Localized patterns and hole solutions in one-dimensional extended systems

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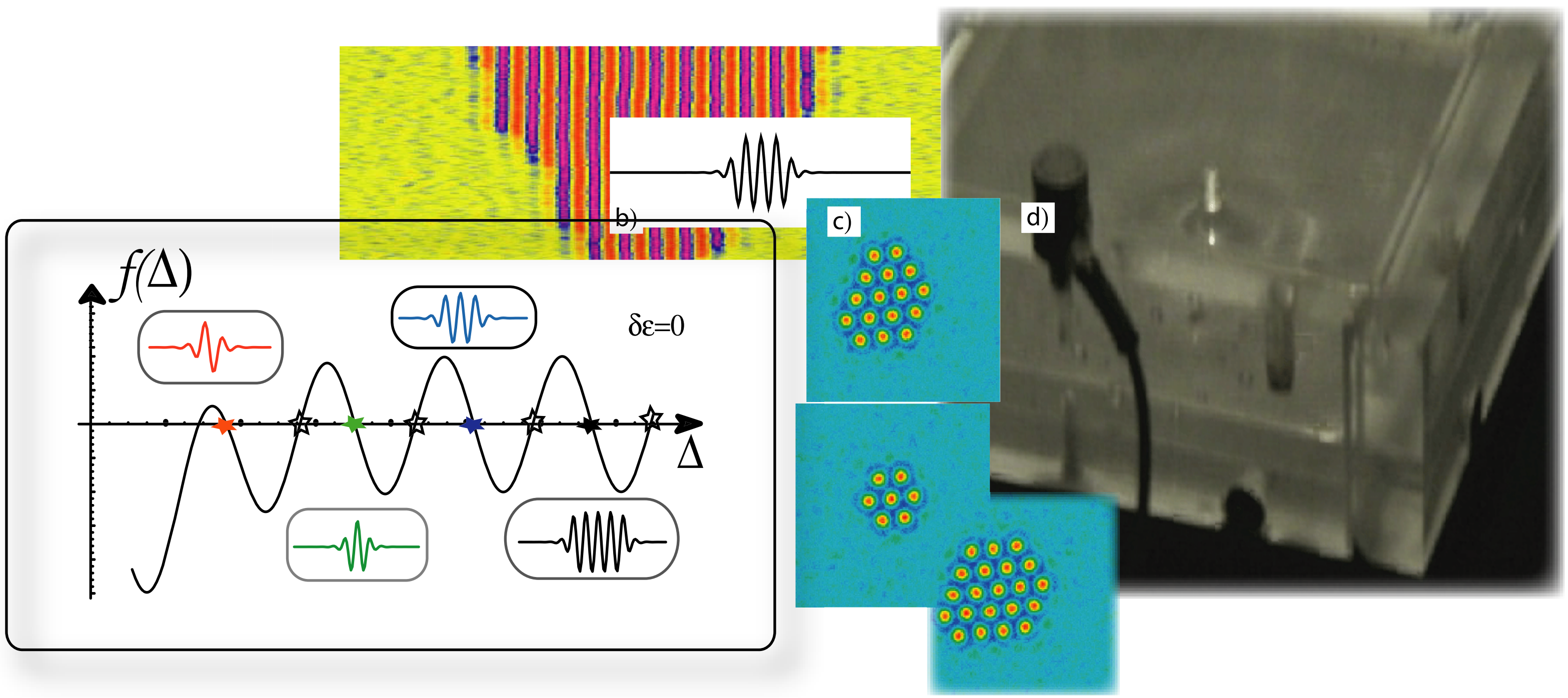

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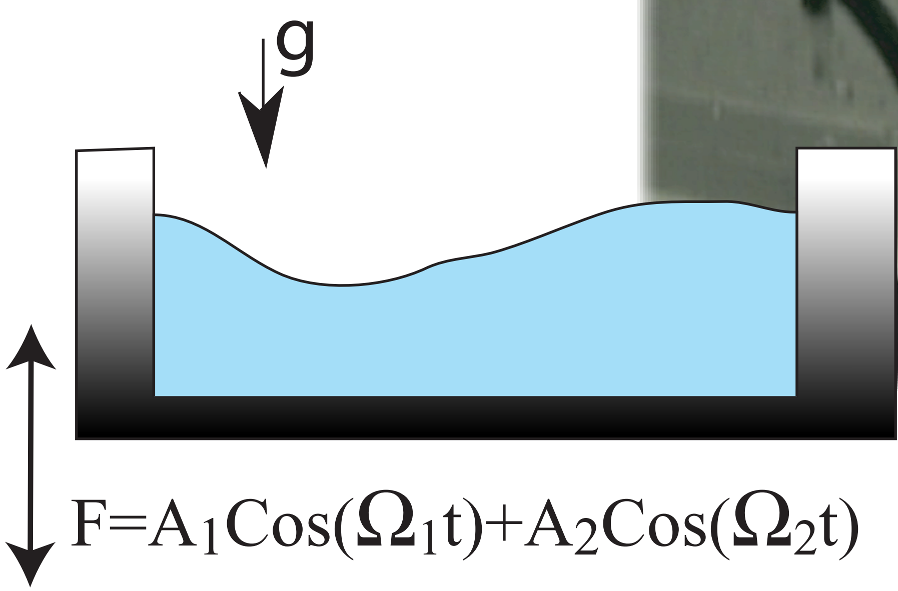
Outline

- Introduction of Localized solution in Nature.
- Localized structure are robust phenomena.
- Universal description of the localized structures

  Amended amplitud equation.
- Front interaction.
- Conclusions.
- Outlook.

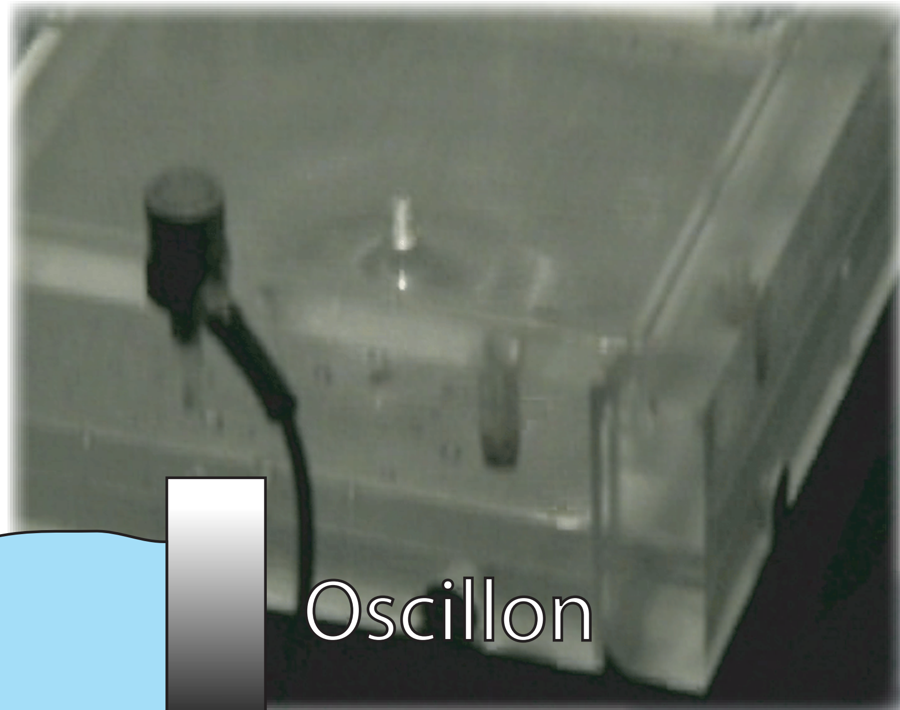
Physical examples and motivation

- Localized structures (oscillons and localized patterns) are observed in a vertically driven Newtonian fluid (water and glycerin).



$F = A_1 Cos(\Omega_1 t) + A_2 Cos(\Omega_2 t)$

$\Omega_1/\Omega_2 = 3/2,\ 5/4, \ldots$

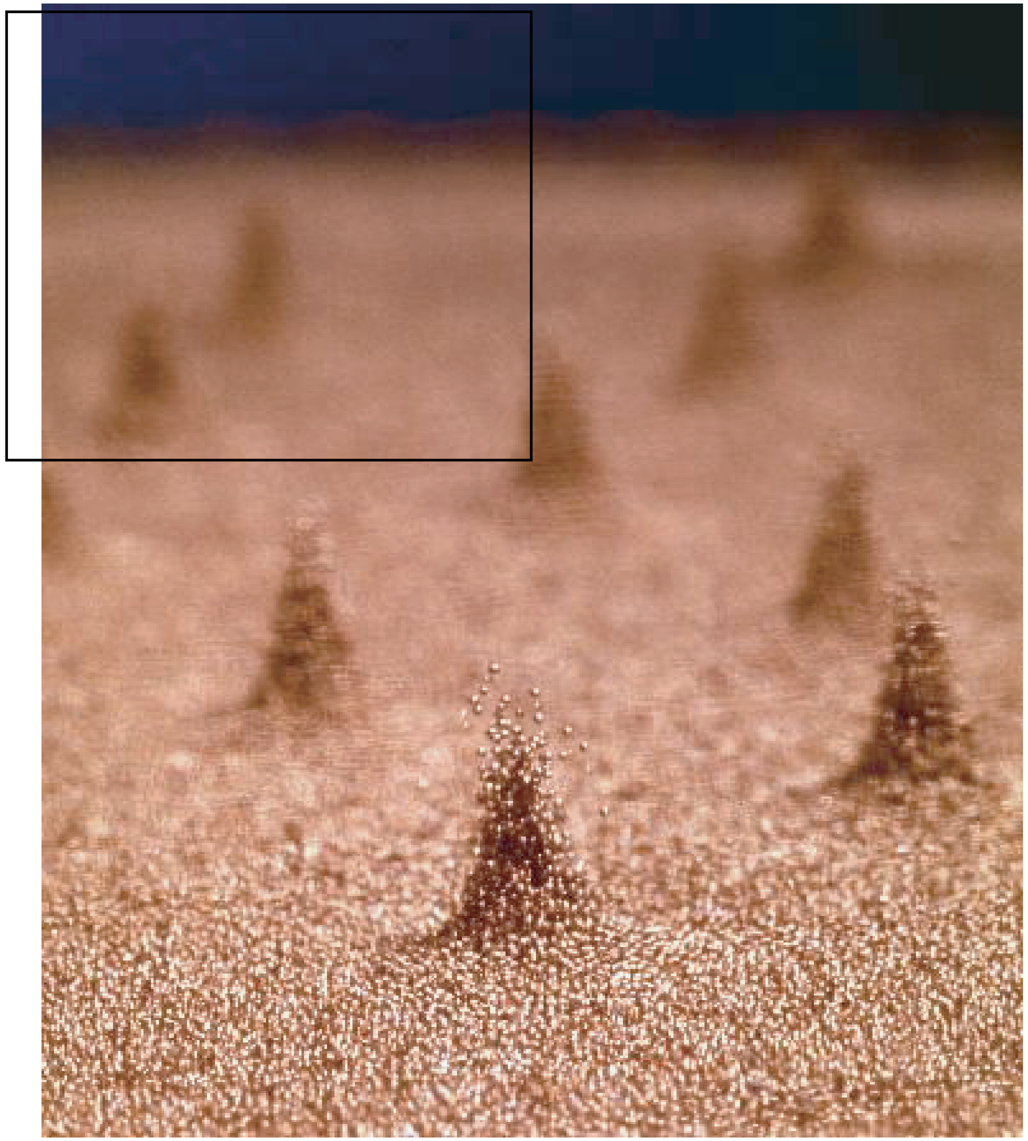


Fineberg et al, Phys. Rev. lett. 83, 3190 (1999).

Collaboration with N. Mujica (DFI) and S. Residori (INLN).

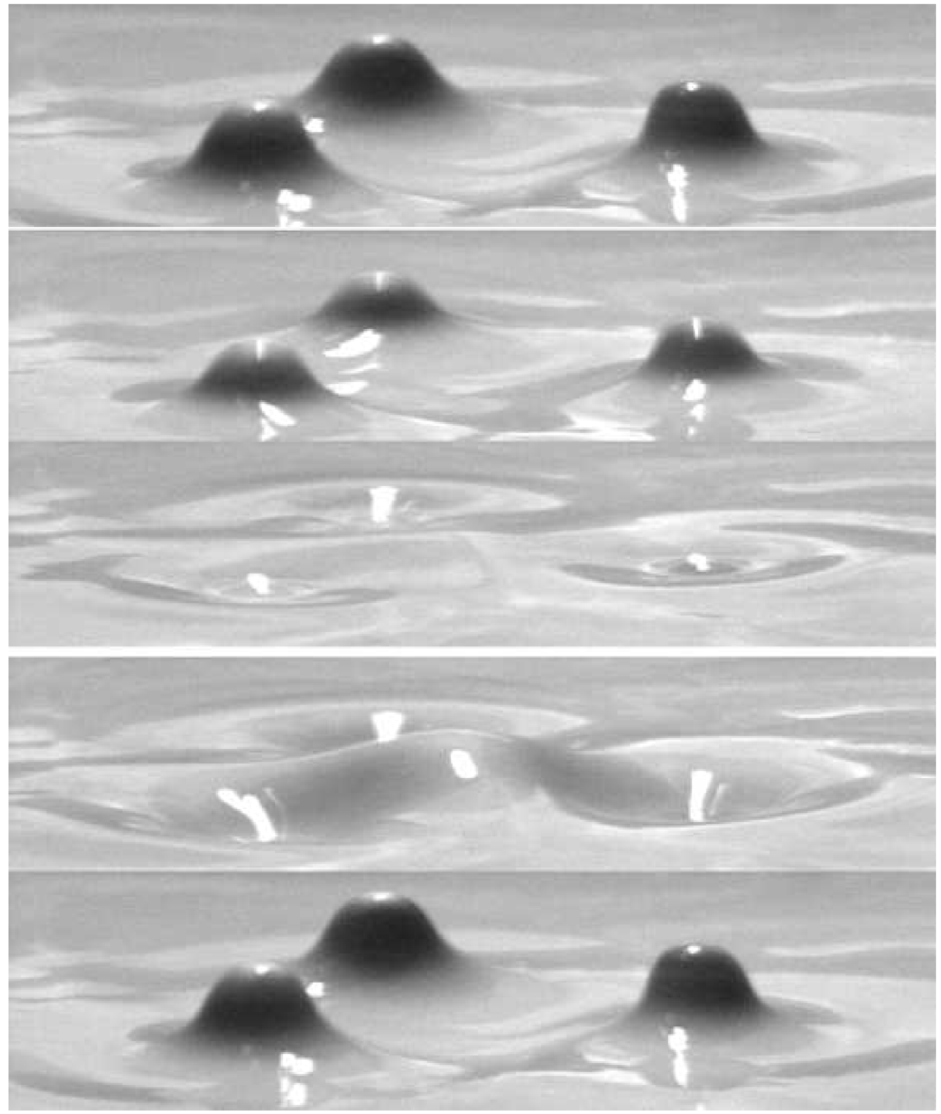
Physical examples and motivation

- Fluidized granular matter



F. Melo et al, Nature, 382, 793 (1996).

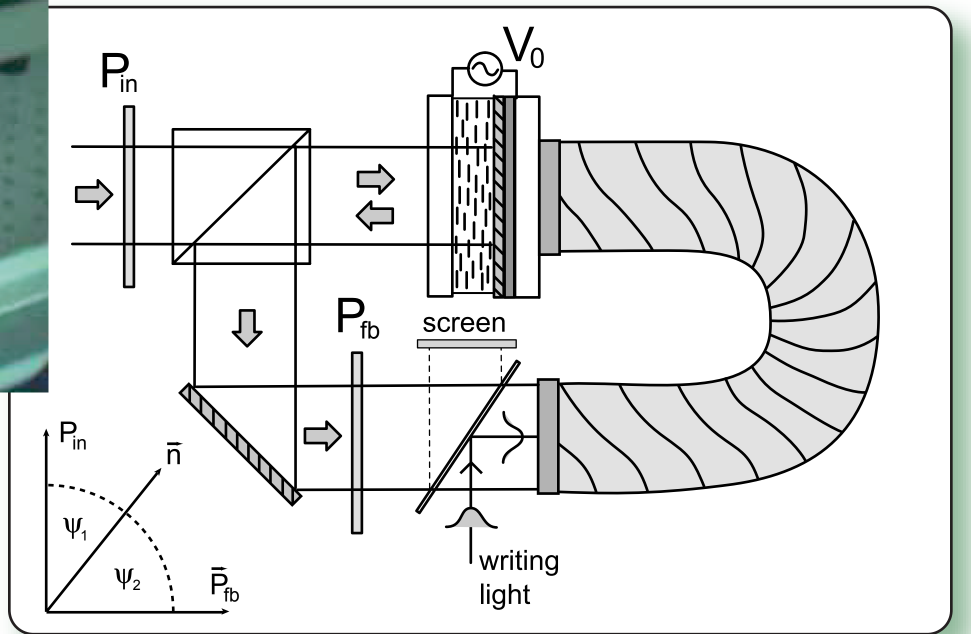
- Vertically vibrated colloidal Suspension

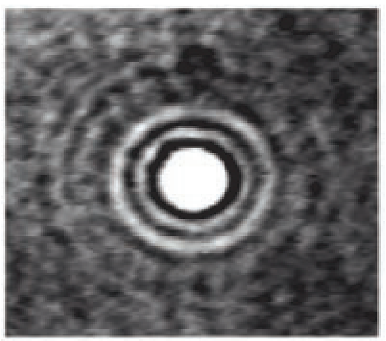


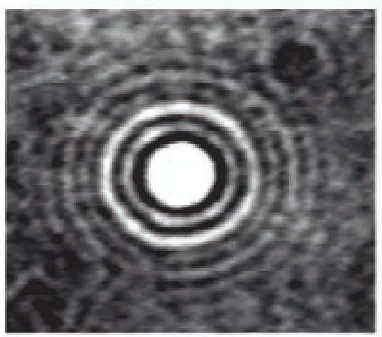
Phys. Rev. Lett. 83, 3190 (1999).

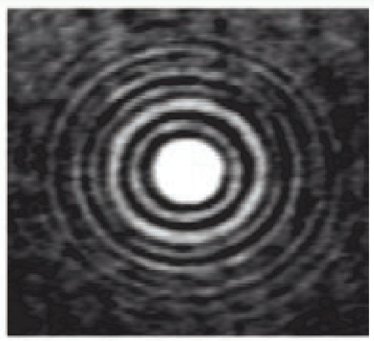
Physical examples and motivation

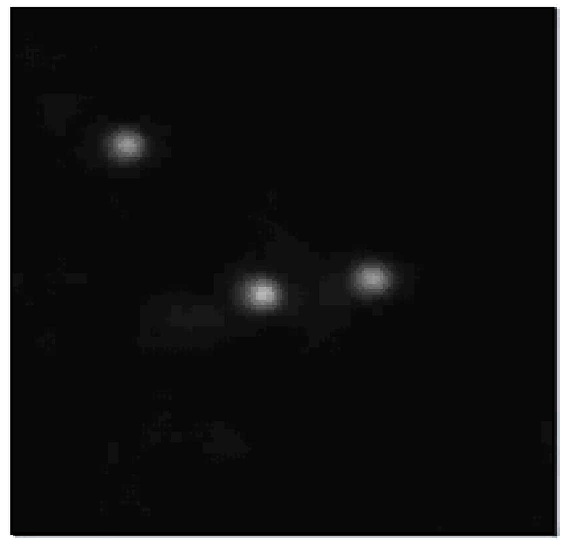
- Liquid crystal light valve with optical feedback







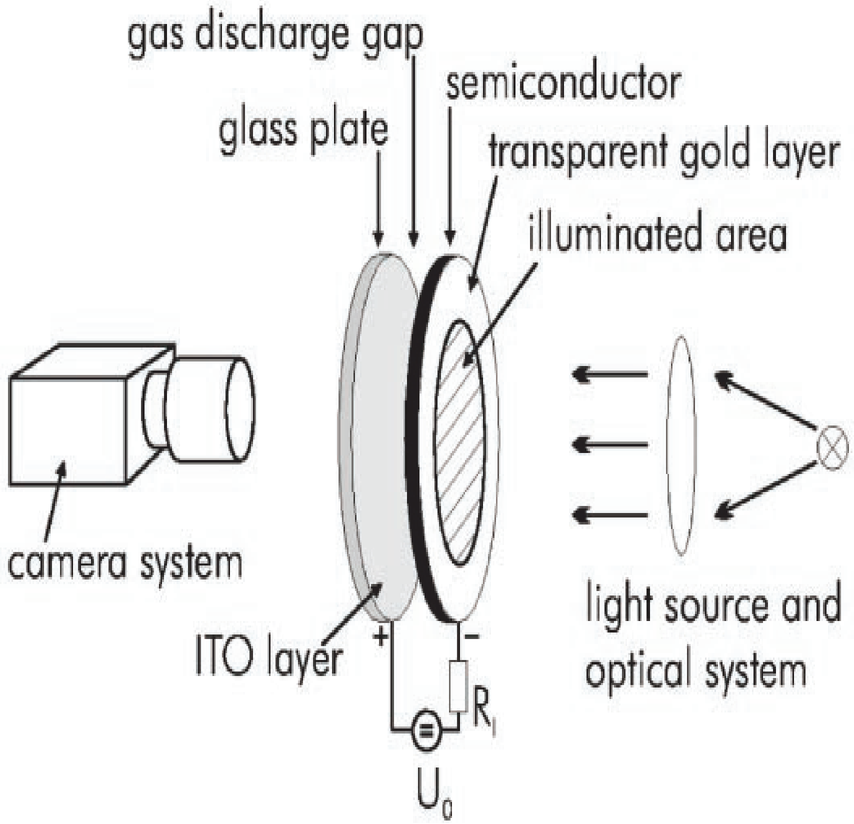


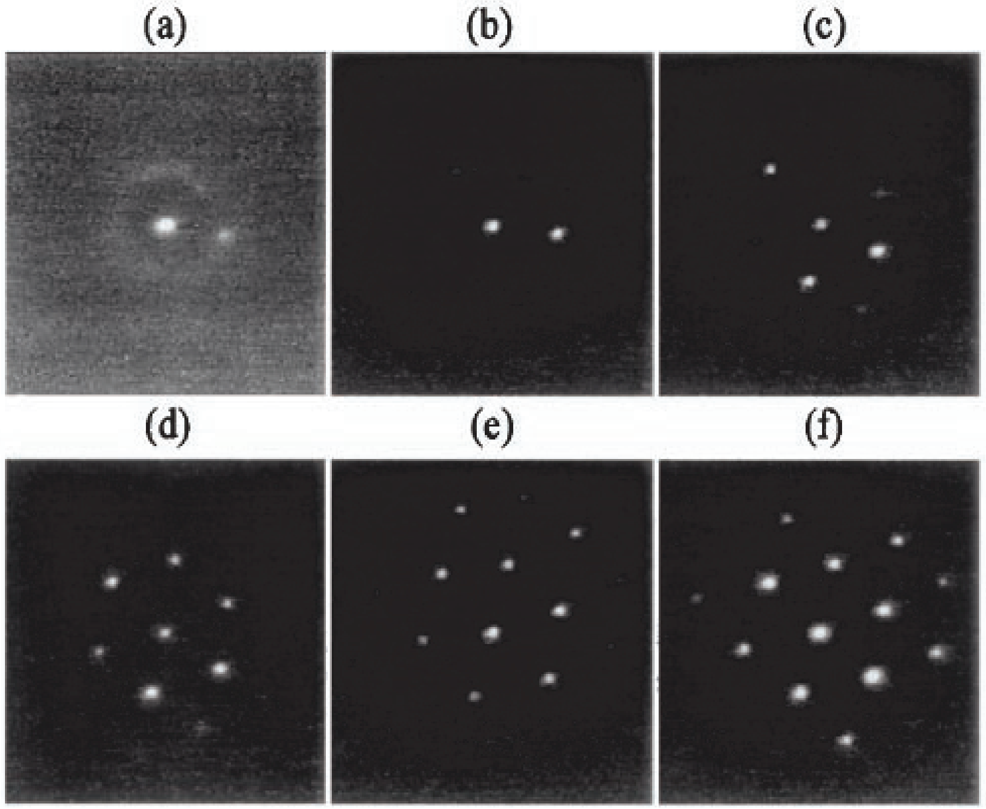


M.G. Clerc, A. Petrossian and S. Residori, Phys. Rev. E 71, 015205 (2005).

Physical examples and motivation

- Gas discharge system

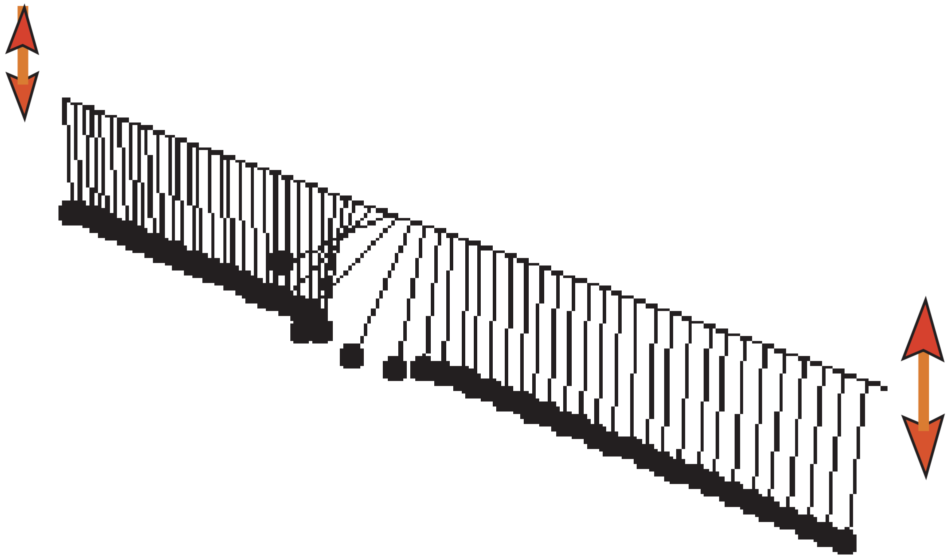


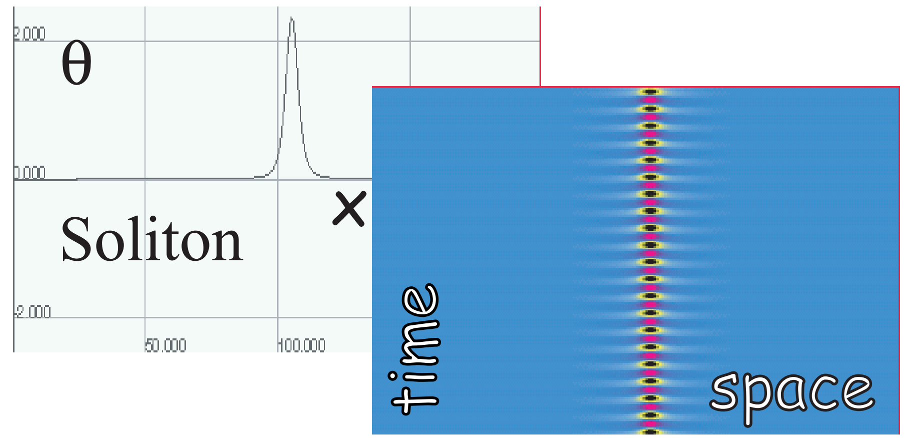


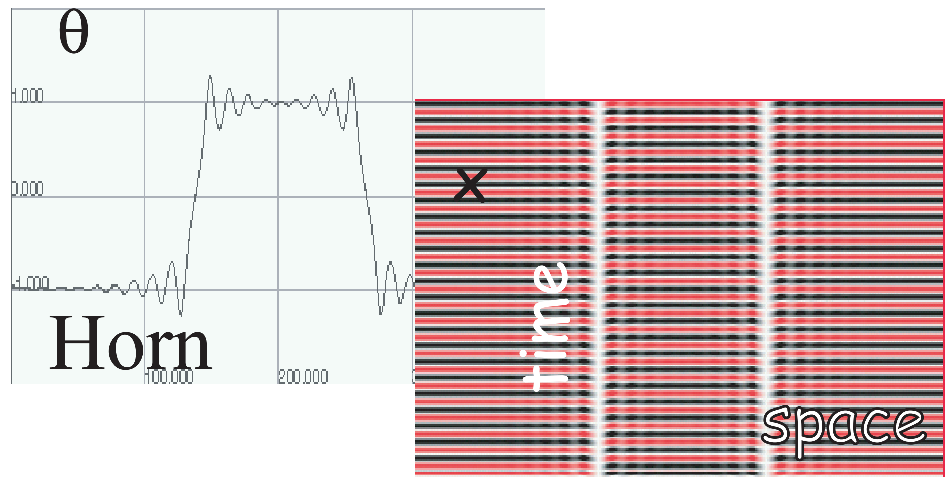
Y.A. Astrov and Y. A. Logvin, Phys. Rev. Lett. 79, 2983 (1997).

Physical examples and motivation

- Chain of pendula driven parametrically.







Continues description

$$\ddot{\theta} = -\left(1 + \gamma \sin\left(2\left[1+\nu\right] t\right)\right) \sin\left(\theta\right) - \mu\dot{\theta} + \partial_{xx}\theta$$

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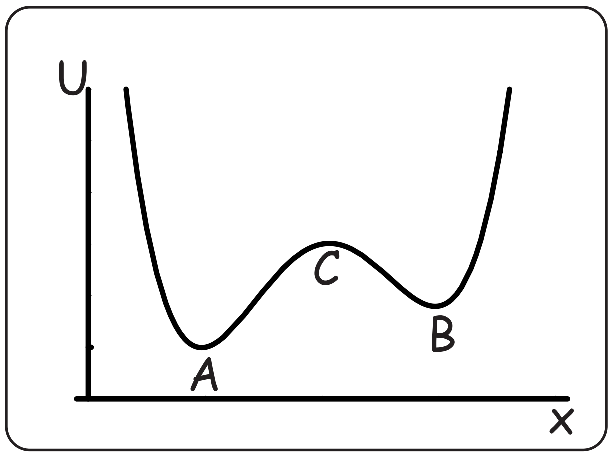
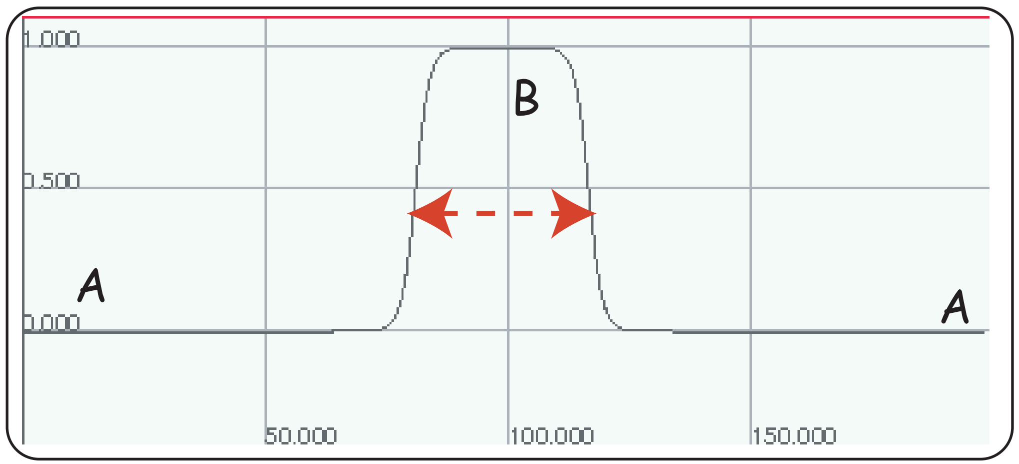
Robust phenomena

- The localized structures are a robust phenomenon!, observed in magnetic materials, liquid crystal, gas discharge, chemical reactions, fluids, granular matter and non linear optic.

Main ingredients of the localized structures

- Bistability between two homogeneous states
- Intrinsic length





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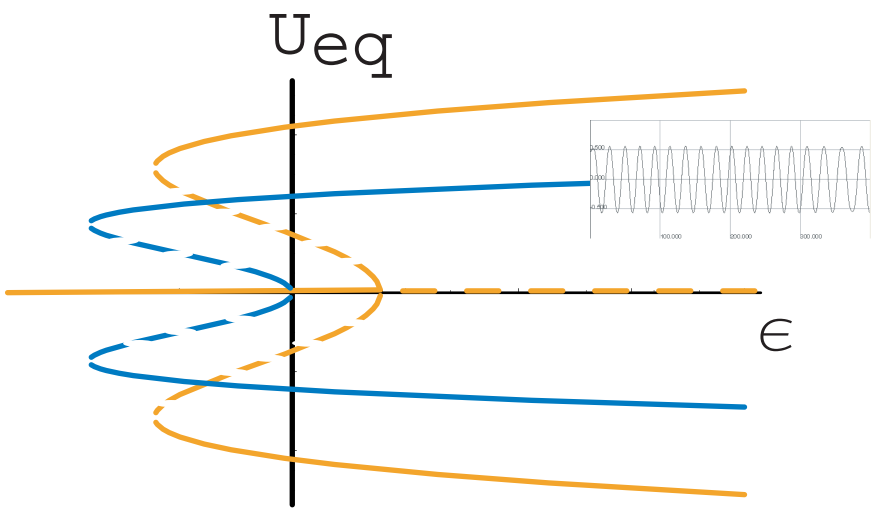
Prototype model of localized structures

- Subcritical Swift-Hohenberg Model

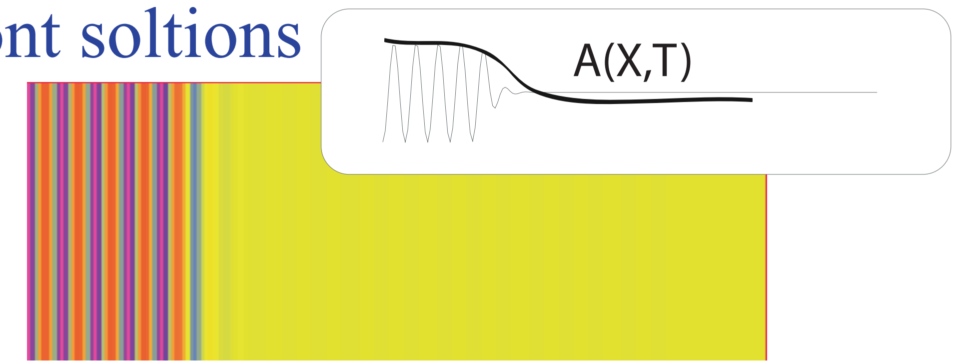
$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - (\partial_{xx} + q^2)^2 u + \sqrt{\eta}\zeta\,(x,t)$$

where $\langle \zeta\,(x,t)\,\zeta\,(x',t') \rangle = \delta\,(x - x')\,\delta\,(t - t')$

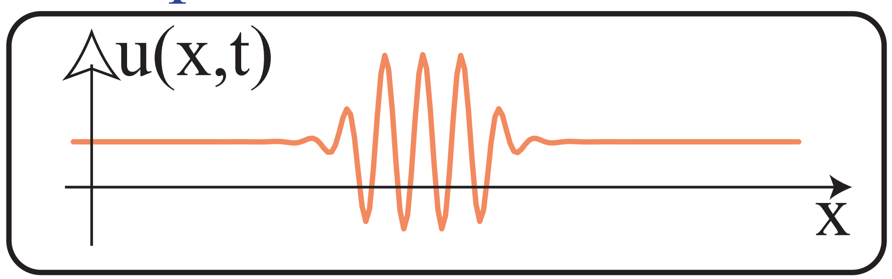
- Bifurcation Diagram (the system exhibits coexistence between uniform and pattern state)



Front soltions



Localized patterns



Prototype model of localized structures

- Subcritical Swift-Hohenberg Model

$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - (\partial_{xx} + q^2)^2 u + \sqrt{\eta}\zeta\,(x,t)$$

Reaction: biestability

where $\langle \zeta\,(x,t)\,\zeta\,(x',t') \rangle = \delta\,(x - x')\,\delta\,(t - t')$

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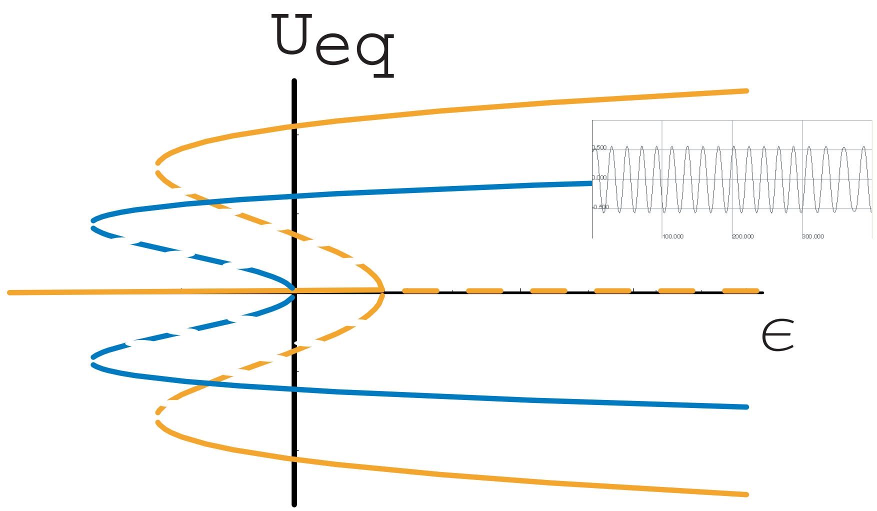
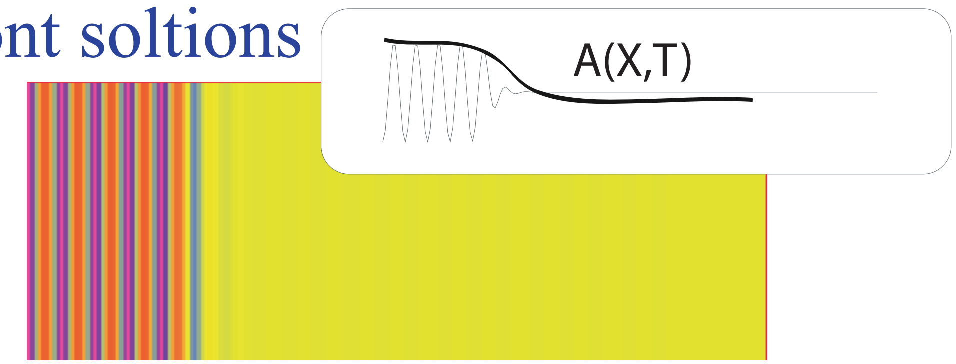


Front soltions



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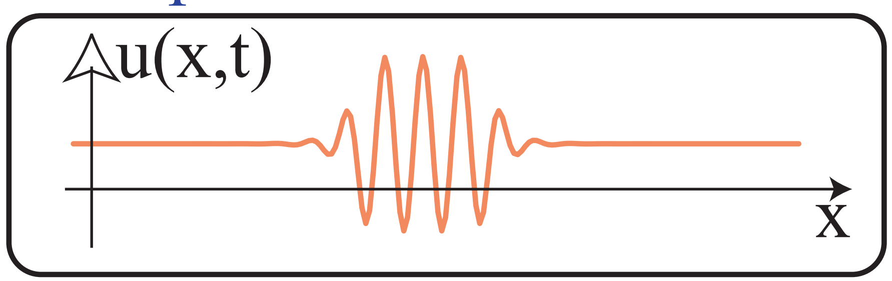

Prototype model of localized structures

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$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - (\partial_{xx} + q^2)^2 u + \sqrt{\eta}\zeta(x,t)$$

Transport: Intrinsic length "q"

where $\langle \zeta(x,t)\zeta(x',t')\rangle = \delta(x-x')\delta(t-t')$

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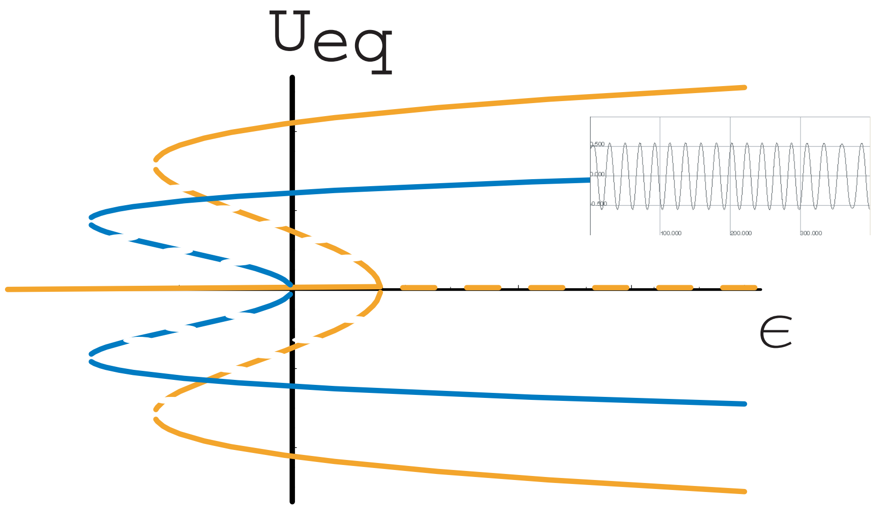
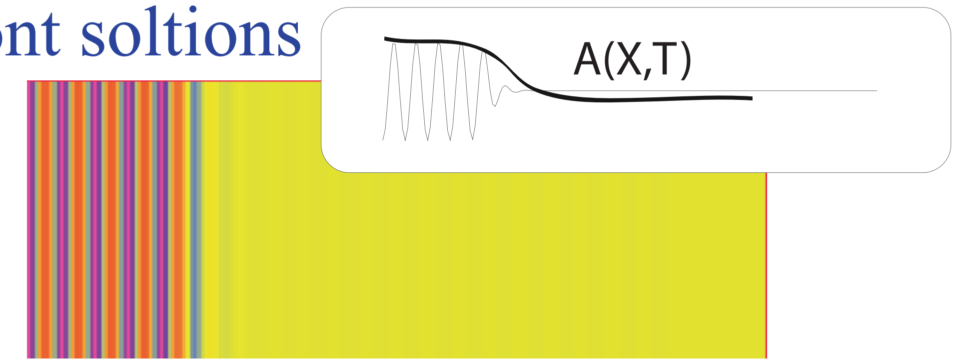
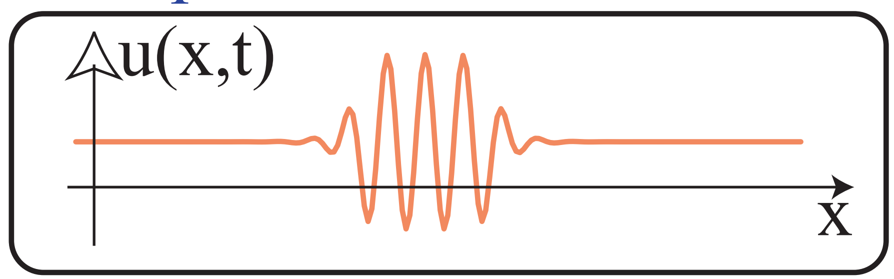


Front soltions



Localized patterns



Prototype model of localized structures

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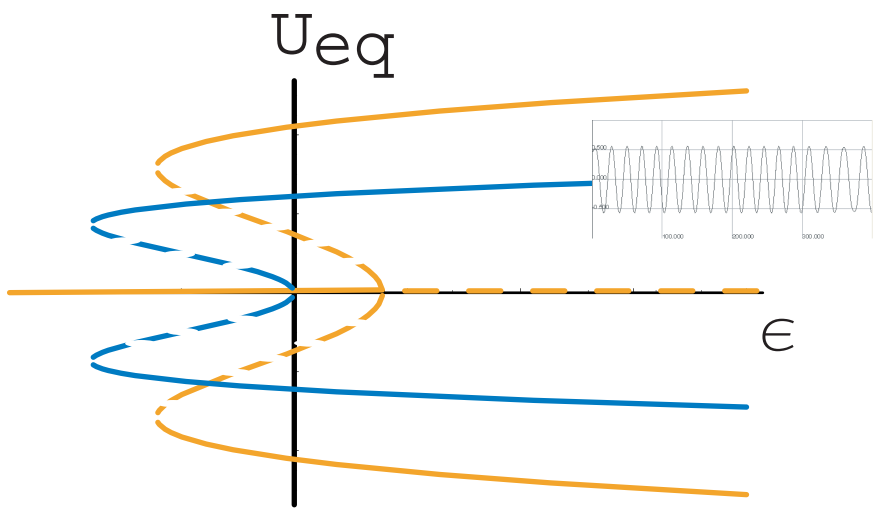
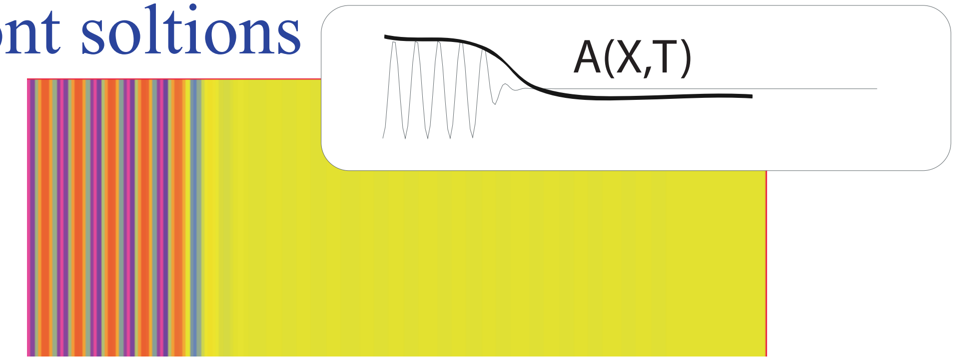
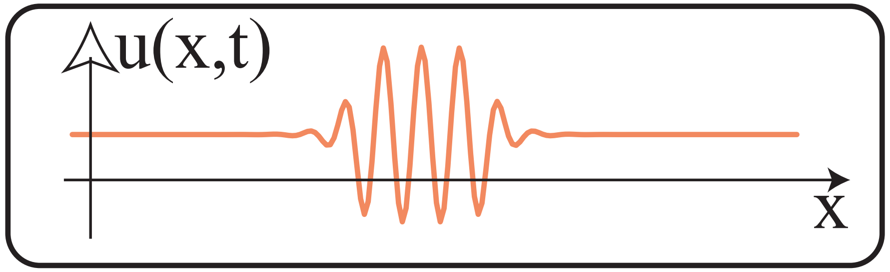


Front soltions



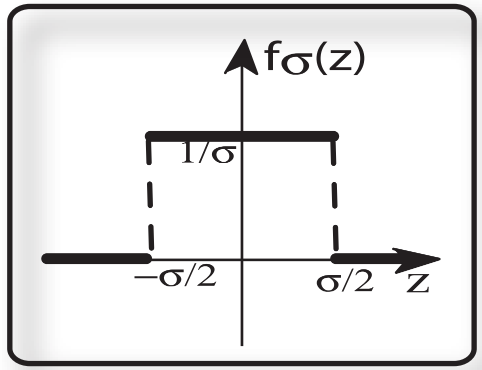
Localized patterns



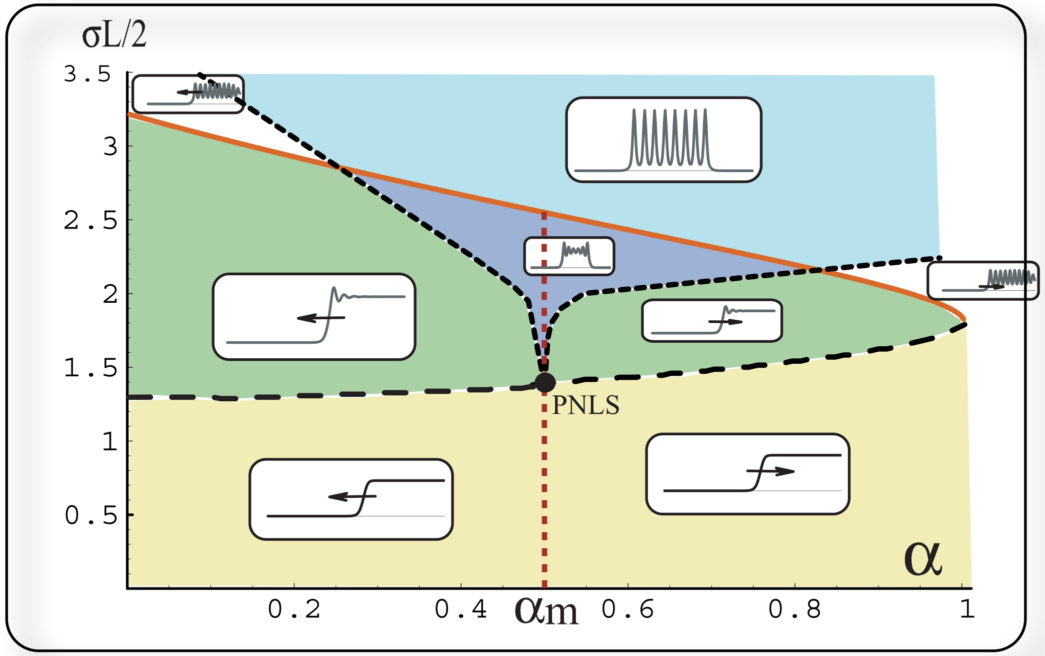
Non-local Nagumo model (Population Dynamic)

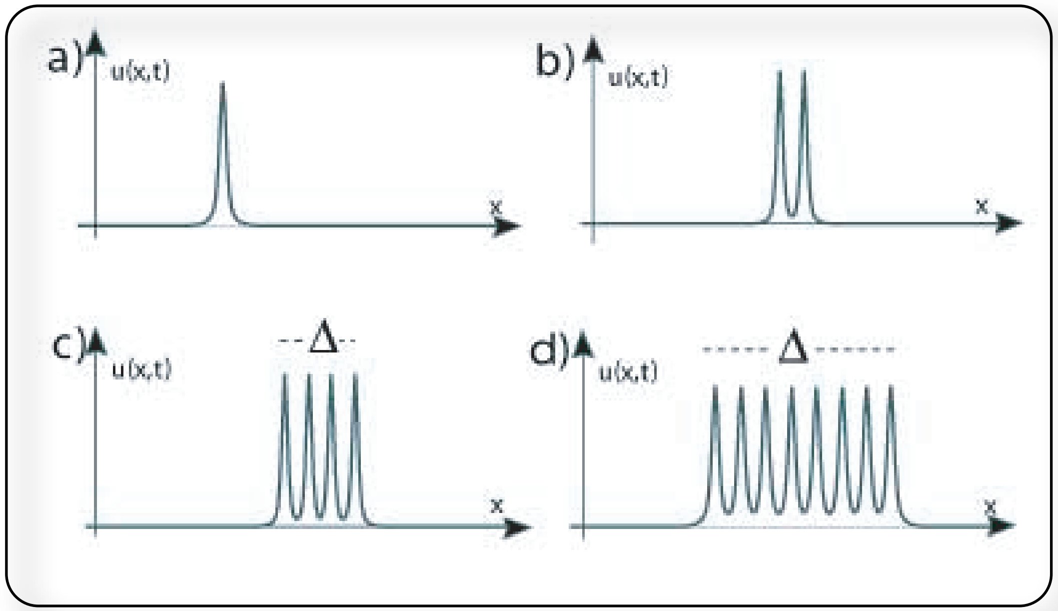
- A simple non-local model that exhibit bistability is

$$\partial_t u = \partial_{xx} u - \alpha u + (\alpha+1)u^2 - u\int_\Omega u'^2 f_\sigma(x,x')dx'$$



where the influence function $f_\sigma(x,x') = f_\sigma(x-x')$, is a even function and it is normalized $\int_\Omega f_\sigma(x,x')dx' = 1$. And $0 < \alpha < 1$.





M.G. Clerc, D. Escaff and V.M. Kenkre. To appear PRE

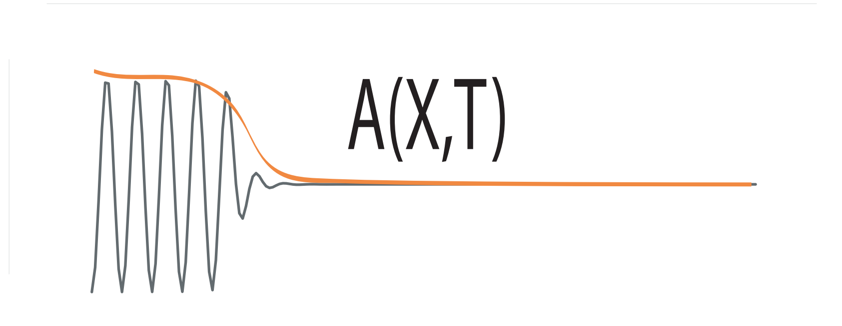
Universal description of localized structure

- Subcritical Swiht-Hohenberg model

$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - (\partial_{xx} + q^2)^2 u + \sqrt{\eta}\zeta(x,t)$$

Using the ansatz

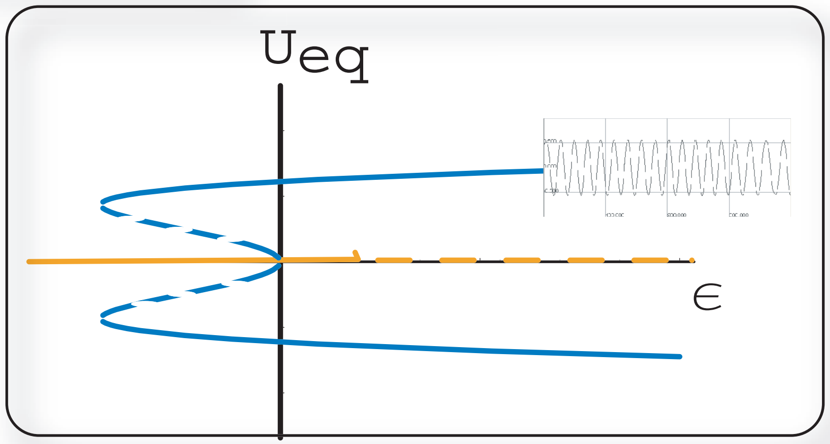
$$u(x,t) = \nu^{1/2} A(X,T) e^{iqx} + \nu^{5/2} W(X,x,T) e^{3iqx} + \text{c.c.} + \text{h.o.t.}$$



one obtains the envelope equation

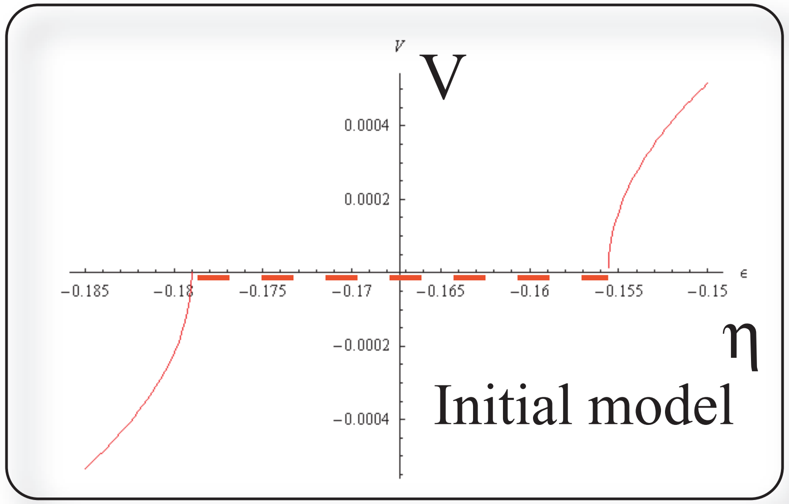
$$\partial_\tau A = \epsilon A + |A|^2 A - |A|^4 A + \partial_{yy} A$$

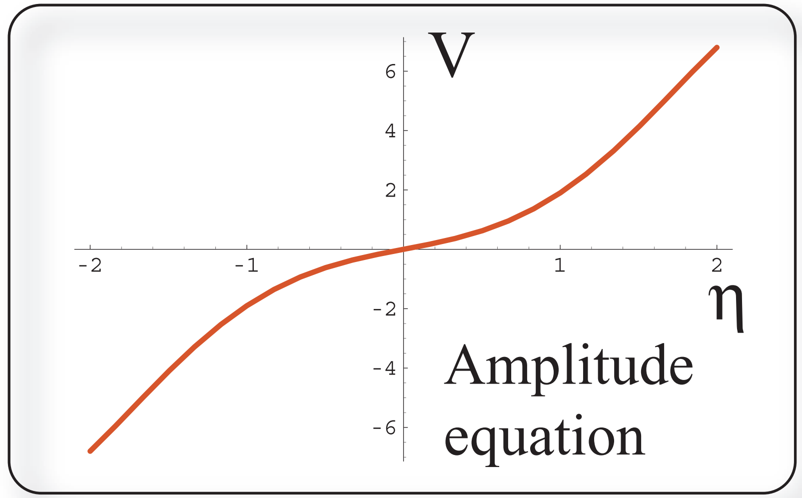
- Patterns state.
- The existence of coexistence of uniform state and pattern.
- Front solution



Problem of amplitude equation

- Front solutions do not have locking phenomena (adiabatic elimation)

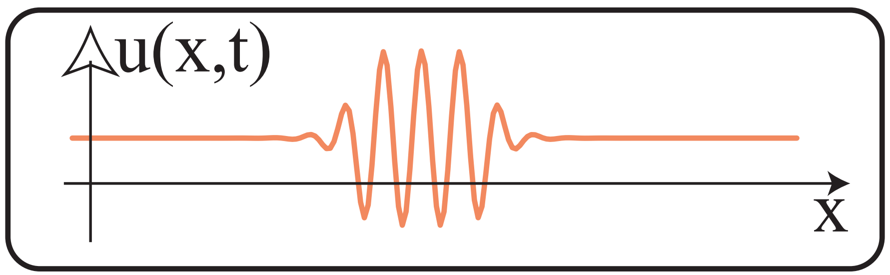


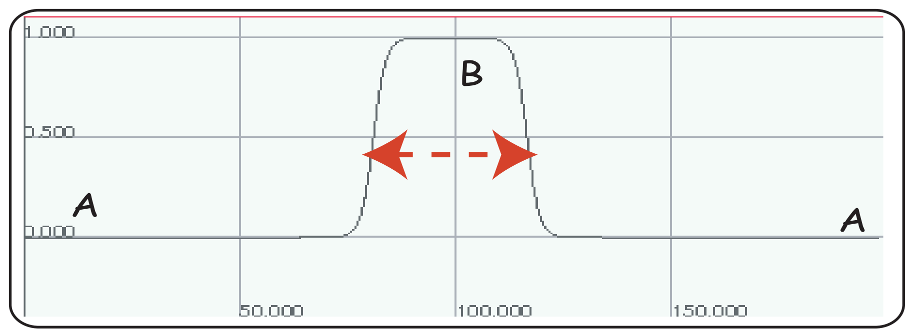


Pining range

Y Pomeau, Physica D 23, 3 (1986)

- Close to the pining range the system exhibits localized patterns and the amplitud equation do not exhibits stable localized solution





Amended amplitude equation

- In the derivation of the amplitude equation, we have assumed that the spatial variation of the amplitude are large in compare to the spatial variation of the pattern. This it is false!

$$u(x,t)=\nu^{1/2}A(X,T)e^{iqx}+\nu^{5/2}W(X,x,T)e^{3iqx}+c.c.+h.o.t.$$

Considered the non resonate terms (rapidly spatial varying perturbation) the amplitude equation reads (amended amplitude equation)

$$\partial_\tau A = \epsilon A + |A|^2 A - |A|^4 A + \partial_{yy} A$$
$$+\left(\frac{A^3}{9\nu} - \frac{A^3|A|^2}{2}\right) e^{\frac{2iqy}{a\sqrt{|\varepsilon|}}} - \frac{A^5}{10} e^{\frac{4iqy}{a\sqrt{|\varepsilon|}}} + \frac{\sqrt{\eta} b}{|\varepsilon|^2} e^{\frac{iqy}{a\sqrt{|\varepsilon|}}} \zeta(y,\tau)$$

Non resonante terms

Generalization

- A dynamical system that exhibits coexistence between a patterns and homogenoeus state, always close to the spatial bifurcation we can introduce

$$u(x,t)=\nu^{1/2}A(X,T)e^{iqx}+\nu^{5/2}W(X,T)e^{3iqx}+c.c.+h.o.t.$$

symmetry arguments $\{x \rightarrow -x,\ A \rightarrow \bar{A}\}$ $\{x \rightarrow x + x_o,\ A \rightarrow Ae^{iqx_o}\}$

The envelope equation satisfies

$$\partial_T A = f\left(|A|^2\right) A + \partial_{XX} A + \sum_{m,n} g_{mn} A^m \bar{A}^n e^{iq(1+n-m)x}$$

One has analogous arguments.

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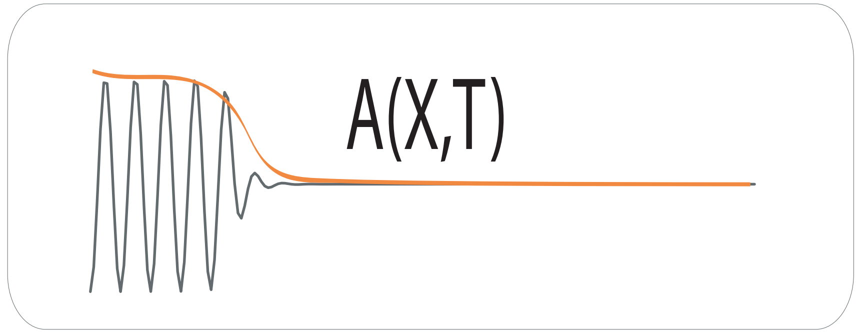

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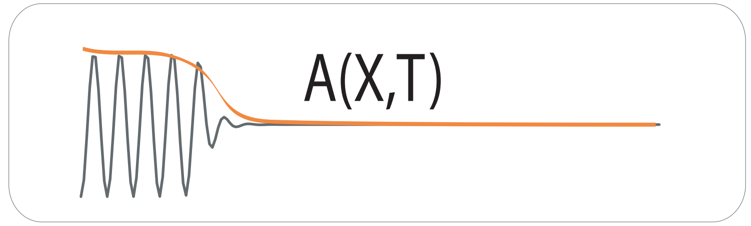
Non resonante terms

Amended amplitude equation

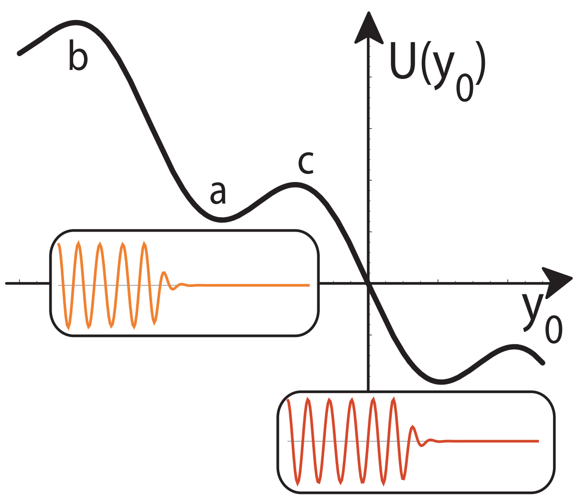
- When the non-resonante termrs are negligible, the system has an analytical front solutions in the Maxwell point



$$A_{\pm} = \sqrt{\frac{3/4}{1+e^{\pm\sqrt{3/4}(y-y_o)}}}\, e^{i\theta}$$

- In order to study the effect of the non-resonante terms in the dynamics of the core front, we use the ansatz

$$A\left(y,\tau\right) = \left(A_{+}(y-y_o(\tau)) + \delta\rho\right)e^{i\delta\Theta}$$



$$\dot{y}_o = -\frac{\partial U\left(y_0\right)}{\partial y_o} + \frac{ab}{|\epsilon|^2}\sqrt{\frac{\eta}{2d}}\zeta\left(\tau\right)$$
$$= \Delta + \Gamma\cos\left(\frac{2q}{d\sqrt{|\epsilon|}}y_o - \varphi\right) + \frac{ab}{|\epsilon|^2}\sqrt{\frac{\eta}{2d}}\zeta\left(\tau\right)$$

M.G. Clerc et al, Phys. Rev. Lett. 94, 148302 (2005).

Amended amplitude equation

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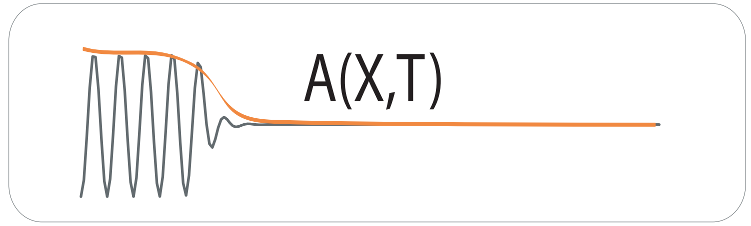

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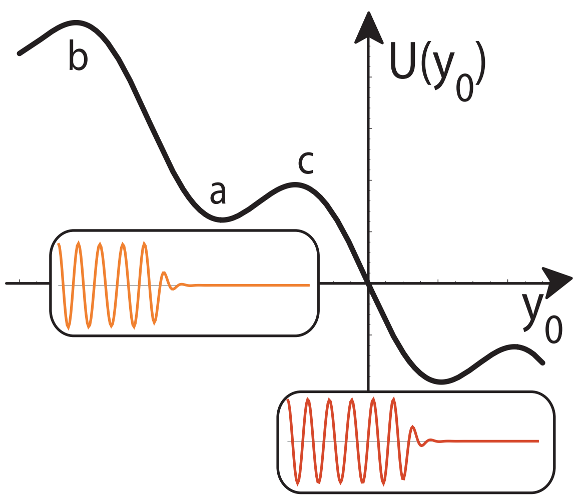

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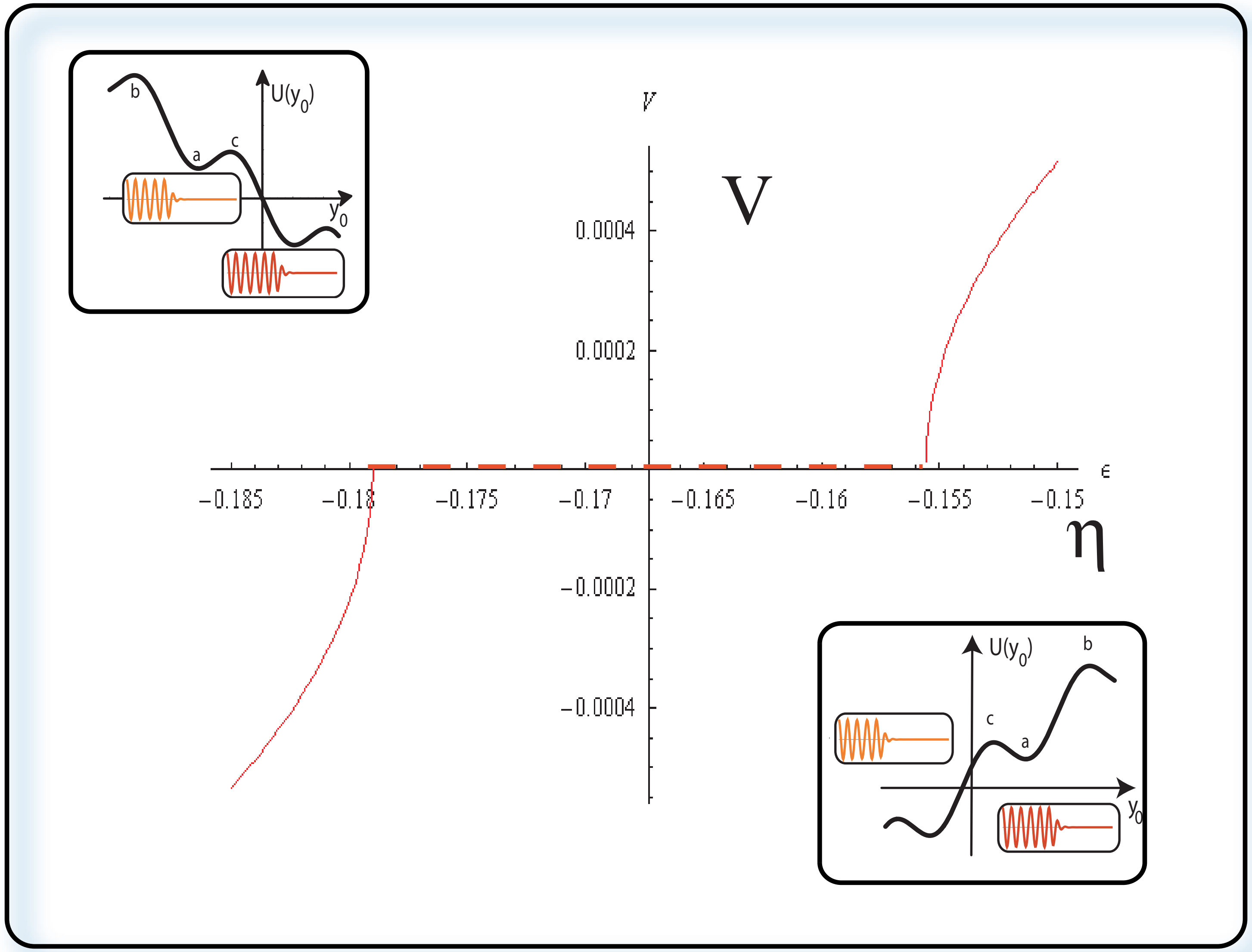
Non-Resonante terms

M.G. Clerc et al, Phys. Rev. Lett. 94, 148302 (2005).

Amended amplitude equation

- The non-resonante terms are responsable of the locking phenomena and pining range



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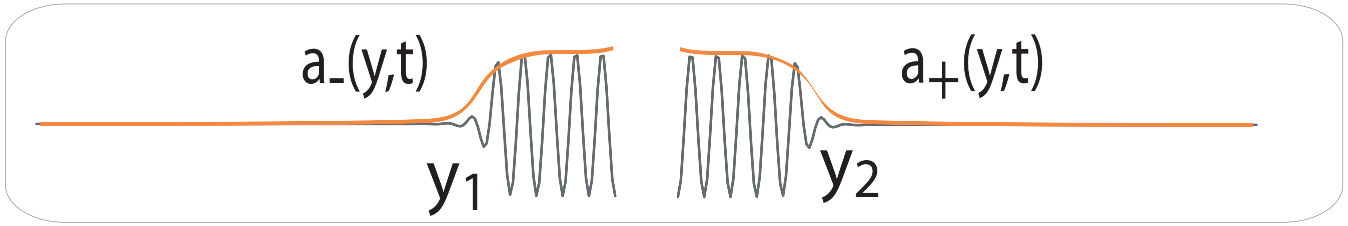
Ansatz Localized Structure

- One can imagine that a localized structure is composed by two front (Front Interactions)



Close to the Maxwell point ($\varepsilon=\varepsilon_m+\delta\varepsilon$), we use the ansatz (We consider all non-resonant terms as perturbations)

$$A_{LP}(y,\tau) = \left[a_-(y-y_1(\tau)) + a_+(y-y_2(\tau)) - \sqrt{\frac{3}{4}} + \rho(y_1,y_2,y,\tau) \right] e^{i\theta(y_1,y_2,y,\tau)} ,$$

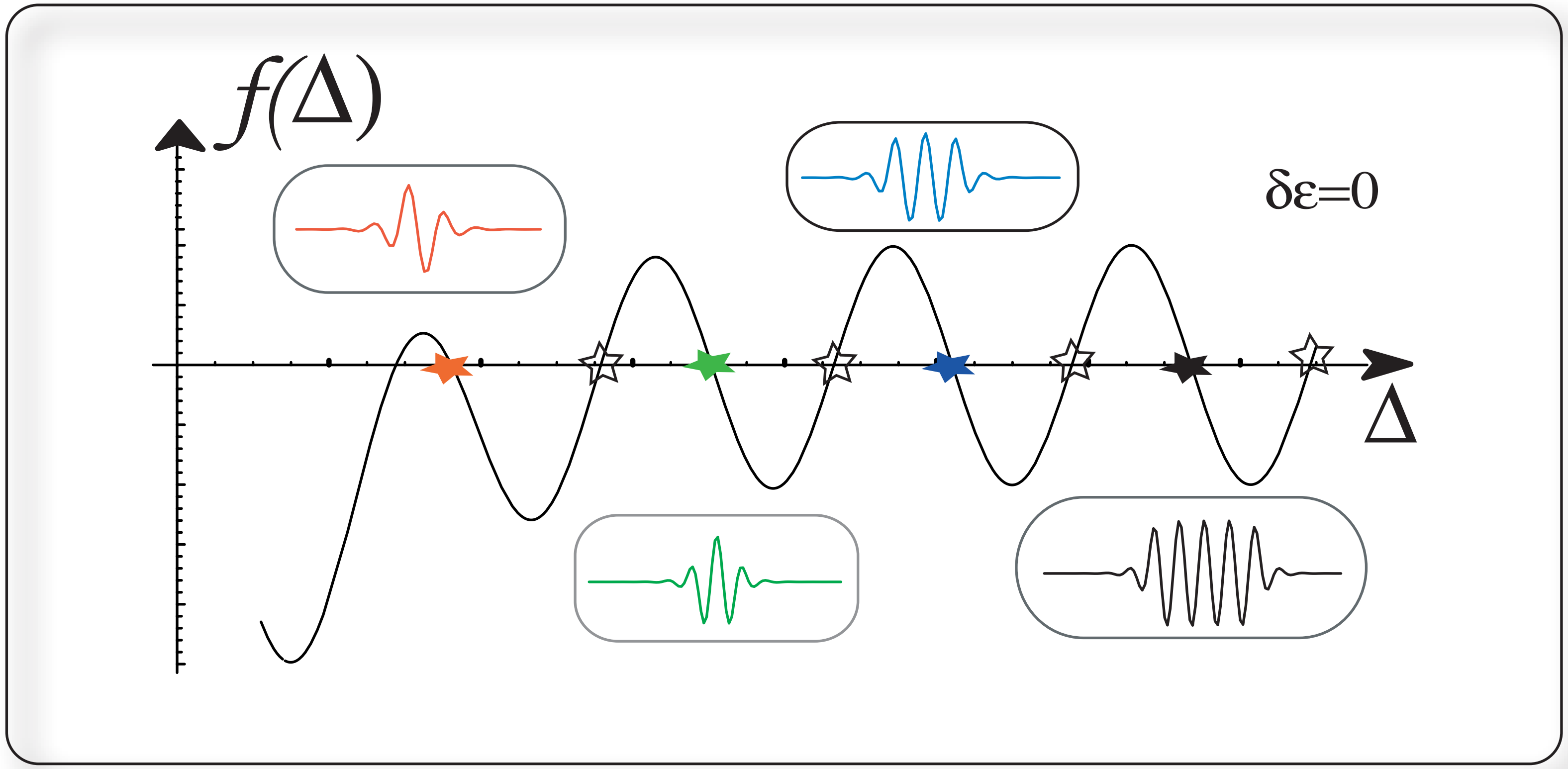
in the amended amplitude equation. Where ρ and θ are small functions. Introducing $\Delta=y_2-y_1$

Front Interaction

- We obtain the following solvability condition for the distance between the fronts

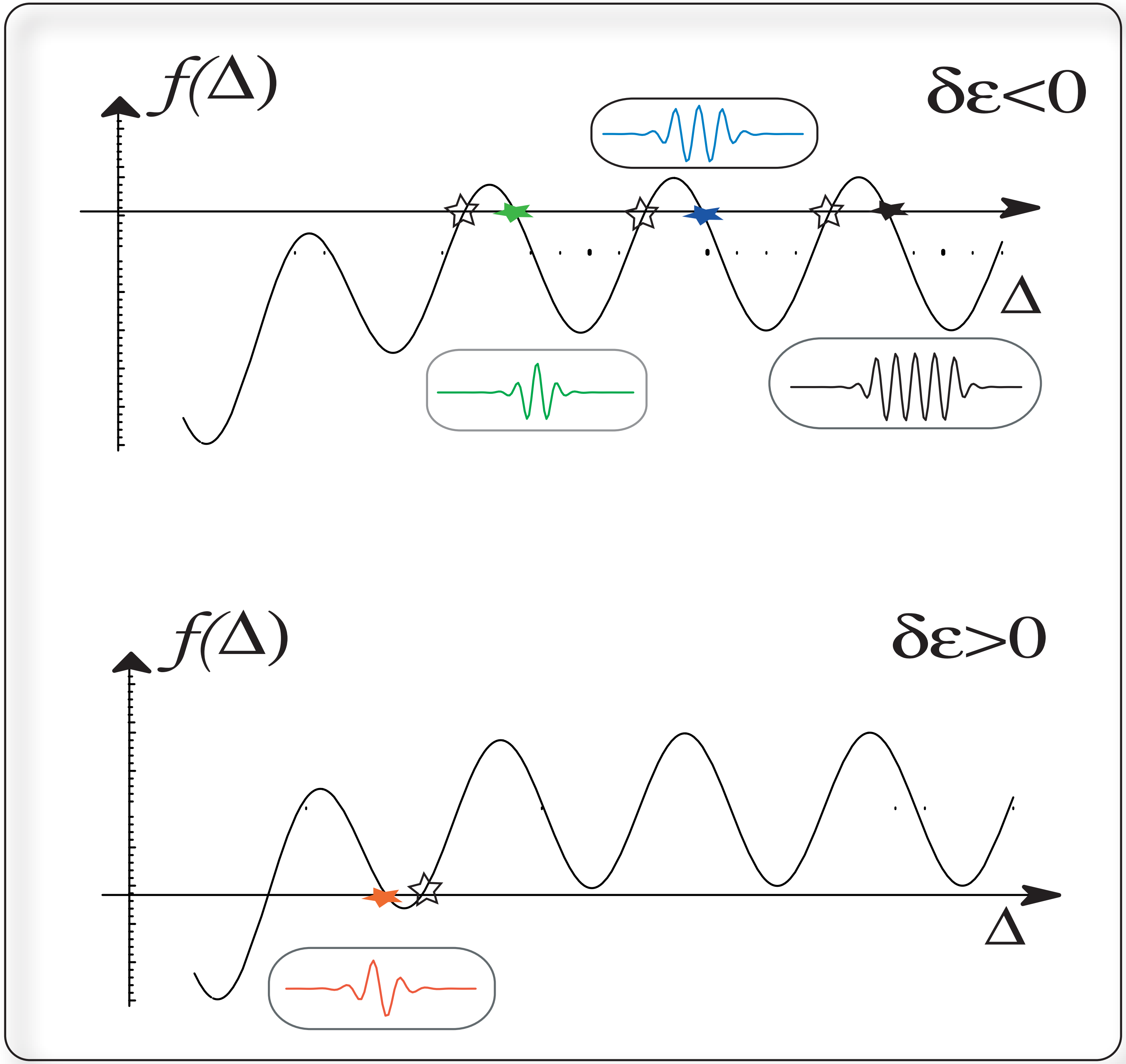
$$\frac{d\Delta}{d\tau} = f(\Delta) \equiv -\alpha \exp\left(-\sqrt{\frac{3}{4}}\Delta\right) + \beta\cos(2q\Delta/\sqrt{\varepsilon}) + 2\delta\varepsilon\,,$$

where $\alpha = 27\sqrt{3}/64$ and $\beta = 64\sqrt{3}q^2\exp(-q4\pi/\sqrt{\varepsilon})/3\varepsilon.$



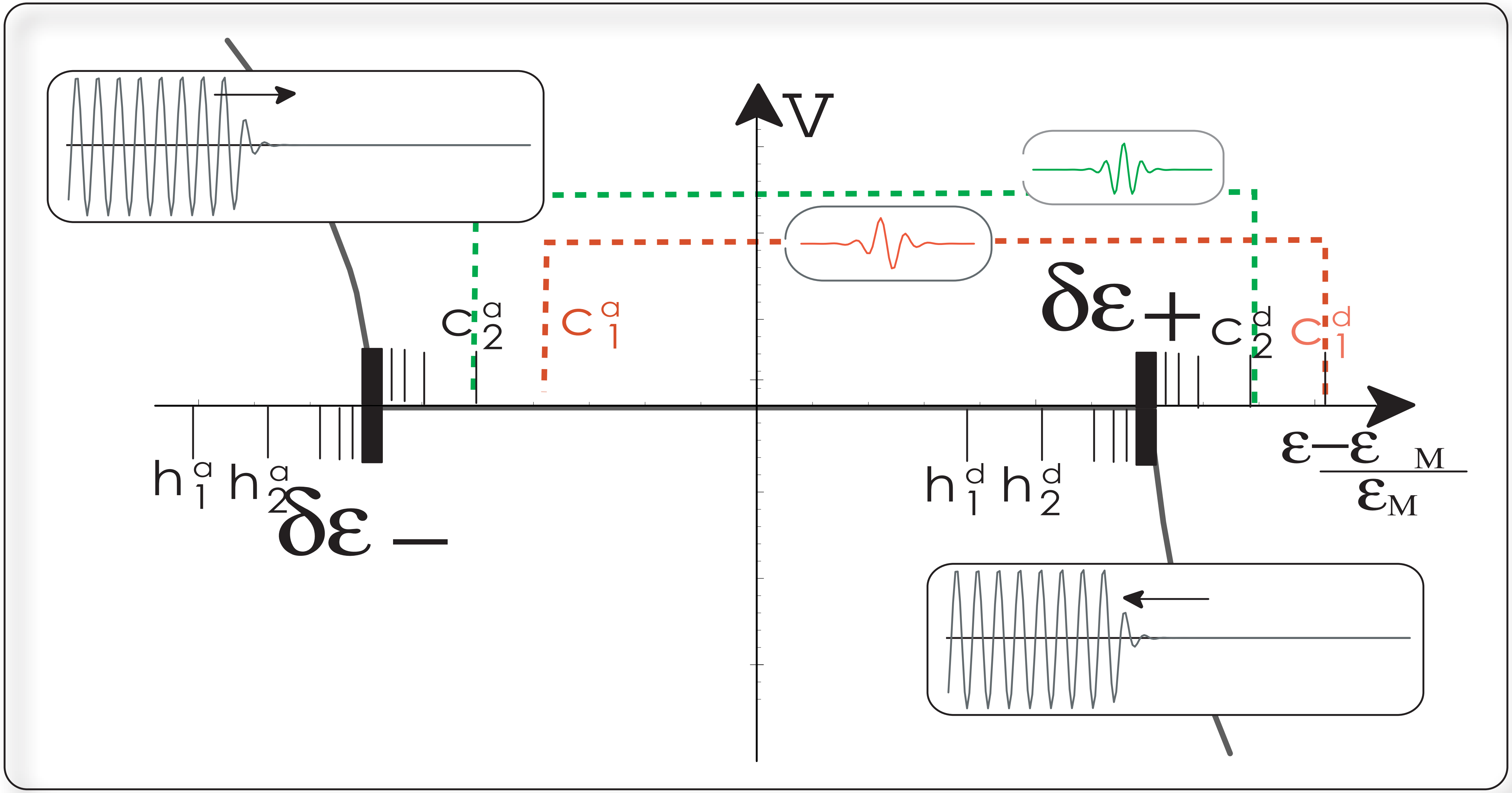
M.G. Clerc and C. Falcon, Physica A 356, 48 (2005).

Bifurcation diagram of Localized structure



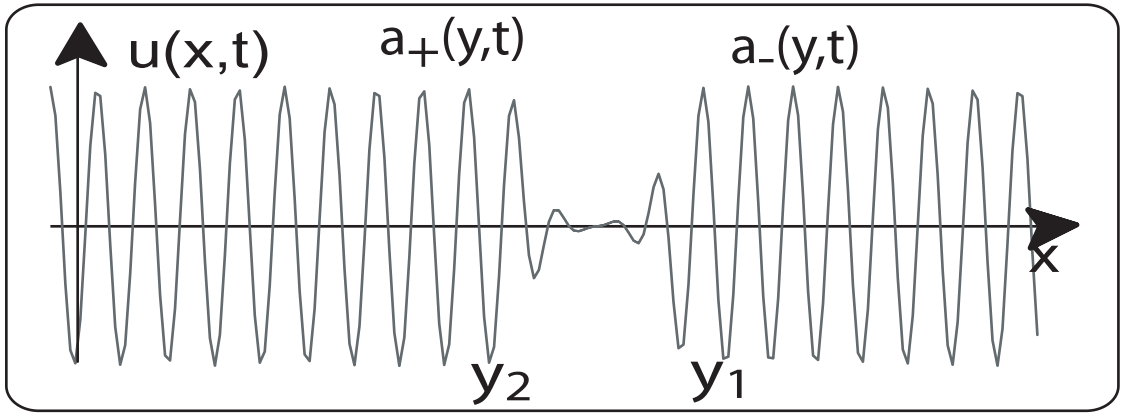
Changing the bifurcation parameter, we observe the localized patterns appear an desappear by saddle ode bifurca tion

Bifurcation diagram of Localized structure



P. Coullet, C. Riera and C. Tresser, Phys.Rev. Lett. 84, 3069 (2000).
M.G. Clerc and C. Falcon, Physica A 356, 48 (2005).

Hole solutions



- One can imagine that a hole solution composed by two front (Front interactions, $y_1>y_2$)

- Close to the Maxwell point ($\varepsilon=\varepsilon_m+\delta\varepsilon$), we use the ansatz (We consider all non-resonant terms as perturbations)

$$A_{LP}(y,\tau) = \left[a_-(y-y_1(\tau)) + a_+(y-y_2(\tau)) \qquad + \rho(y_1,y_2,y,\tau)\right] e^{i\theta(y_1,y_2,y,\tau)},$$

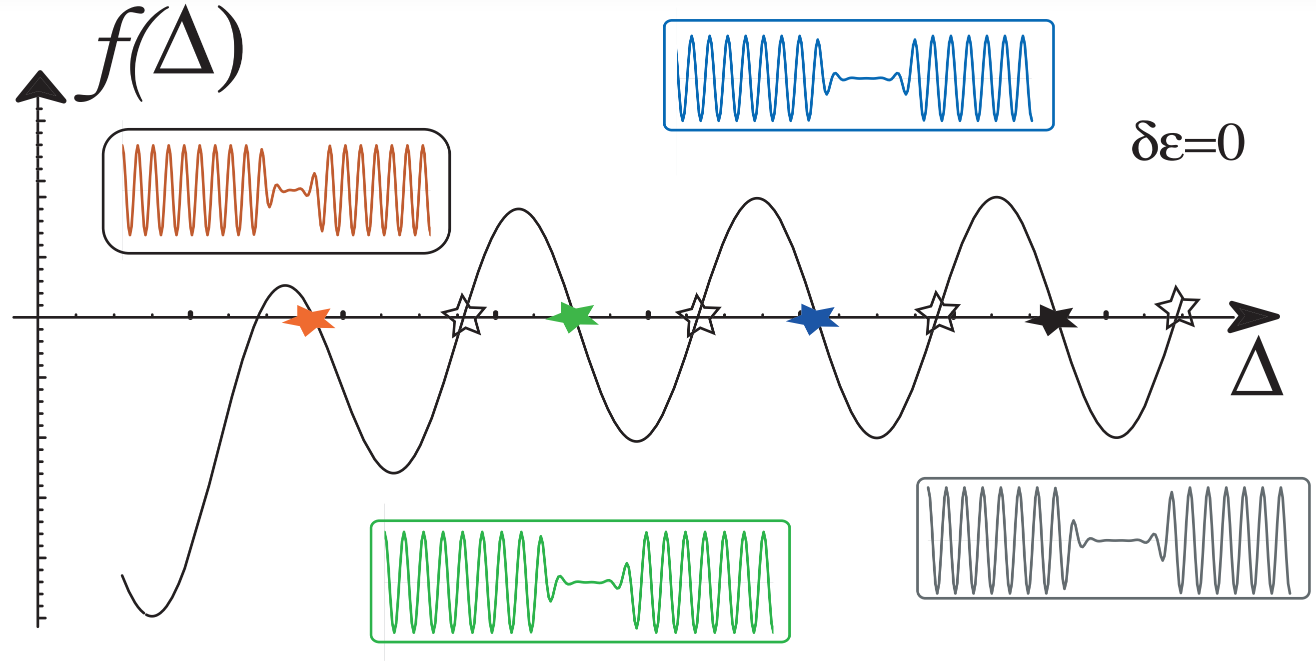
in the amended amplitude equation. Where ρ and θ are small functions. Introducing $\Delta=y_1-y_2$

Hole solutions

- We obtain the following solvability condition for the distance between the fronts

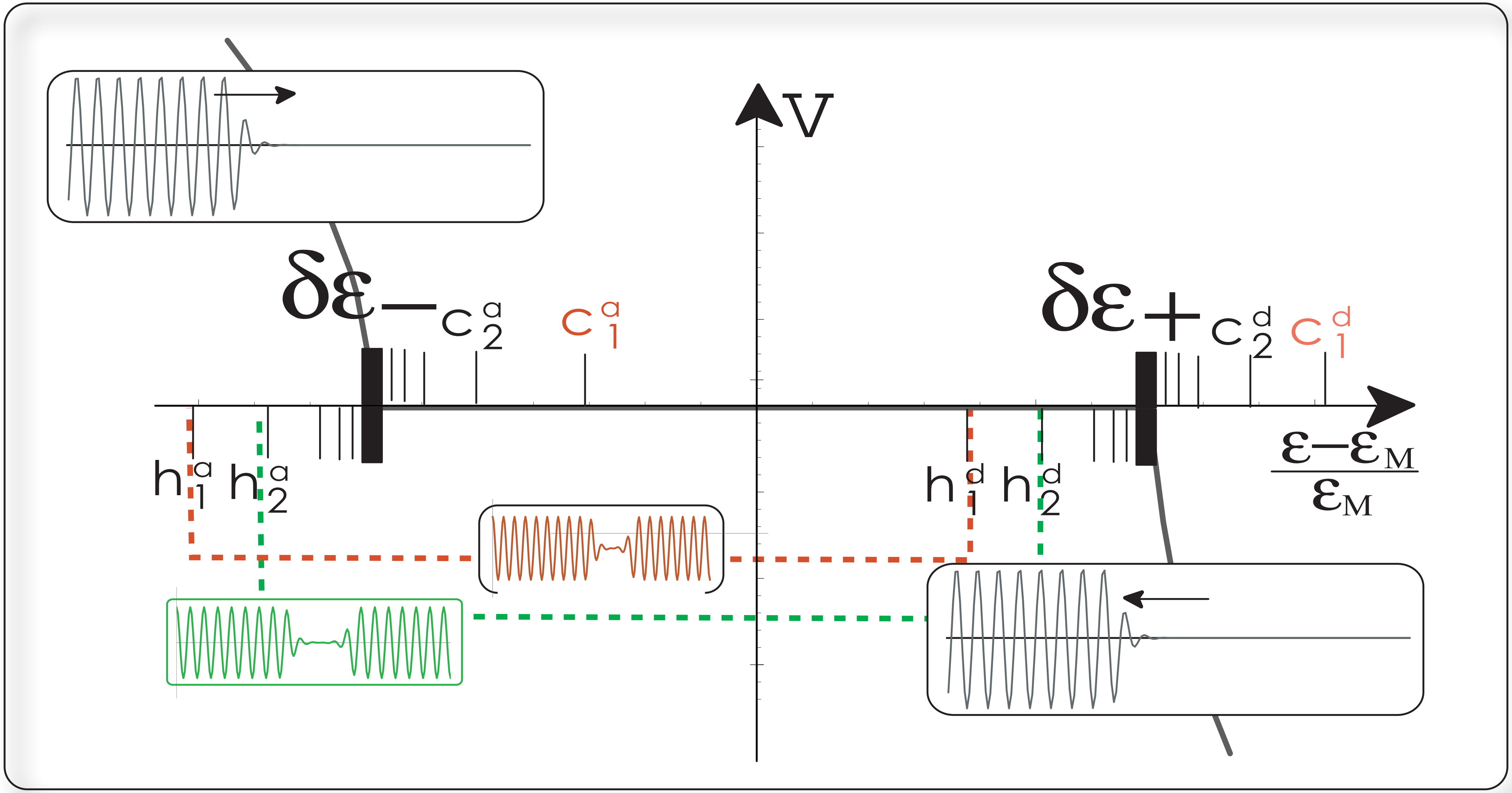
$$\frac{d\Delta}{d\tau} = f(\Delta) \equiv -\alpha \exp\left(-\sqrt{\frac{3}{4}}\Delta\right) + \beta\cos(2q\Delta/\sqrt{\varepsilon}) - 2\delta\varepsilon\,,$$

where $\alpha = 27\sqrt{3}/64$ and $\beta = 64\sqrt{3}q^2\exp(-q4\pi/\sqrt{\varepsilon})/3\varepsilon.$



M.G. Clerc and C. Falcon, Physica A 356, 48 (2005).

Bifurcation diagram of Localized structure



P. Coullet, C. Riera and C. Tresser, Phys.Rev. Lett. 84, 3069 (2000).
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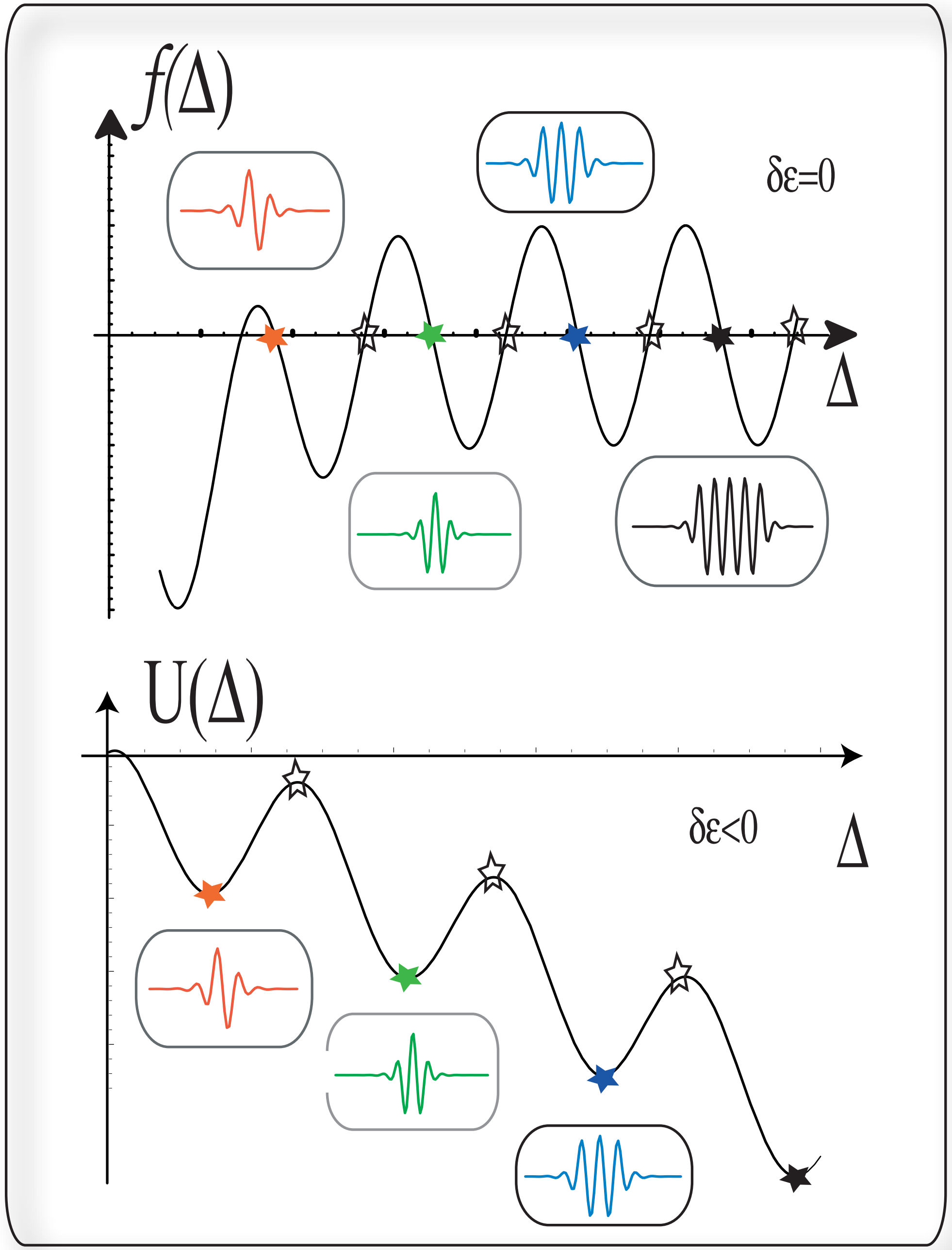
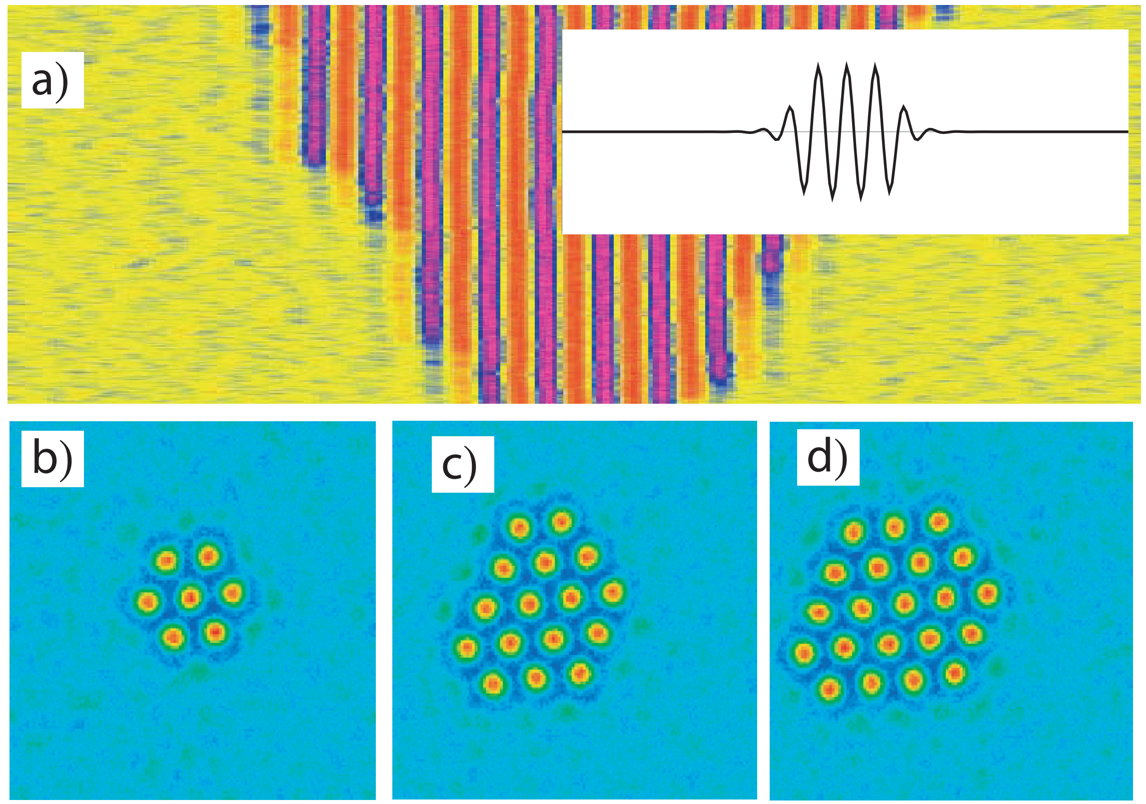
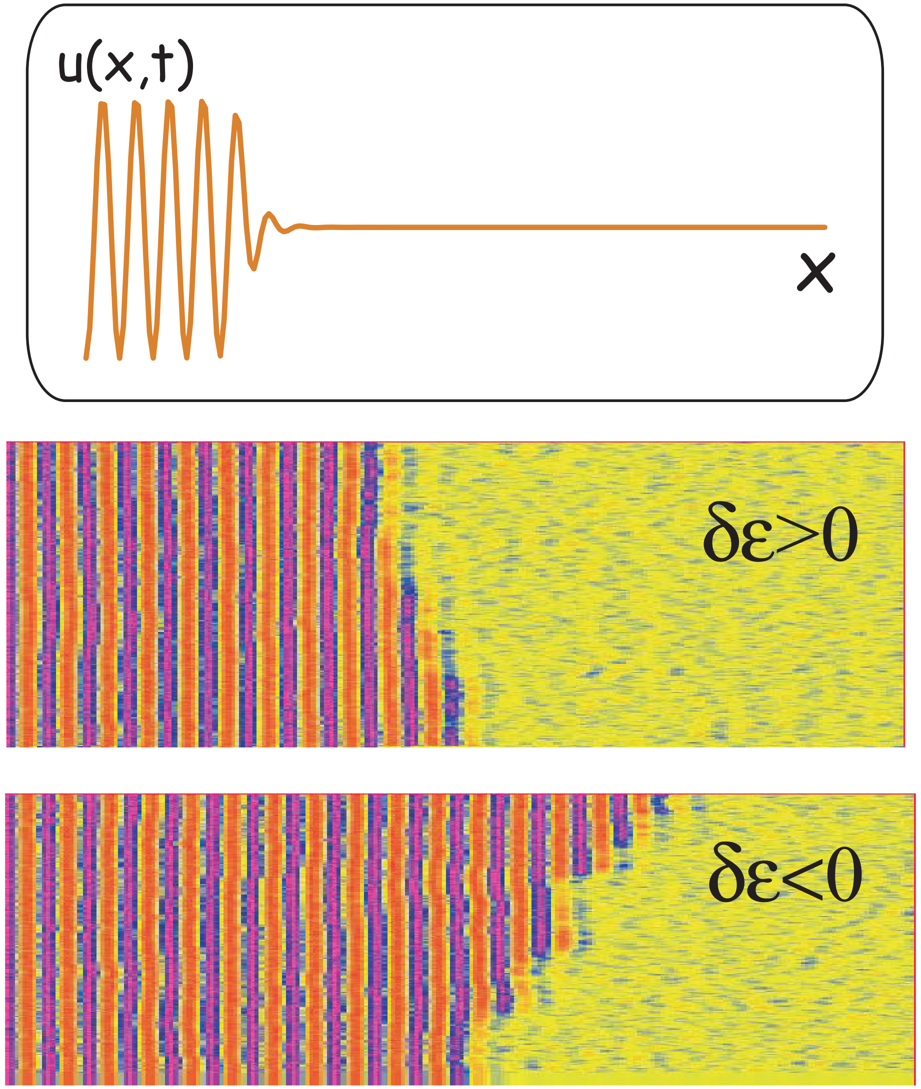
Noise induces front propagation

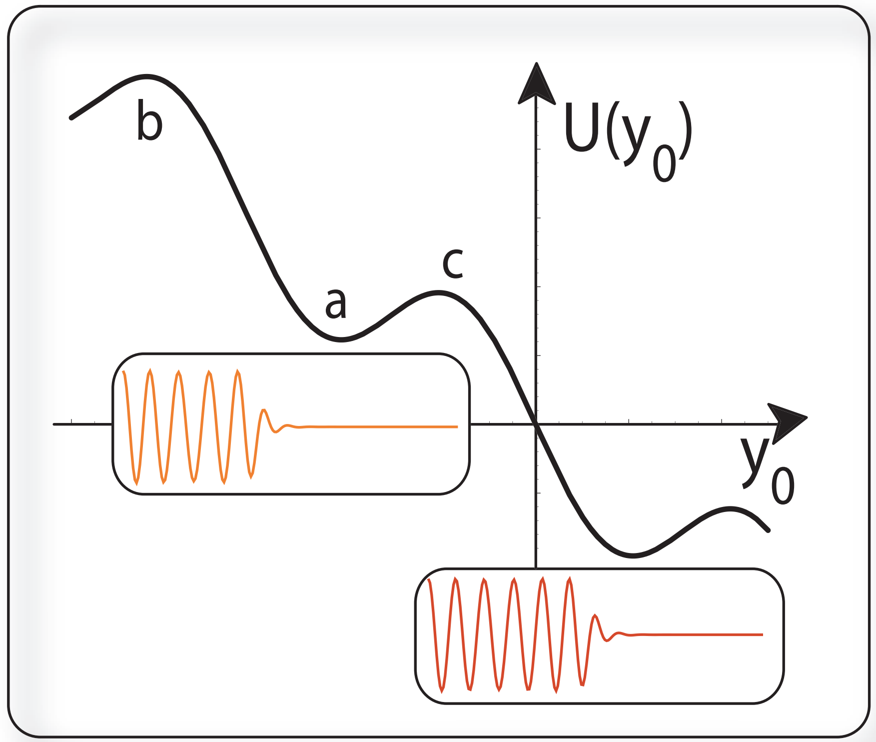




Noise induces that the pattern (uniform) states invades the uniform (pattern) one, depending of bifurcation parameter

Noise induces front propagation





M.G. Clerc and C. Falcon, Physica A 356, 48 (2005).

Outline

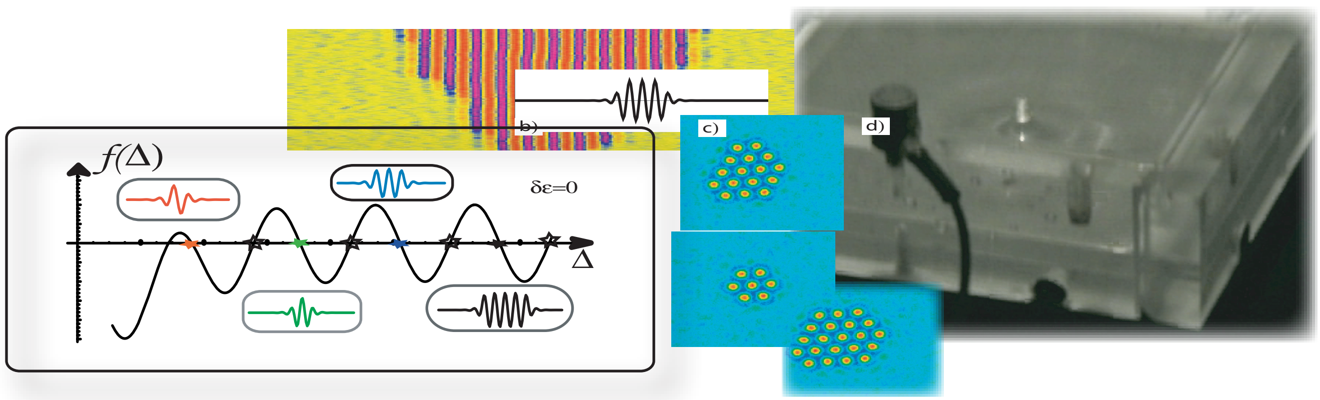
- Introduction of Localized solution in Nature.
- Localized structure are robust phenomena.
- Universal description of the localized structures

  Amended amplitud equation.
- Front interaction.
- **Conclusions.**
- Outlook.

Conclusion

- We have shown on the basis of the front interactions the existence, stability properties, dynamical evolution and bifurcation diagram of localized patterns and hole solutions in one-dimensional extended systems.

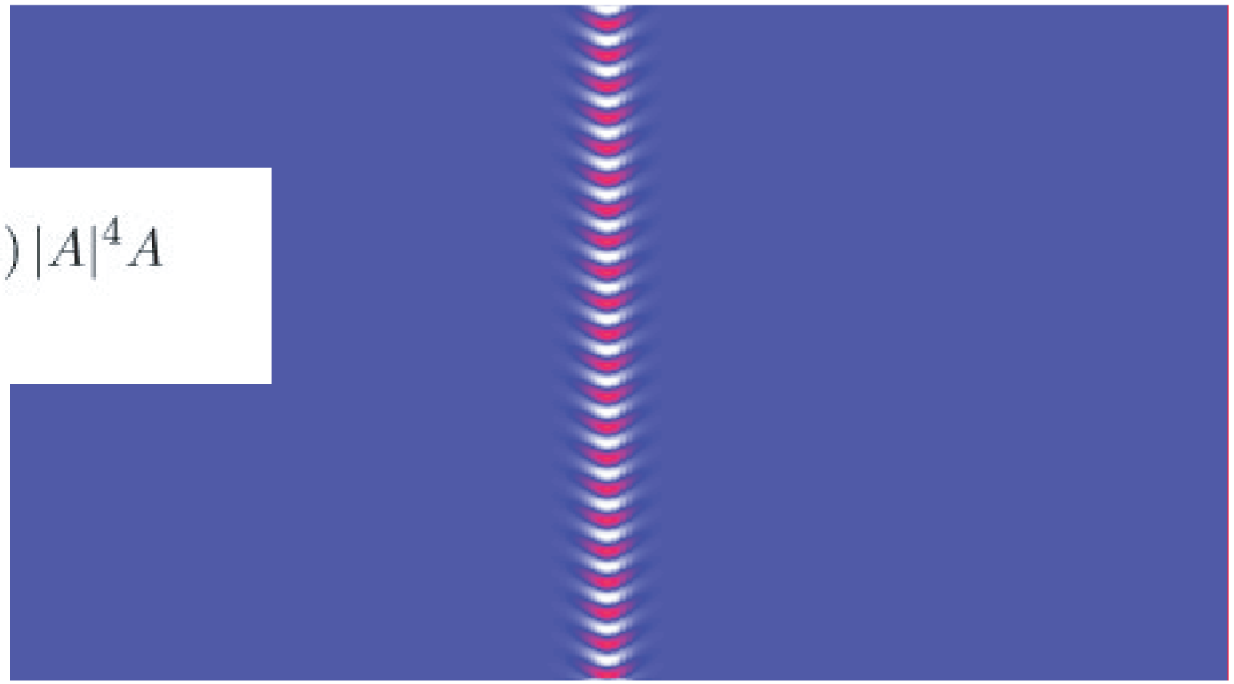
- The conversion of random fluctuations into direct motion of front core is responsible of the front propagation.

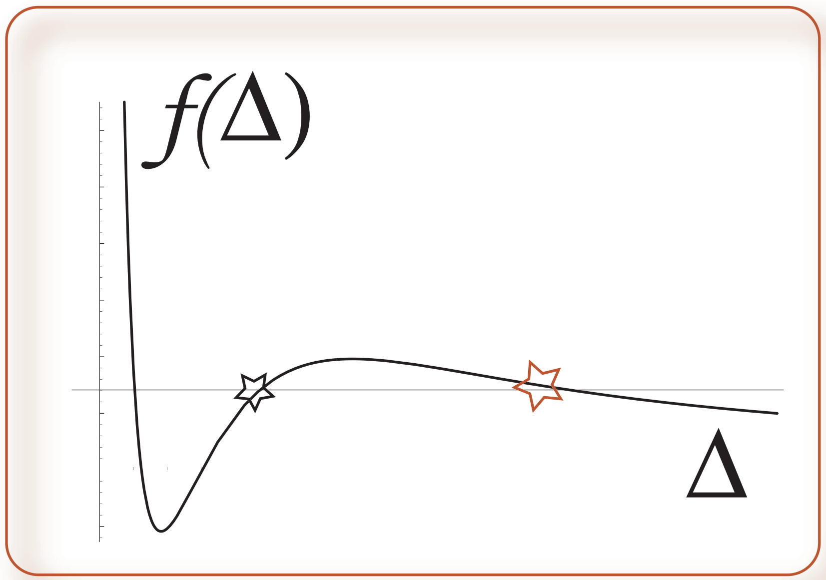


Outlook

- Using the same method, front interaction, we can understand another type of localized solution. In particular pulses in Complex subcritical Ginzburg-Landau equation

$$\partial_t A = \mu A + (\beta_r + i\beta_i)\,|A|^2 A - (\gamma_r + i\gamma_i)\,|A|^4 A + (\alpha_r + i\alpha_i)\,\nabla^2 A$$





In collaboration O Descalzi