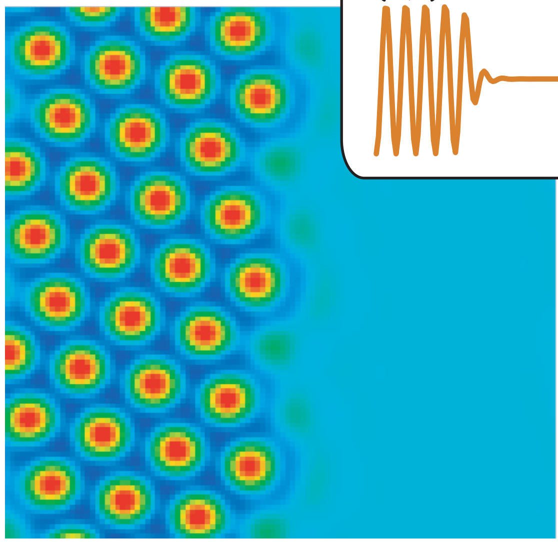
- Newtonian Fluids (two frequencies)

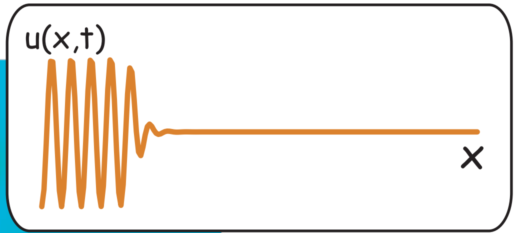


PRL, 85, 756 (2000).

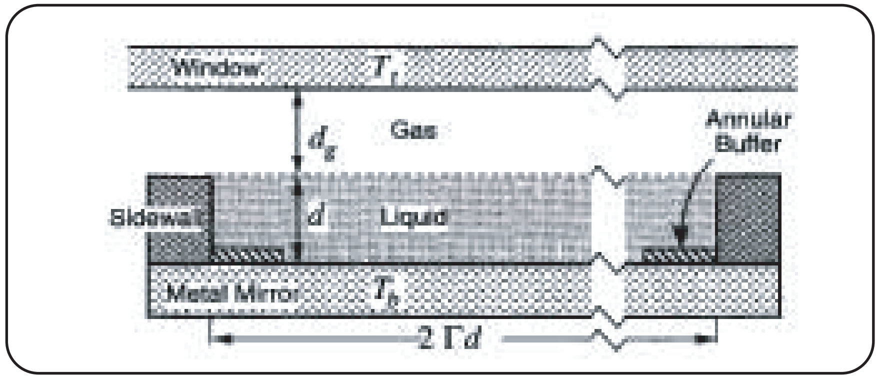
Fronts and experiments

- Simulation

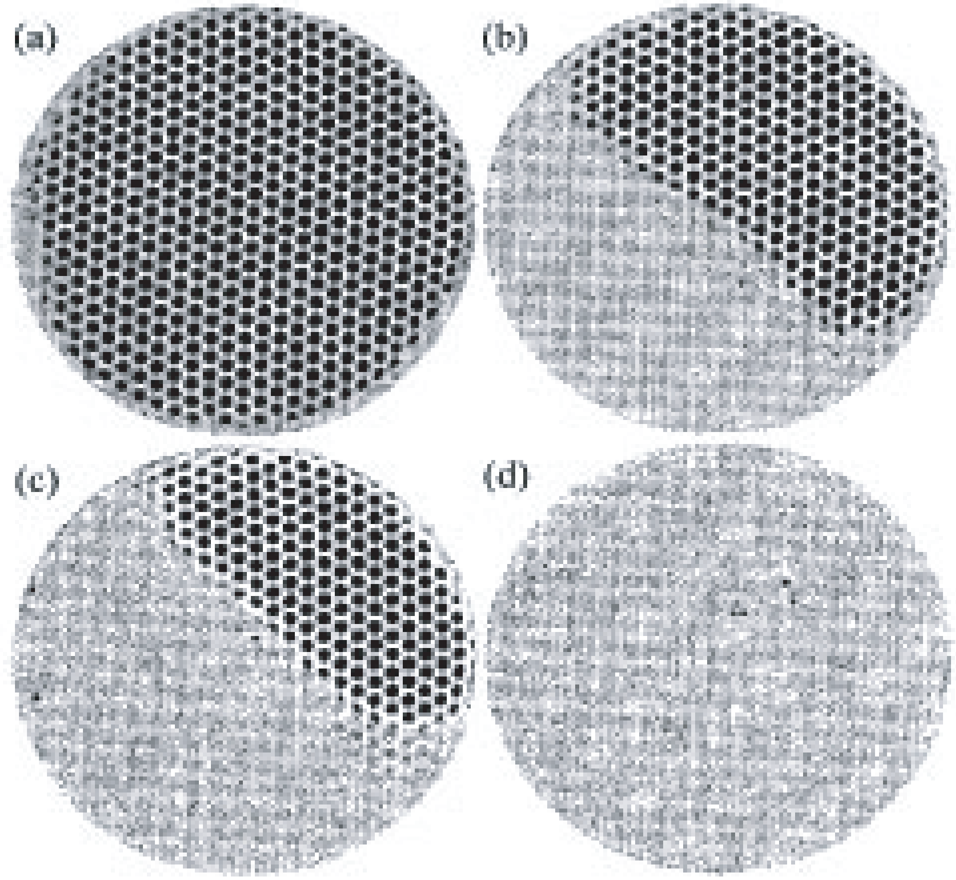




- Benard-Marangoni



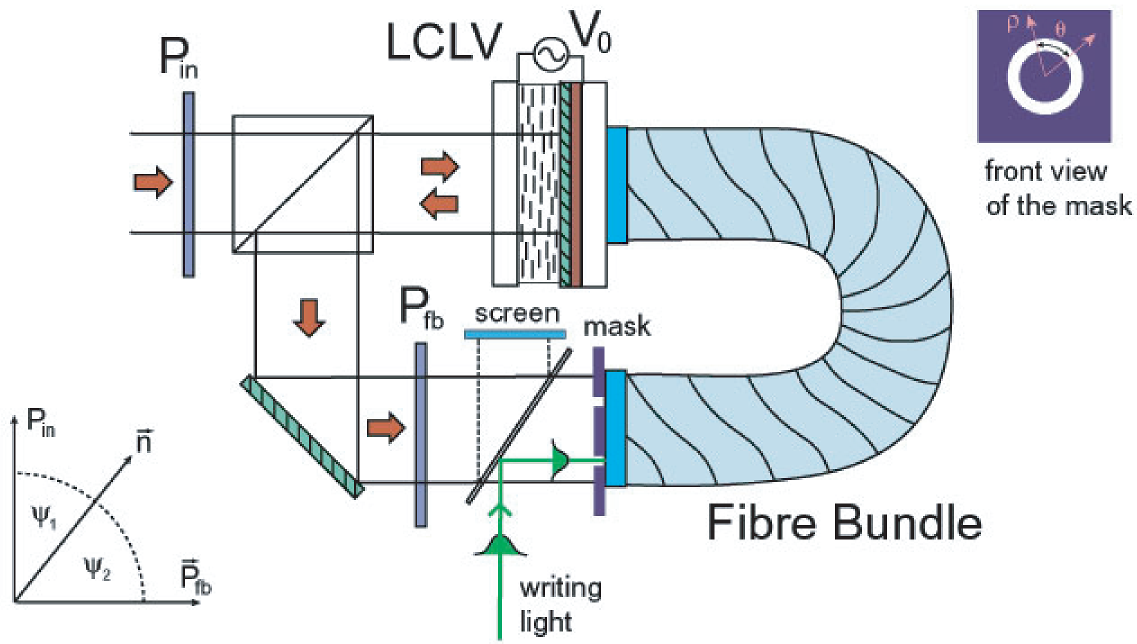
Front propagation

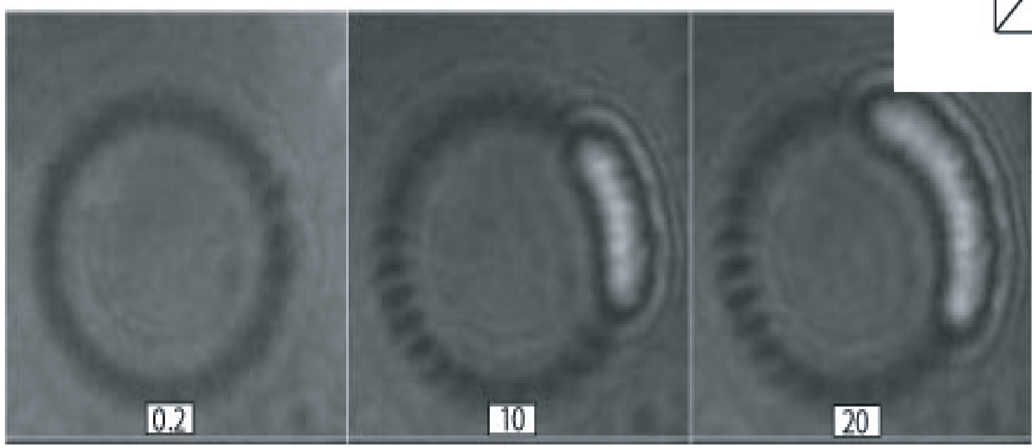


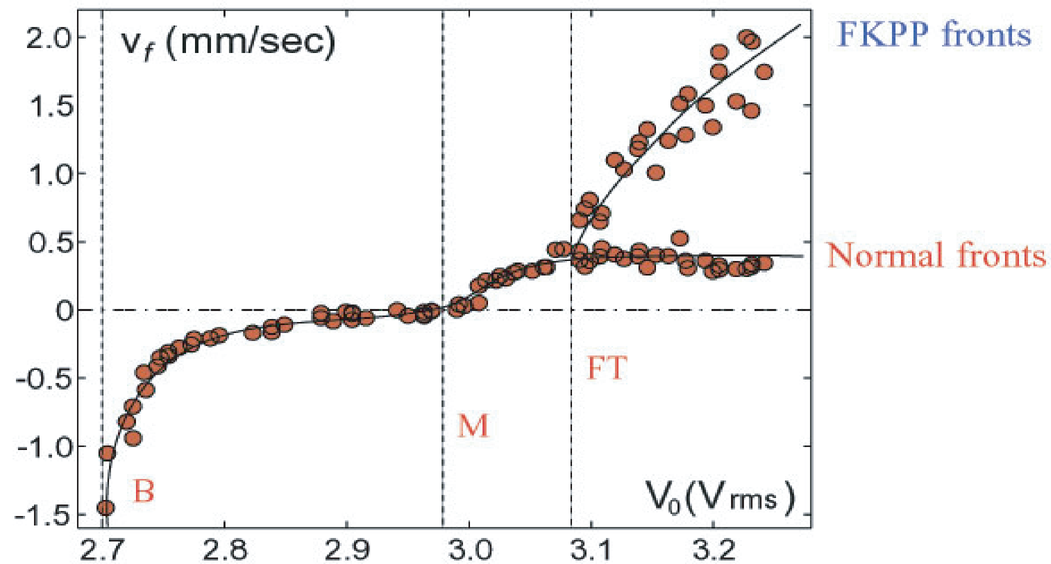
MF Schatz et at, Phys. Rev. Lett. 75, 1938 (1995)

Experimental measurement of front velocity

- Liquid crystal light valve with optical feedback (1-D experiment)



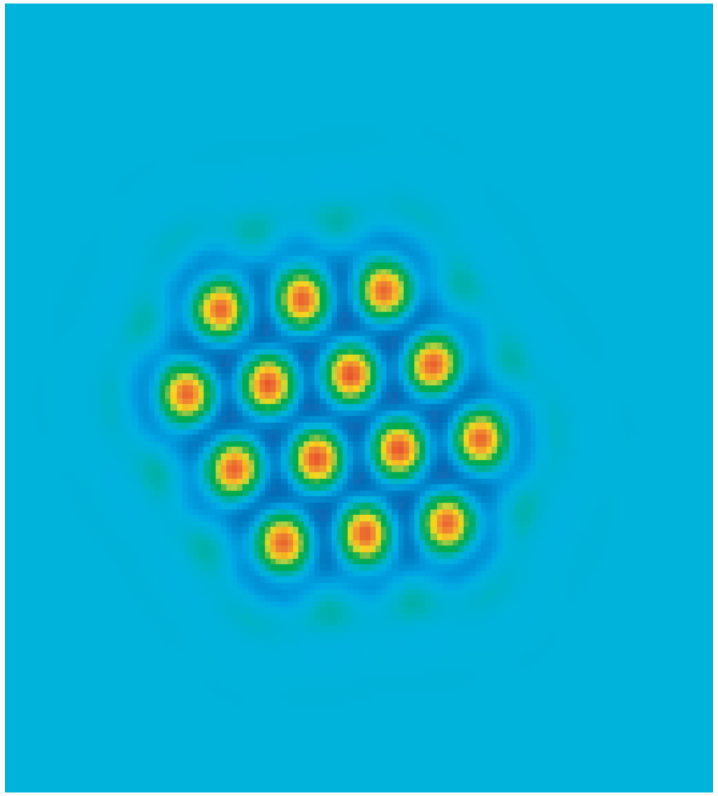




M.G. Clerc et al, Eur. Phys. J. D 28, 435 (2004).

Main ingredients of localized structures

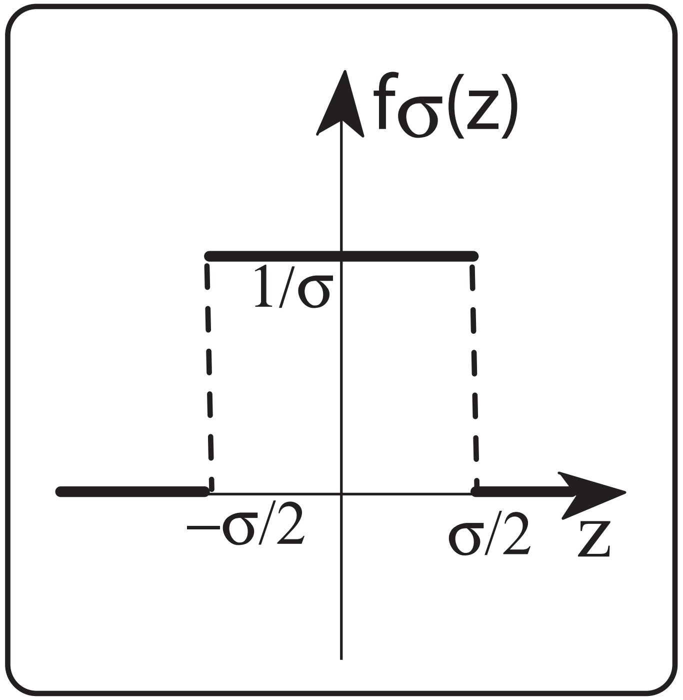
- Coexistence between two steady states (two homogenous state, homogeneous states and spatially periodical one and so forth)
- Intrinsic length



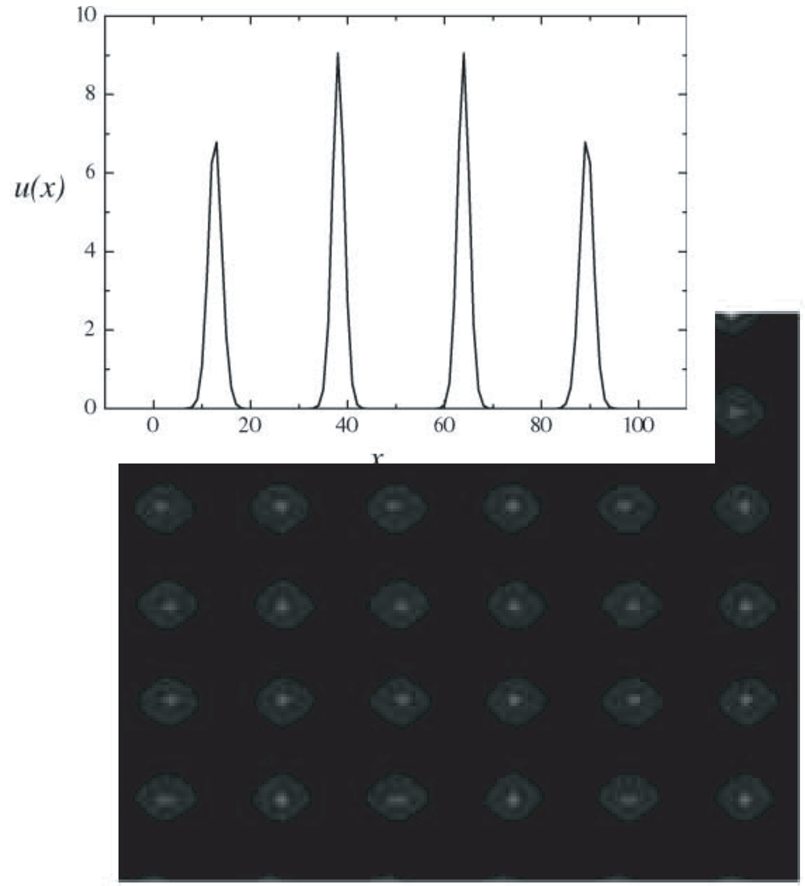
Patterns in Non-local Fisher model

- $$\frac{\partial u(\vec{x},t)}{\partial t} = D\nabla^2 u(\vec{x},t) + a\,u(\vec{x},t) - bu(\vec{x},t)\int_\Omega u(\vec{y},t) f_\sigma(\vec{x},\vec{y}).$$

Where $f_\sigma(|x-y|)$ is typically



- Numerical simulation (1D and 2D)



M. A. Fuentes, M. N. Kuperman, and V.M. Kenkre, Phys. Rev. Lett. 91, 158104 (2003);

C.Lopez, and E. Hernandez-Garcia, Physica D, 199, 223 (2004).

- There is one steady state!

Non-local Nagumo model

- A simple non-local model that exhibit bistability is

$$\boxed{\partial_t u = \partial_{xx} u - \alpha u + (\alpha+1)u^2 - u\int_\Omega u'^2 f_\sigma(x,x')dx'}$$

where the influence function $f_\sigma(x,x') = f_\sigma(x-x')$, is a even function and it is normalized $\int_\Omega f_\sigma(x,x')dx' = 1$. And $0 < \alpha < 1$.

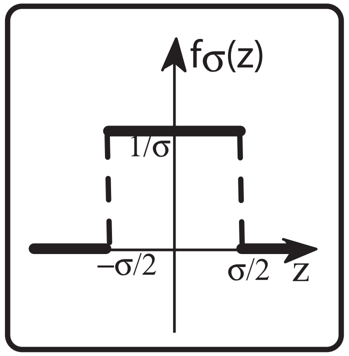
- The model is variational

$$\partial_t u = -\frac{\delta F}{\delta u} \implies \dot{F} = -\int_\Omega (\partial_t u)^2 d^2x$$

where

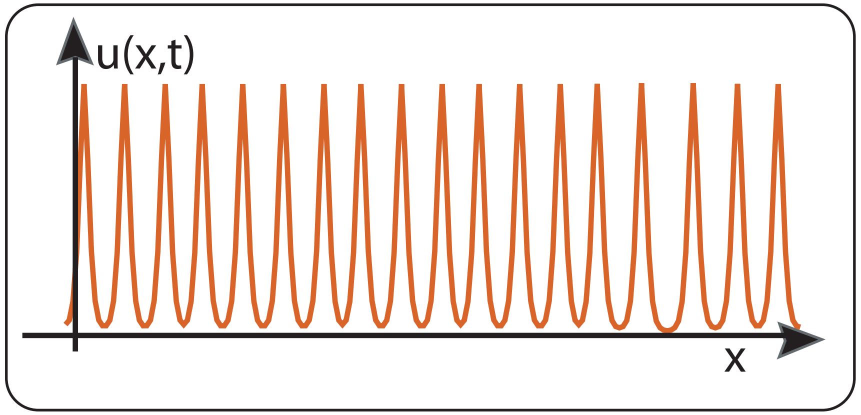
$$F[u] = \int_\Omega \left\{ \frac{1}{2}(\partial_x u)^2 + \frac{\alpha}{2}u^2 - \frac{(\alpha+1)}{3}u^3 \right\} dx + \frac{1}{4}\int_\Omega\int_\Omega u^2 u'^2 f_\sigma(x,x')dxdx'$$

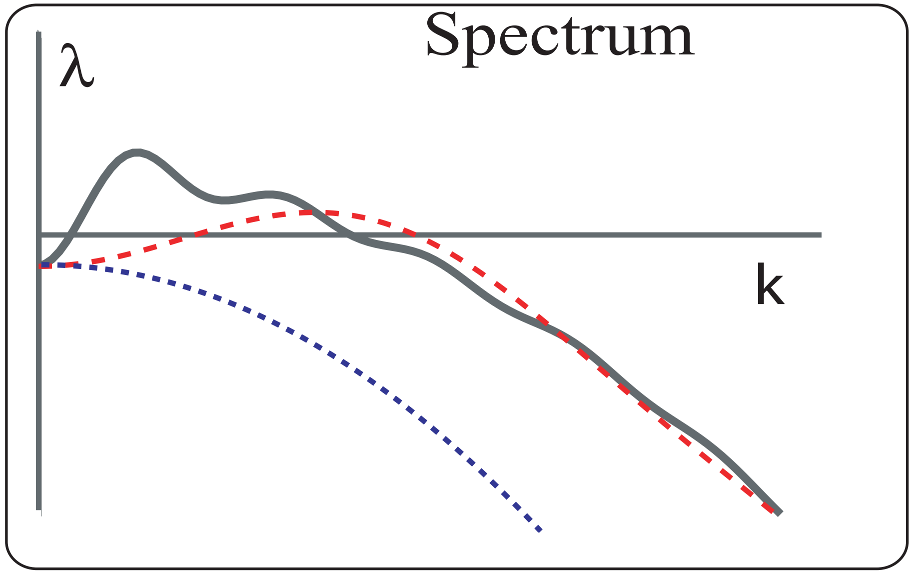
- This model has three steady homegeneous states u=0 (stable), u=α (unstable) and u=1.
- For the sake of simplicity we consider



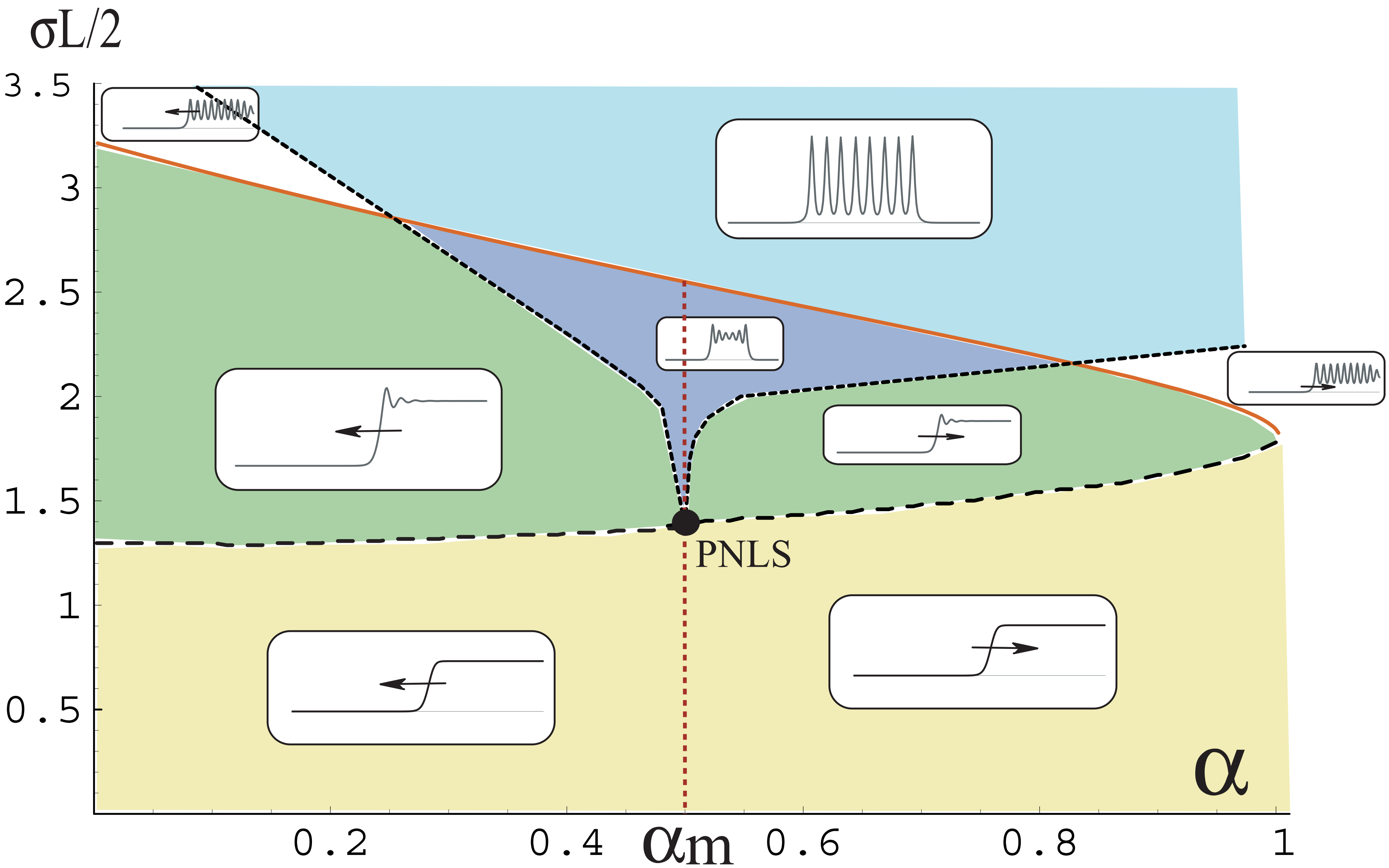
$$f_\sigma(z) = \theta(\sigma + z)\theta(\sigma - z)/2\sigma$$

- For this influence function the system is characterize by two parameter {σ,α}
- The steady state u=1 exhibits an spatial instability





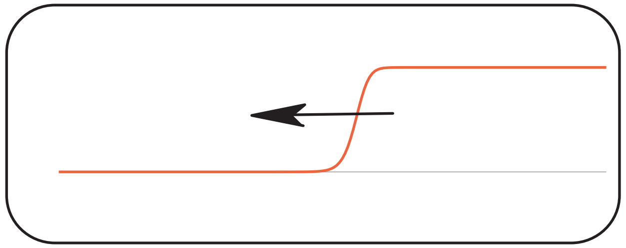
Bifurcation diagram of non-local Nagumo model



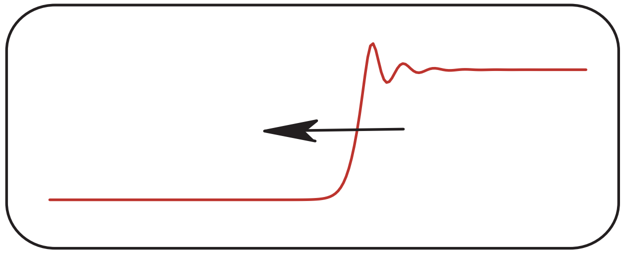
Remarks of particle-type solutions in non-local Nagumo model.

- The system exhibits three type of front solution

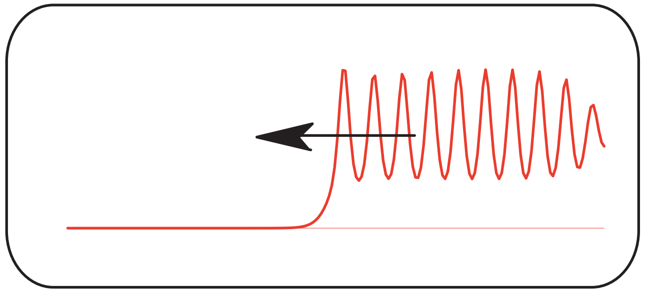
  - homogeneous-homogenous states without spatial oscilation



  - homogeneous-homogenous states with spatial oscilation

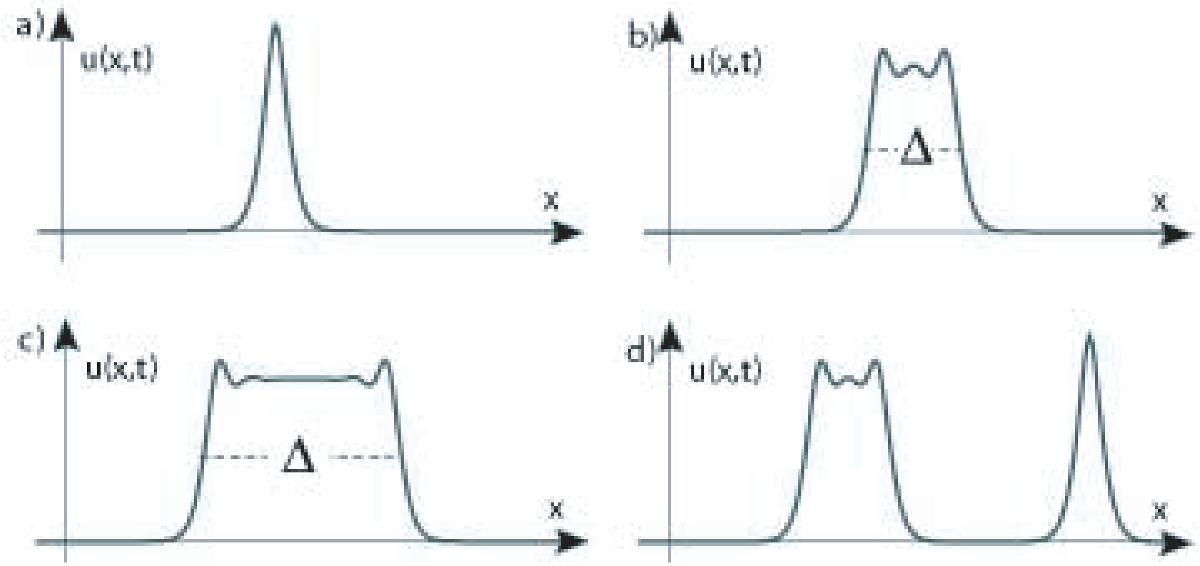


  - homogeneous- spatially periodic state



- The system exhibits two type of Localized structure

- *Horm solutions*: solution bewteen two homogenoeus states, which exhibits spatial damped oscillation.



- These particle-type solutions are consequence of the kink-antikink interaction

$$\boxed{\dot{\Delta} = \alpha \cos(\kappa \Delta) e^{-\beta \Delta} + \eta,}$$

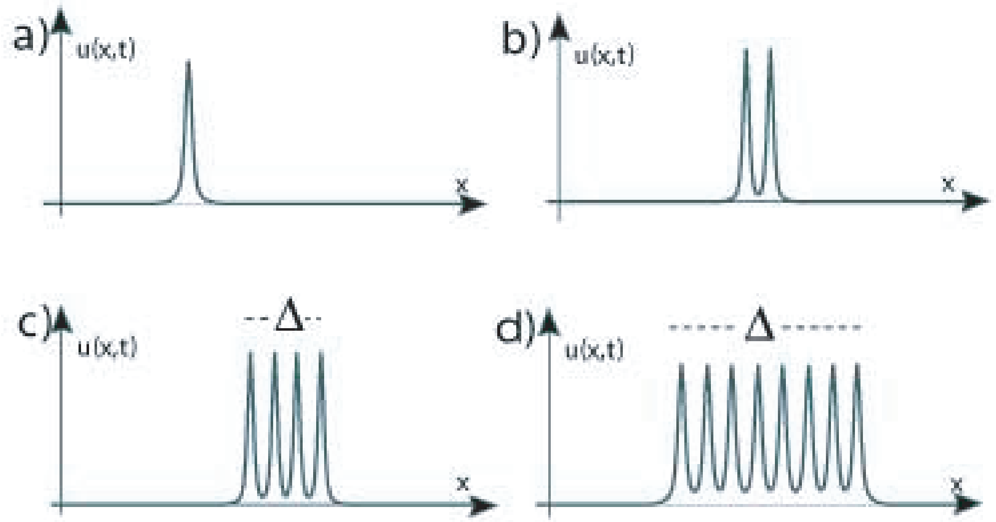
P. Coullet, C. Elphick and D. Repaux, Phys. Rev.Lett. 58, (1987) 431.

where Δ is the distance between the kink and antikink, {κ,β} are the wave-number and exponent of the decreasing damped spatial oscillations, η is the parameter that measu-rements the separation of Maxwell point (η is proportional to a-1/2) and α is a parame-ter of order one.

- Around of the Maxwell point appearance a familly of horm solutions. The length of these solutions are roughly multiple of 2π/κ.

The system exhibits two type of Localized structure

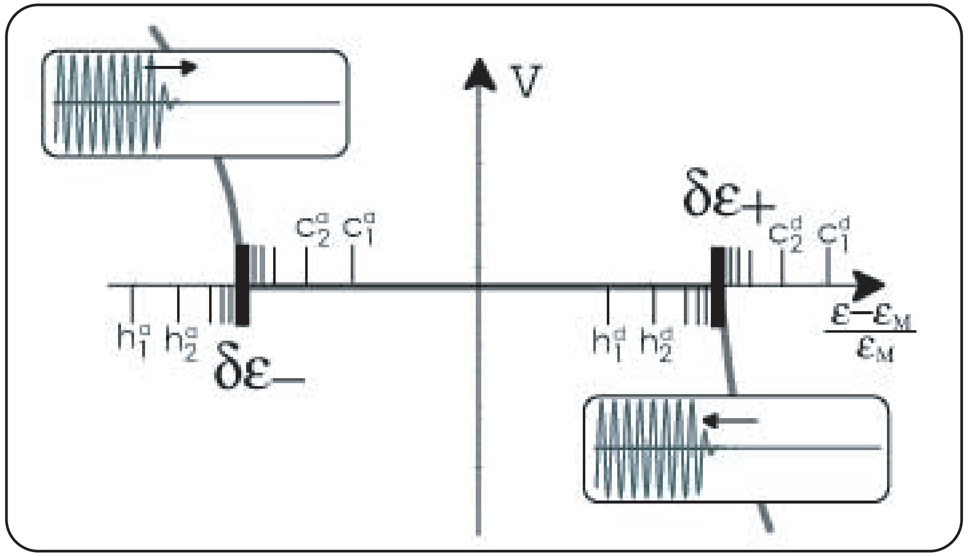
- *Localized patterns*: solution bewteen an homogenoeus and spatially period states.
- These particle-type solutions are consequence of the kink-antikink interaction



$$\dot{\Delta} = -\alpha\Delta\exp(-\beta\Delta) + \gamma\cos(\kappa\Delta) + \eta$$

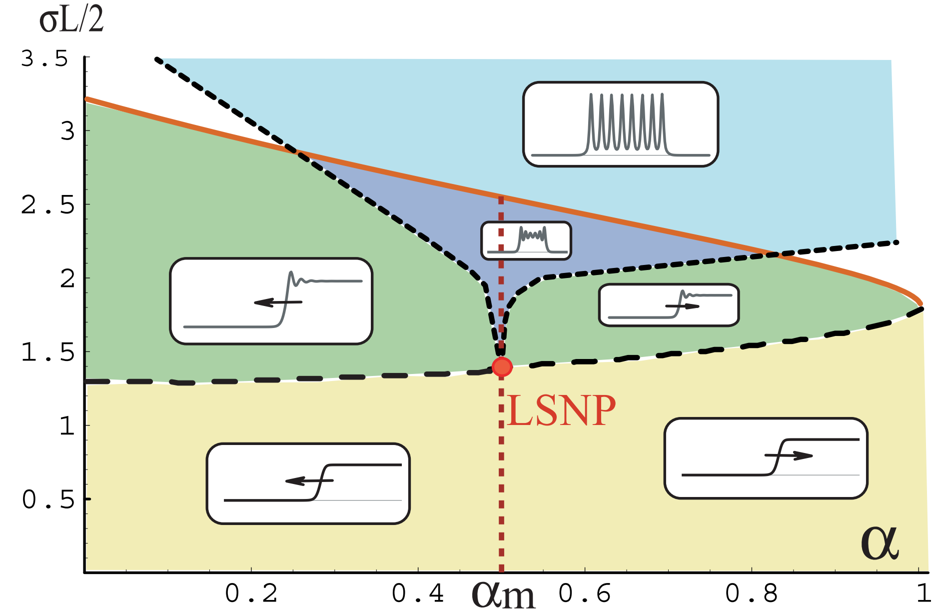
M.G. Clerc, and C. Falcon, To appear in Physica A.

- Around of the pinning range there is a familly of localized patterns. The length of these solutions are roughly multiple of 2π/κ.

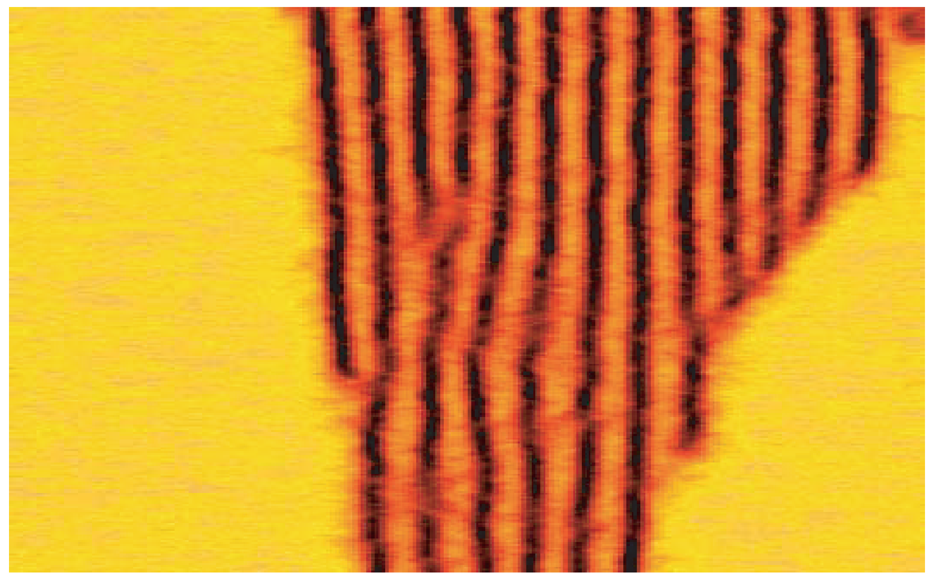


Rev. Lett. 84, 3069 (2002).

- Determination of the point in the parameter space where the particle-type solution appear, localized structures nascent point (LSNP).



- Aditive noise induce front propagation



M.G. Clerc, C. Falcon, and E. Tirapegui, to appear in Phy. Rev. Lett.

Conlusions

- A variational non-local model, non-local Nagumo equation, exhibits coexistence between spatially periodic state and homogeneous ones. Hence, the system has particle-type solution like localized patterns, horm solutions, and front connection.
- Characterization of phase diagram, and the mechanism of localized structure appear.
- Determination of critical point of localized structures appear in the parameter space, *localized structure nascent point*.