Front propagation in liquid crystal light valve with feedback

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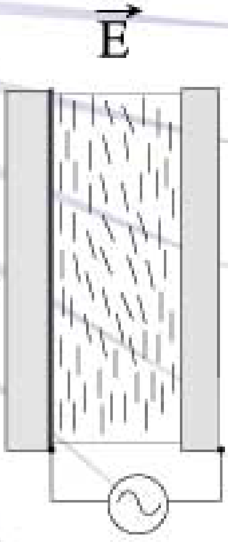
Outline

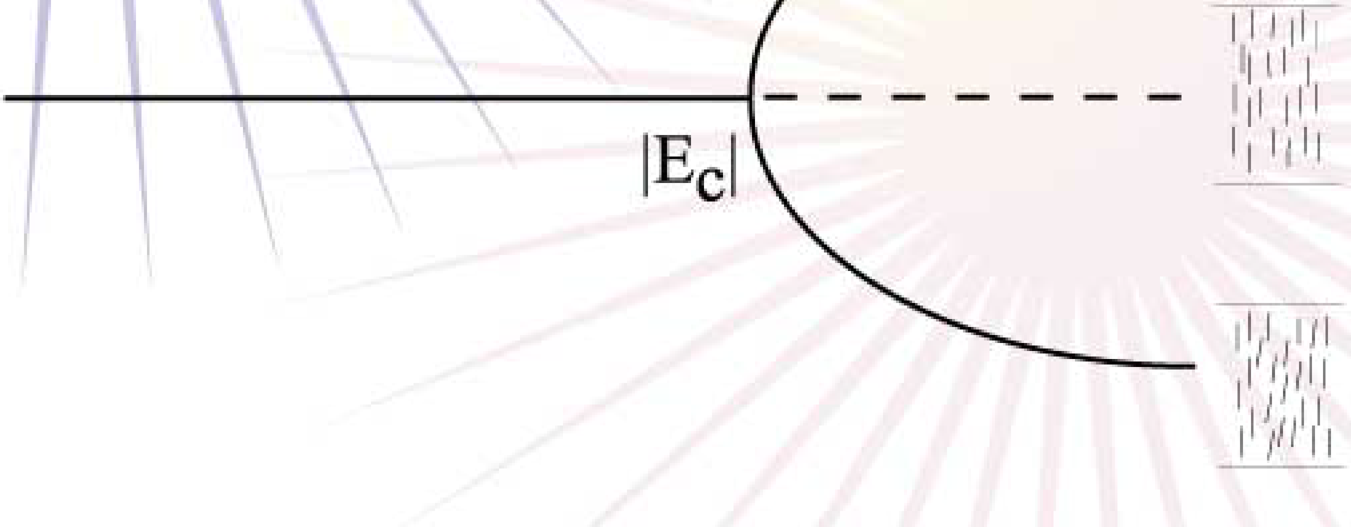
- Freederickz transition.
- Liquid cristal light valve (LCLV), experimental setup.
- Feedback renders the Freederickz transition in first order one.
- Main features first-order transition
- Front propagation.
- Experimental measure.
- Conclusion.

Freederickz Transition

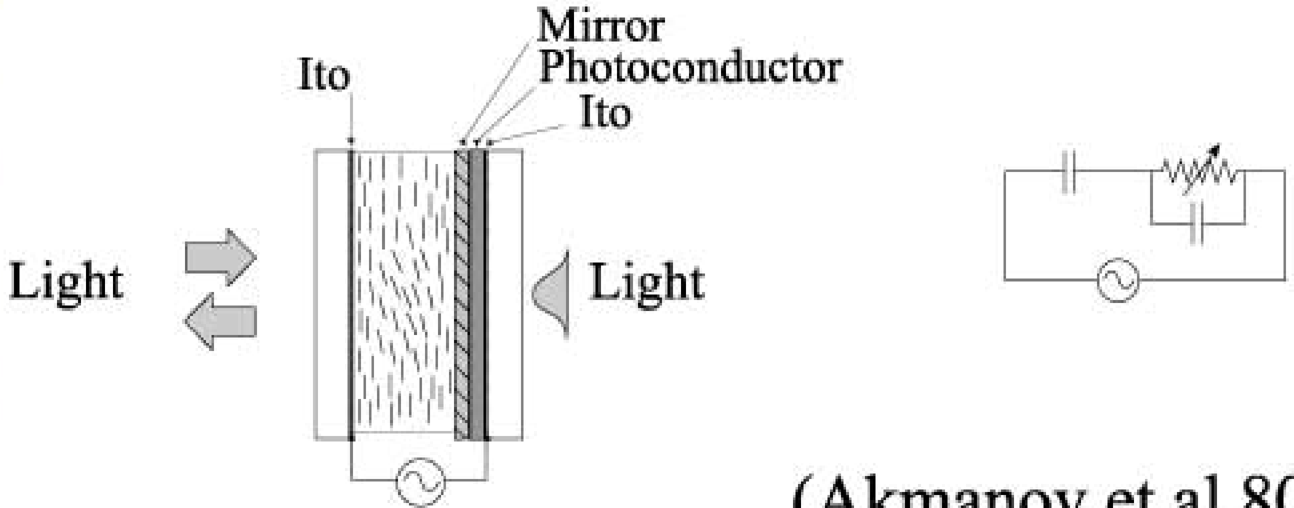


Liquid crystal under the influence of magnetic or electric fields can exhibit distortion of planar or homeotrope aligned. This transition is usually is a second order one.

Bifurcation Diagram: liquid crystal film with planar aligned with positive dielecrtric constant.

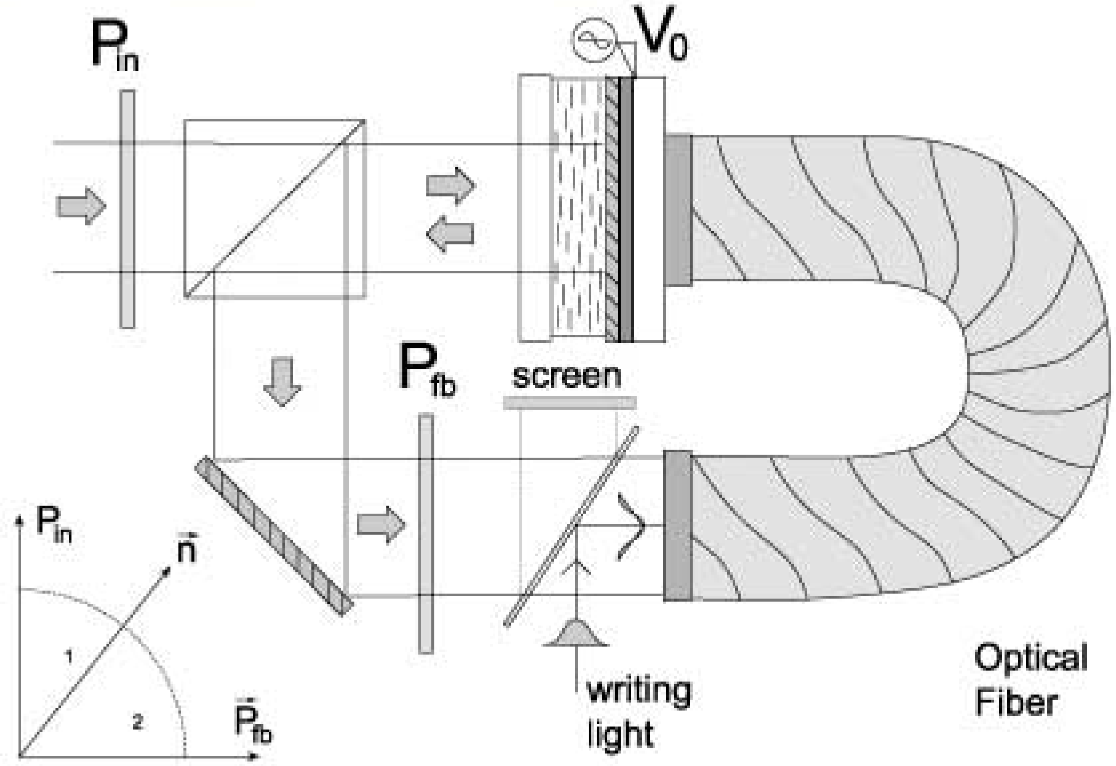


Liquid Crystal Light Valve (LCLV)



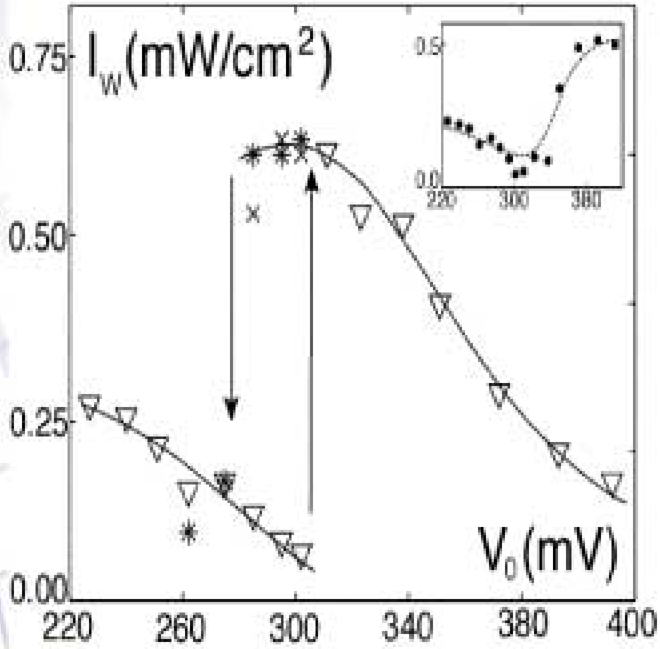
(Akmanov et al 80')

LCLV with feedback

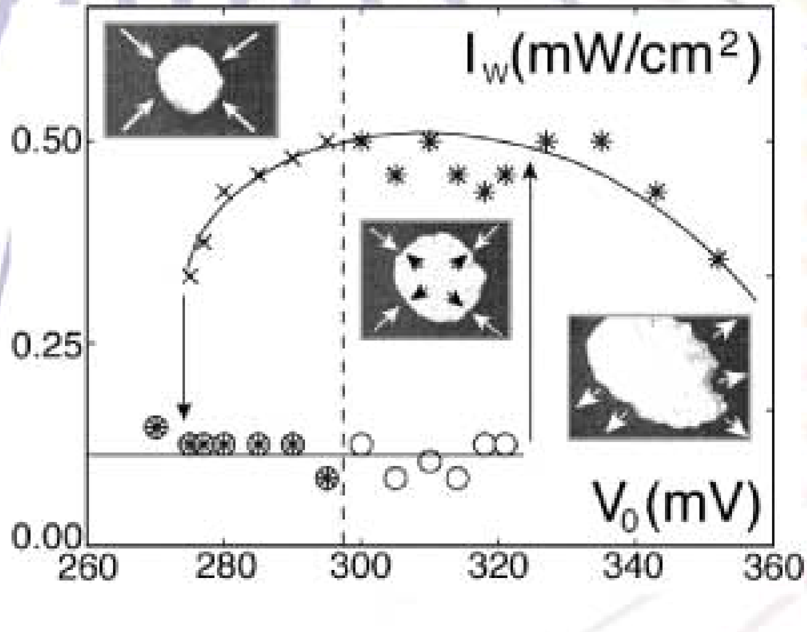


Feedback renders the FT to a first-order one.

- Ligth intensity as a function of r.m.s. value of applied voltage V_o (with and without feedback)
- Critical opalescence.



Spatial behaviors close to Freederickz transition



- Bistability.
- Maxwell point.
- Transition point.
- Front propagation.
- Normal front.
- FKPP front.
- Gibbs-Thomson effect.

Theoretical description

Nematic liquid crystal layer on which an electric field is applied, the director satisfies

$$\gamma \vec{n} \wedge \partial_t \vec{n} = -\vec{n} \wedge \frac{\delta \mathcal{F}}{\delta \vec{n}}, \qquad \vec{n}.\vec{n} = 1$$

where γ is the rotational viscosity and the Frank free energy

$$\mathcal{F} = \frac{1}{2} \int K_1(\vec{\nabla}.\vec{n})^2 + K_2(\vec{n}.(\vec{\nabla} \wedge \vec{n}))^2 + K_3(\vec{n}.(\vec{\nabla} \wedge \vec{n}))^2$$
$$-\epsilon_\perp \vec{E}^2(\vec{n}) - \epsilon_a(\vec{n}.\vec{E}(\vec{n}))^2 d\vec{r}.$$

In the case isotropic case $K_1 = K_2 = K_3 = K$

$$\begin{aligned}\gamma \partial_t \vec{n} &= K(\nabla^2 \vec{n} - \vec{n}(\vec{n}.\nabla^2 \vec{n})) \\ &\quad + \epsilon_a(\vec{n}.\vec{E})(\vec{E} - \vec{n}(\vec{n}.\vec{E})) + \tfrac{\epsilon_\perp}{2} \tfrac{\partial \vec{E}^2}{\partial \vec{n}} \\ &\quad - \tfrac{\epsilon_\perp}{2}(\vec{n}.\tfrac{\partial}{\partial \vec{n}})\vec{E}^2 + \tfrac{\epsilon_a}{2}(\vec{n}.\vec{E})\vec{n}.((\vec{n}.\tfrac{\partial}{\partial \vec{n}})\vec{E})\end{aligned}$$

The total electric field applied to the liquid crystal layer depends on the response of the photoconductor

$$E(n) = E_0 + \alpha I_w(\vec{n}), \qquad \alpha \simeq 4$$

where the first term of the right size is related to the voltage applied to the film and

$$I_w = |\cos\psi_1 \cos\psi_2 + \sin\psi_1 \sin\psi_2 e^{-2ikd\Delta n \cos^2\theta})|^2 I_{in}$$

with I_{in} input intensity,

$\Delta n = 0.2$ diference of index of refraction,

$\psi_1 = \psi_2 = 45^o$ Polarization angles,

$d = 30\mu m$ thickness of liquid crystal layer,

$k = 2\pi/\lambda$ wave number, λ=633 nm.

therefore

$$E(n) = E_0 + A + B\cos(\beta\cos^2\theta)$$

where

$$A = \frac{1}{4}(\cos 2(\psi_1 - \psi_2) + \cos 2(\psi_1 + \psi_2) + 2)\alpha I_{in}$$

$$B = \frac{1}{4}(\cos 2(\psi_1 - \psi_2) - \cos 2(\psi_1 + \psi_2))\alpha I_{in}$$

The stability analysis of the director equation shown that the first Fourier mode becomes unstable

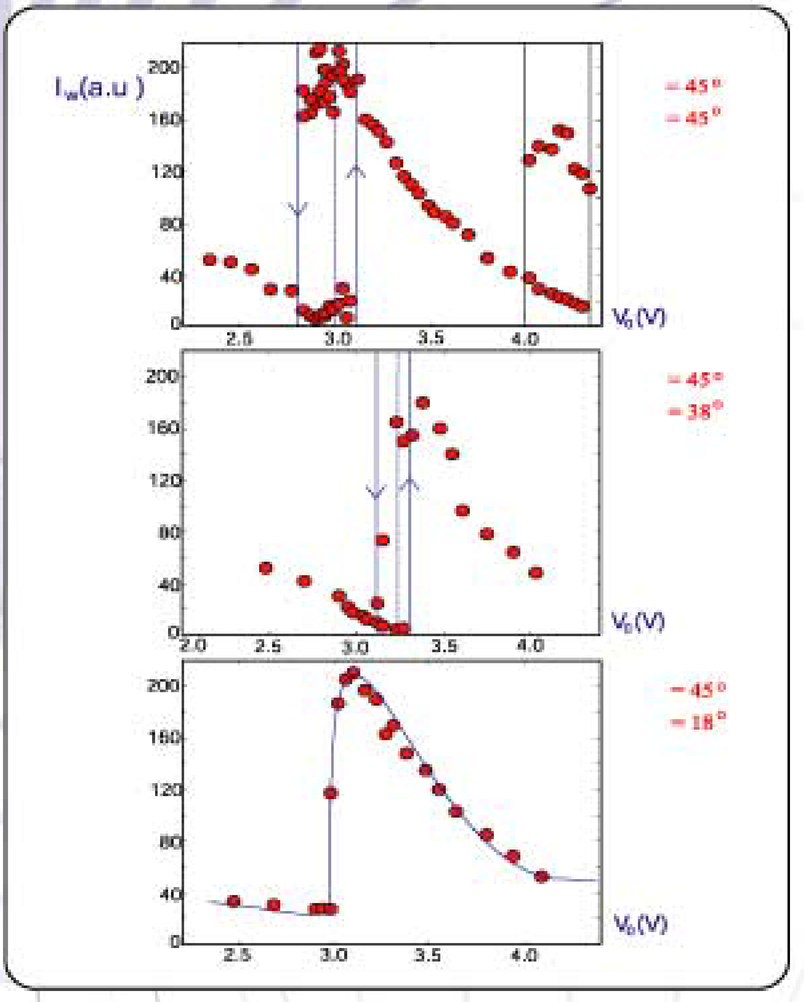
$$\vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} 0 \\ n_y = u(x,y)\sin(\pi z/d) \\ n_z = 1 - u^2 \sin^2(\pi z/d)/2 \end{pmatrix}$$

where the amplitude u(x,y;t) satisfies

$$\partial_t u = c_1 u + c_3 u^3 + c_5 u^5 + K\nabla_\perp^2 u$$

where
$$\begin{aligned}
c_1 &= \gamma^{-1}(-\tfrac{K\pi^2}{d^2} + (A + E_0 + B\cos(\beta))(\epsilon_a(A+E_0) \\
&\quad +B(\epsilon_a \cos(\beta) + 2\epsilon_\perp \beta \sin(\beta)))) \\
c_3 &= \gamma^{-1}(\tfrac{1}{3}\tfrac{K\pi^2}{d^2} - \tfrac{2}{3}\epsilon_a(E_0+A)^2 + \tfrac{2}{\pi^2}\epsilon_\perp(2B\beta\sin(\beta))^2 \\
&\quad -\tfrac{2}{3\pi^2}(12\beta^2\epsilon_\perp + \pi^2\epsilon_a)(B\cos(\beta))^2 \\
&\quad +\tfrac{\beta}{\pi^2}(\epsilon_a(8+\pi^2) - \epsilon_\perp\pi^2)(A + E_0 + B\cos(\beta))B\sin(\beta) \\
&\quad -\tfrac{4}{3\pi^2}(6\beta^2\epsilon\perp + \pi^2\epsilon_a)(E_0 + A)B\cos(\beta)) \\
c_5 &= \gamma^{-1}((\tfrac{2B\beta}{\pi^2})^2((4+\pi^2)\epsilon_a - \pi^2\epsilon_\perp)(\sin(\beta)^2 - \cos(\beta)^2) \\
&\quad -\tfrac{B\beta}{12\pi^4}(192\beta^2\epsilon_\perp + (9\pi^2+64)\pi^2\epsilon_a)(E_0+A)\sin(\beta) \\
&\quad -(\tfrac{2\beta}{\pi^2})^2((4+\pi^2)\epsilon_a - \pi^2\epsilon_\perp)(E_0+A)B\cos(\beta) \\
&\quad -\tfrac{B^2}{12\pi^4}(9\pi^2+64)\epsilon_a + 768\beta^2\epsilon_\perp)\beta\sin(\beta)\cos(\beta)).
\end{aligned}$$

Qualitative and quantitative agreement with the experimental measure and the parameters

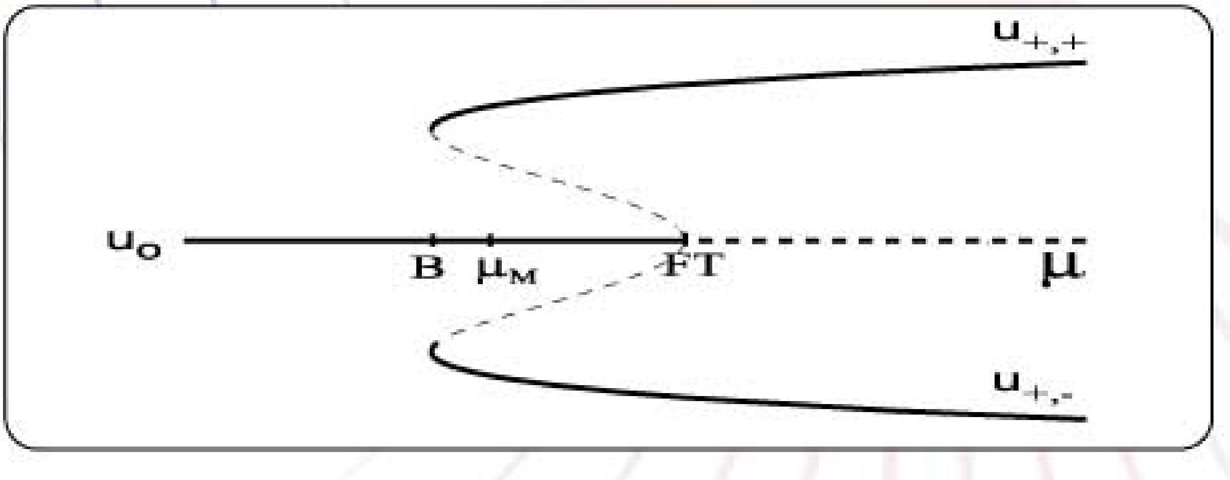


Normal form

In the case of first-order transition with symmetry reflection the system is described (dimensionless)

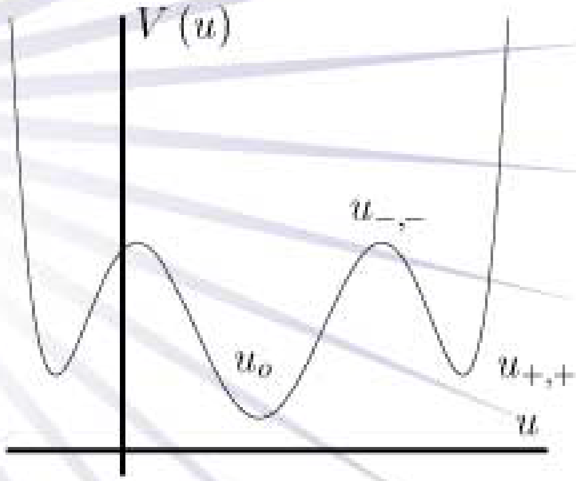
$$\partial_t u = \mu u + u^3 - u^5 + \vec{\nabla}^2 u$$

The bifurcation diagram



- The system is potential $\partial_t u = -\frac{\partial V}{\partial u} + \vec{\nabla}^2 u$

$$V(u) = -\mu\frac{u^2}{2} - \frac{u^4}{4} + \frac{u^6}{6}.$$



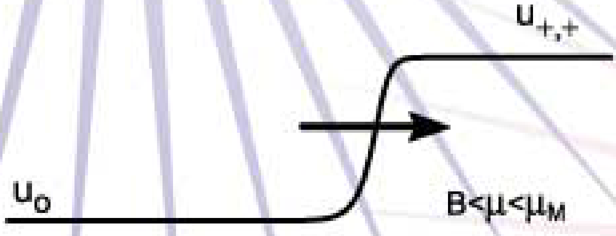
- The stationary solution

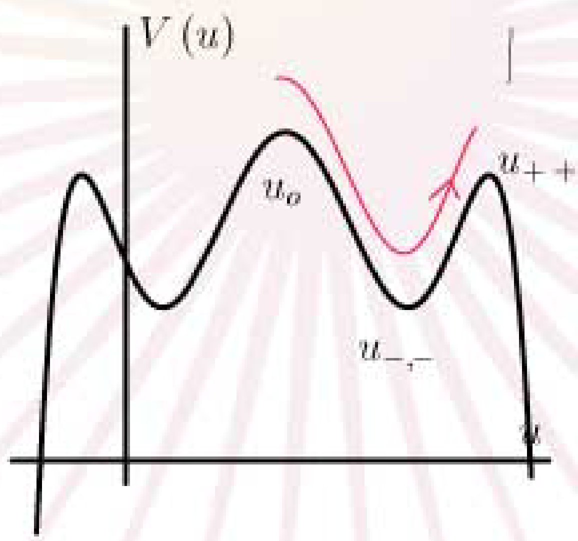
$$u_o = 0, \quad u_{-,\pm} = \pm\frac{\sqrt{1-\sqrt{1+4\mu}}}{2}. \qquad u_{+,\pm} = \pm\frac{\sqrt{1+\sqrt{1+4\mu}}}{2},$$

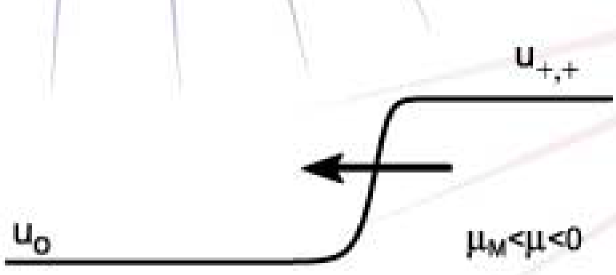
Front Solution

"Heteroclinic connection of stationary states in the stationary or inertial spatial system". Using the ansatz u(x;t)=u(x-ct)

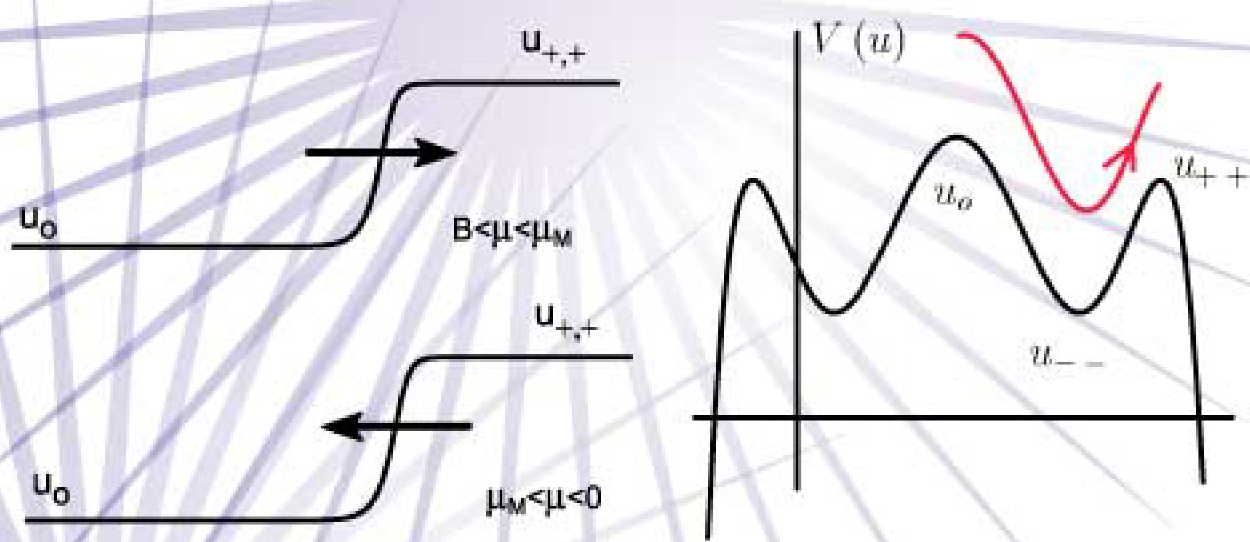
$$\partial_{zz}u + c\partial_z u = -\frac{\partial(-V)}{\partial u}$$



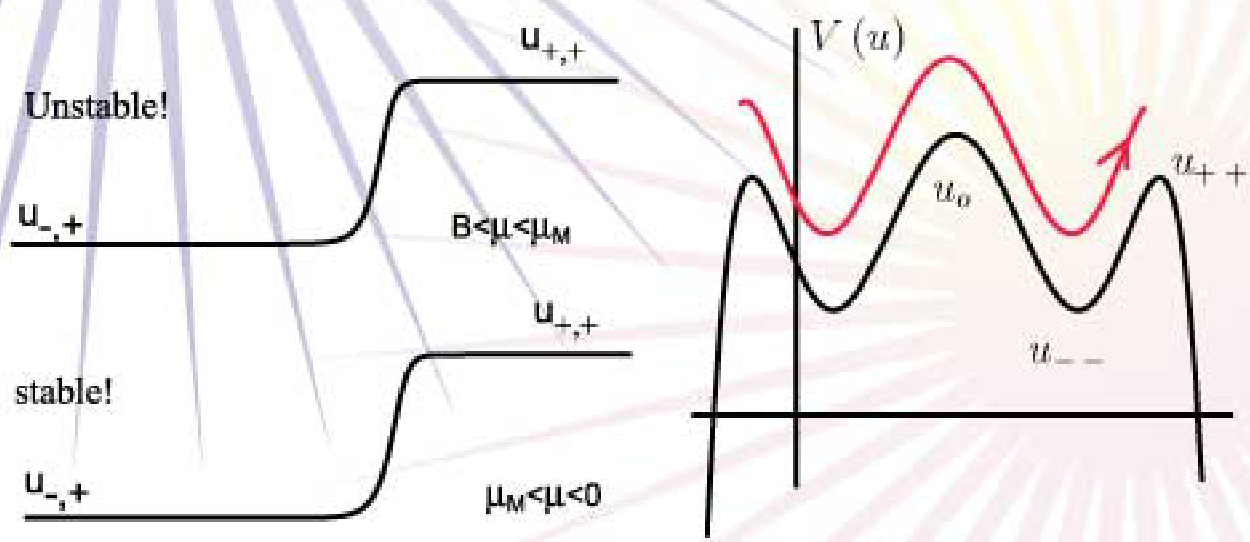




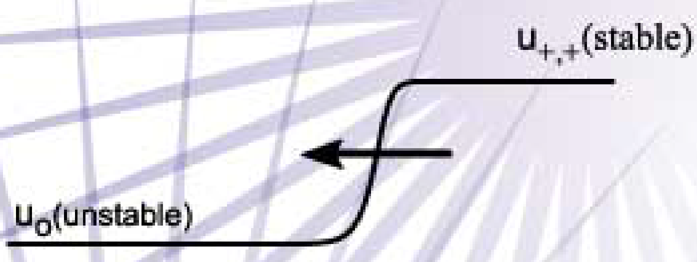
- Normal front

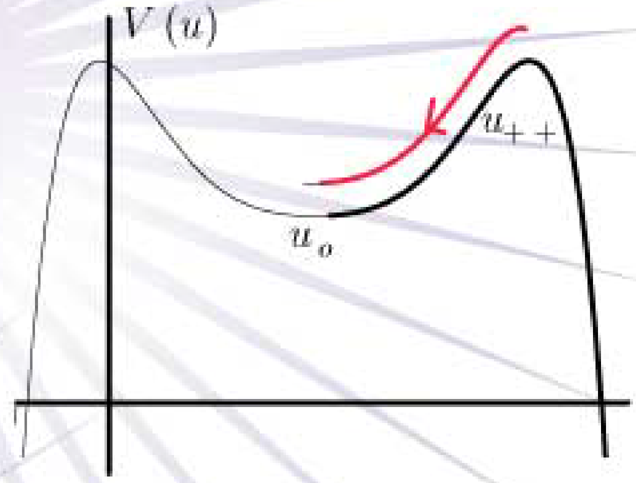


- Wall



FKPP front

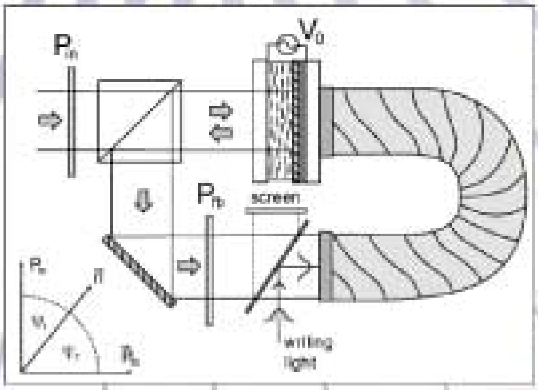


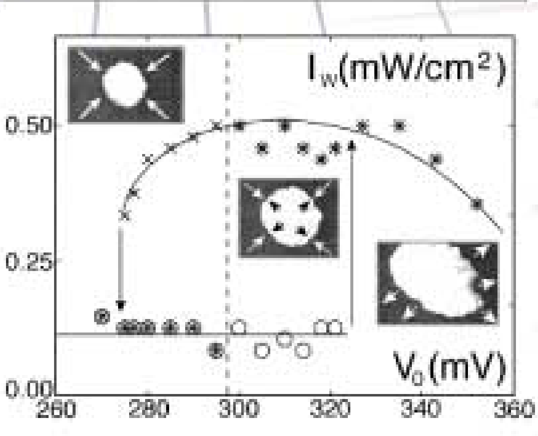


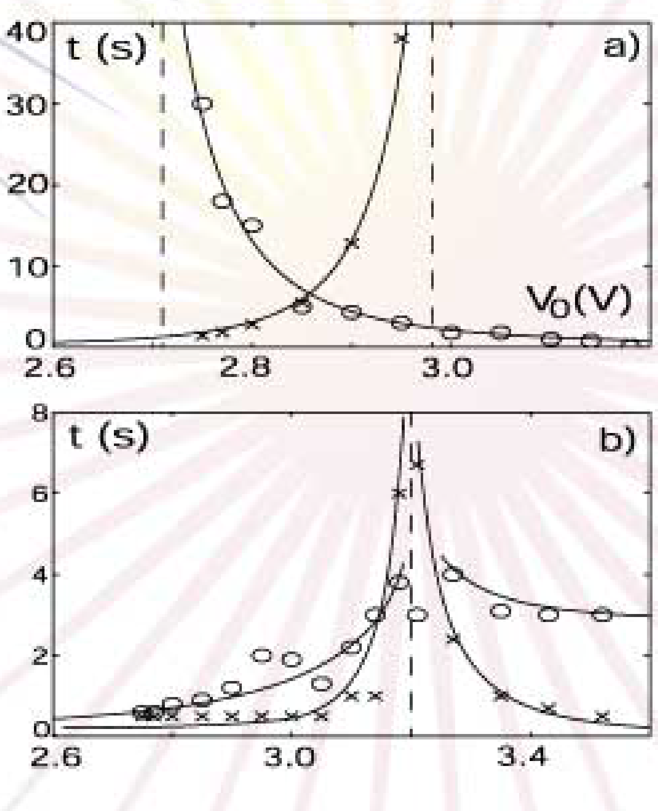
- There is minimum velocity
- The initial condition determine the velocity.
- $\mu>0$ the linear analysis determine the dispertion relation

Front Propagation (Experimental measures)

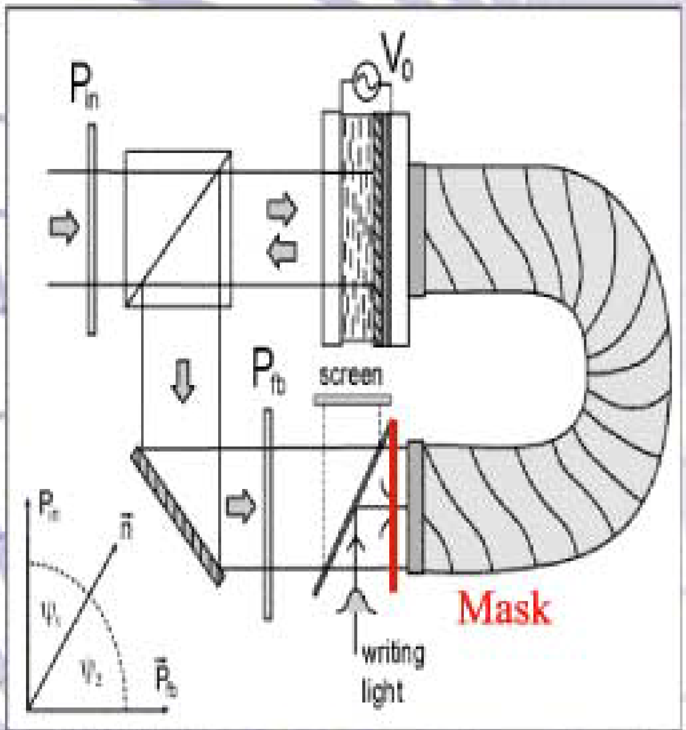
Bistability, Maxwell point, Freederickz transition

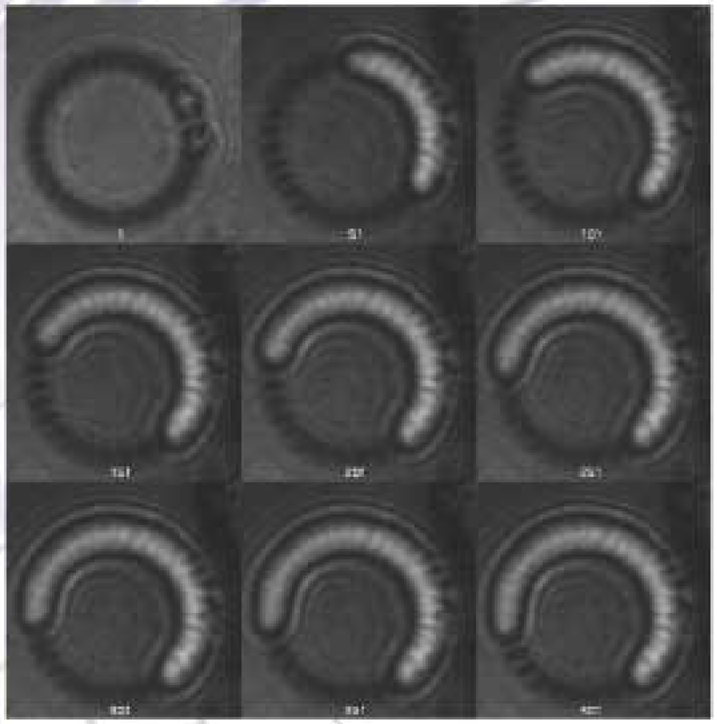




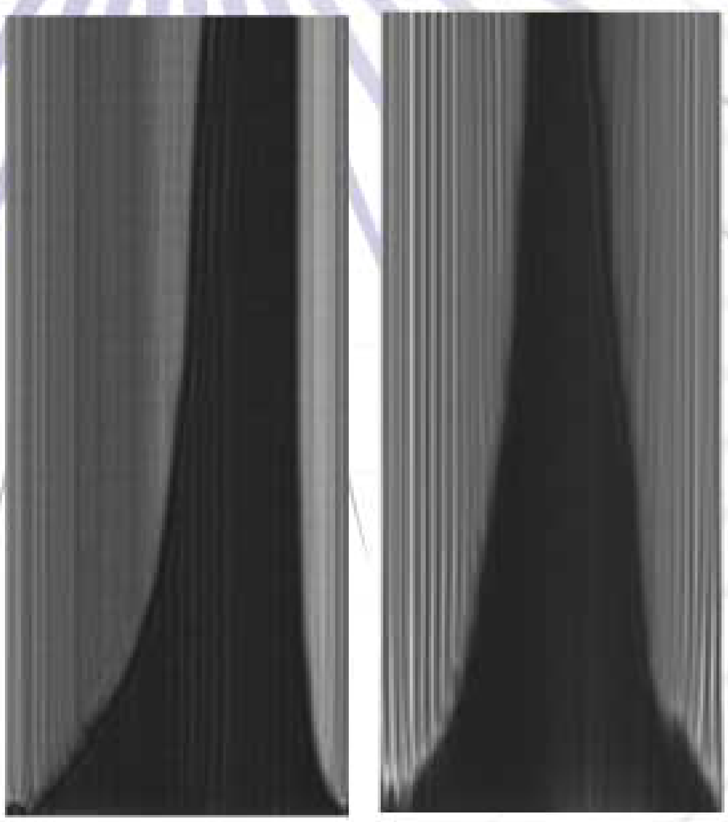


Experimental setup (1D)

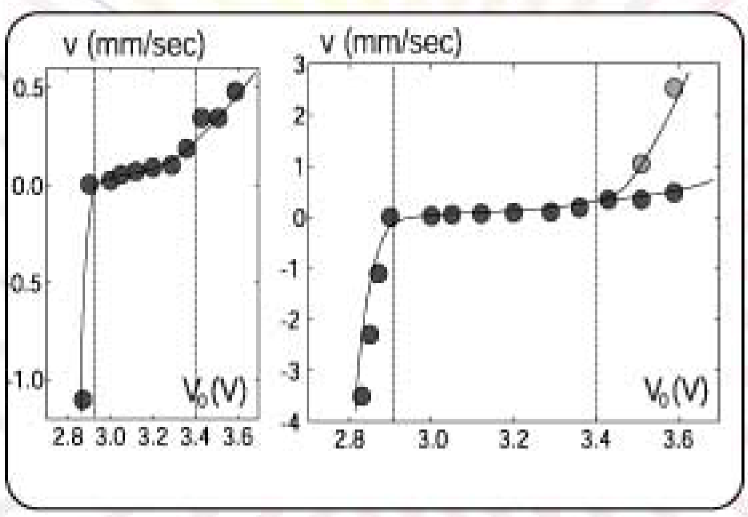




spatio-temporal diagram



Front velocity



Conclusion

- The feedback renders the FT into a first order one.
- An amplitude equation, derived close to FT, exhibits a fair qualitative agrement with the experimental observation.
- Bistability, Maxwell point, freedrickz transition and front velocityof Normal and FKPP front (1D,2D) have been measured.

Outlook

- Noise induced spatio-temporal intermittence

$$\partial_t u = \mu u + u^3 - u^5 + \nabla_\perp^2 u + u\eta(x,t)$$