Interface dynamics of a front connecting stripe pattern with uniform state

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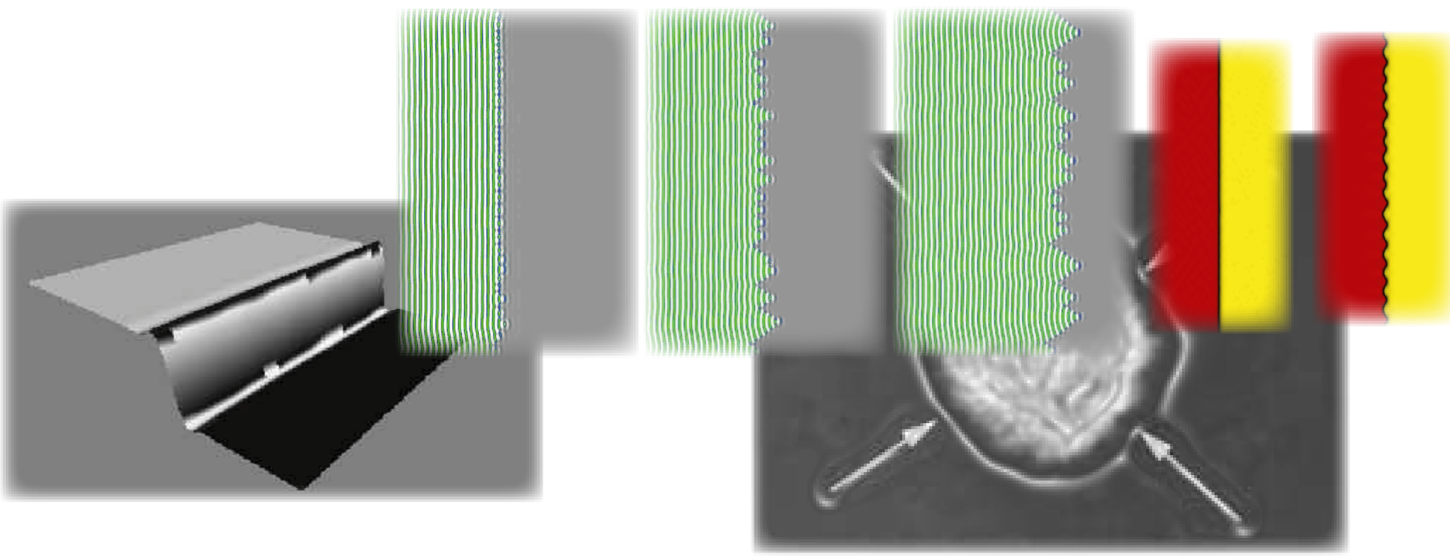

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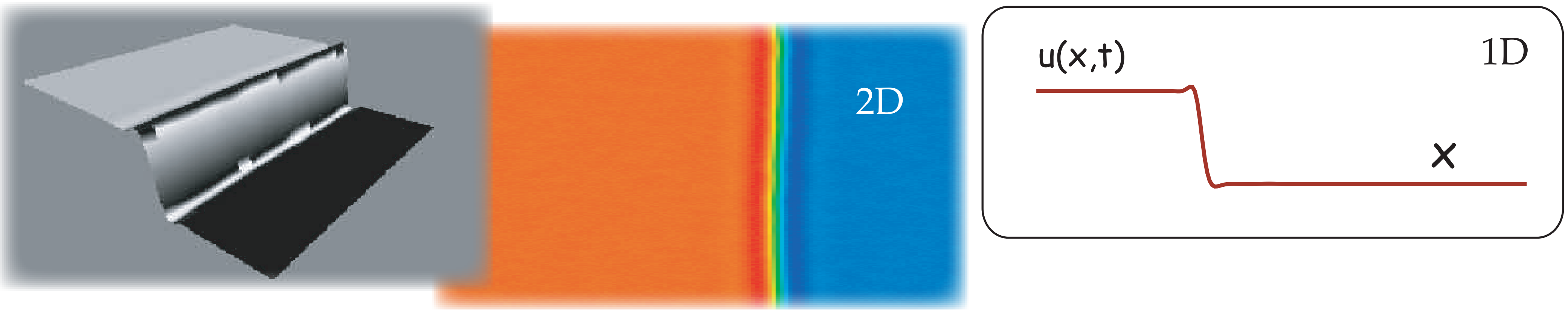
Ring Program ACT 15, PBCT & CIMAT

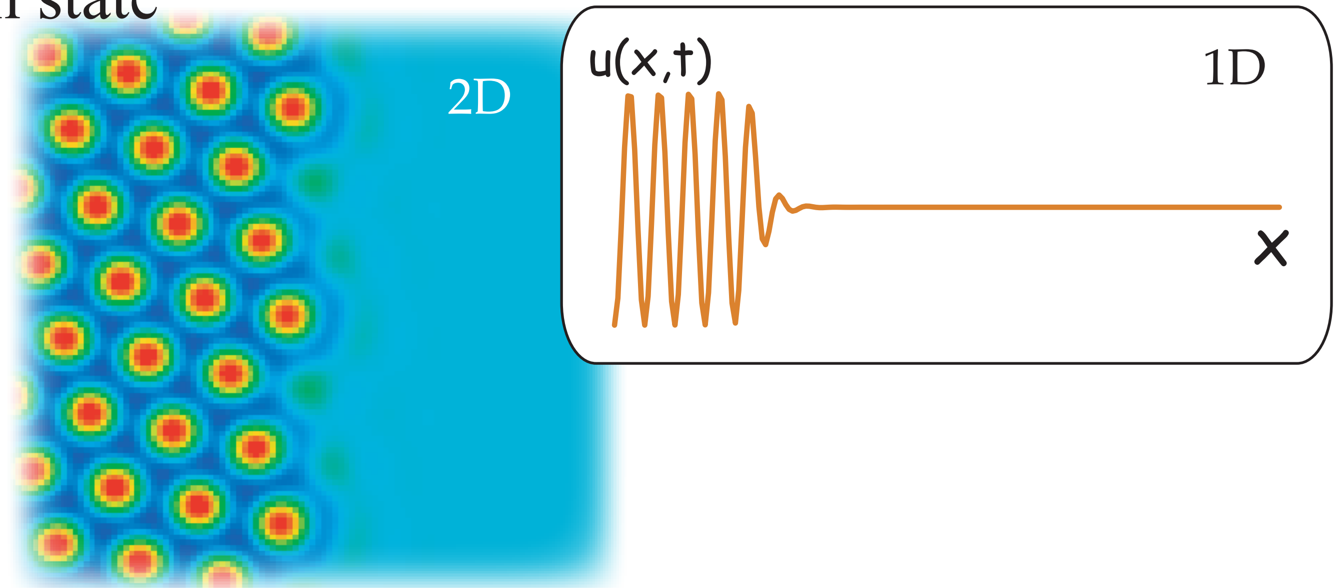
Front solution

- "Stationary solution that links two steady states".
- "In 1D the front solution are heroclinic connection of stationaries states in the stationary extended system or moving reference frame" (multi-stability).

Two uniform states

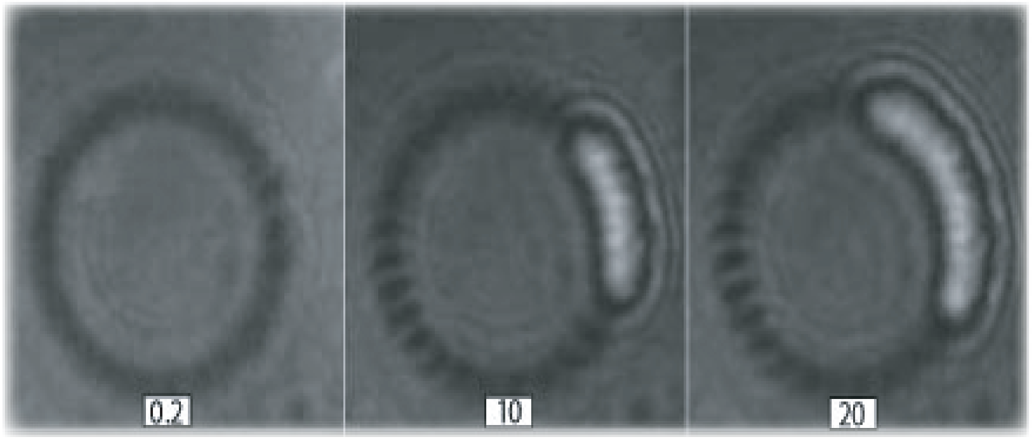


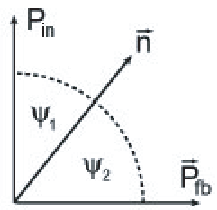
Pattern and uniform state

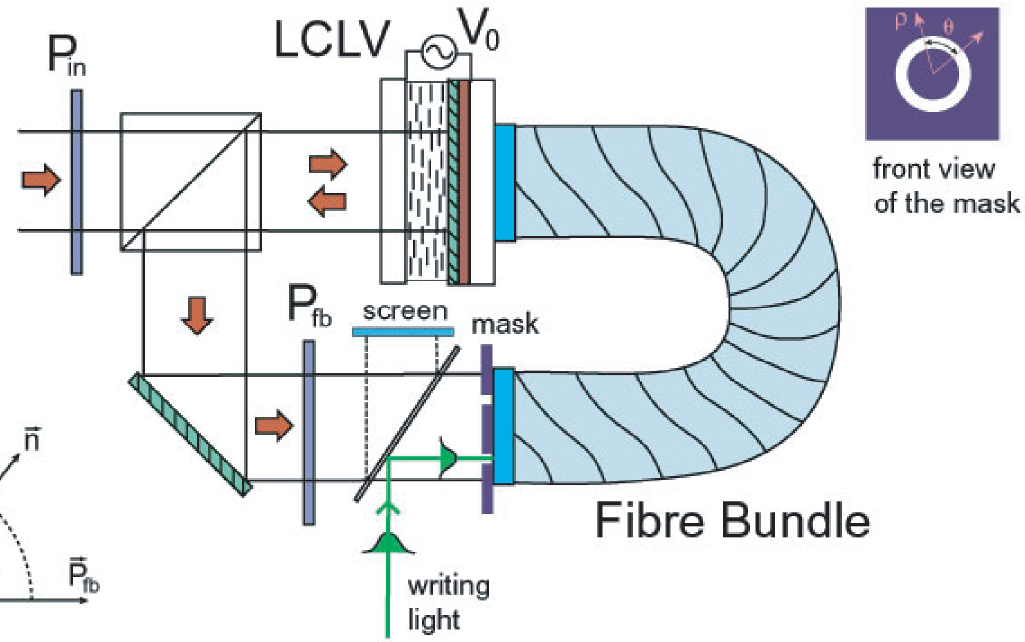


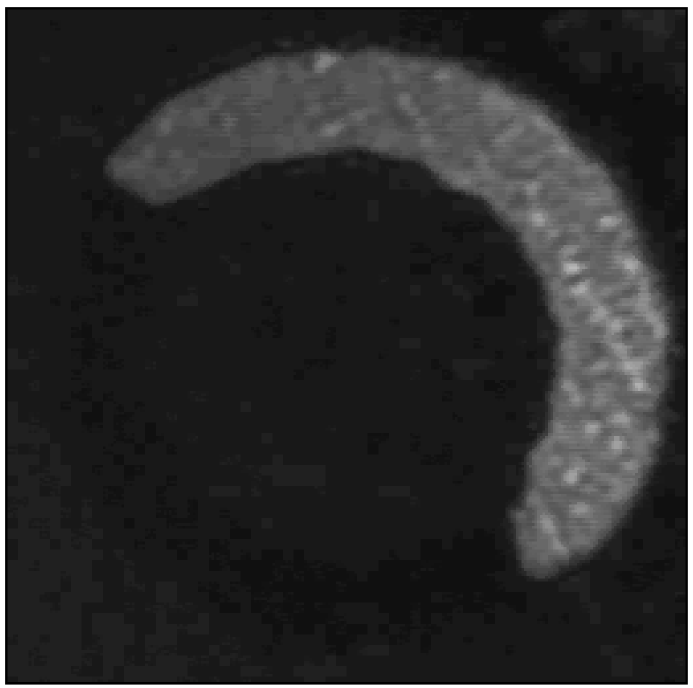
Front and experiments

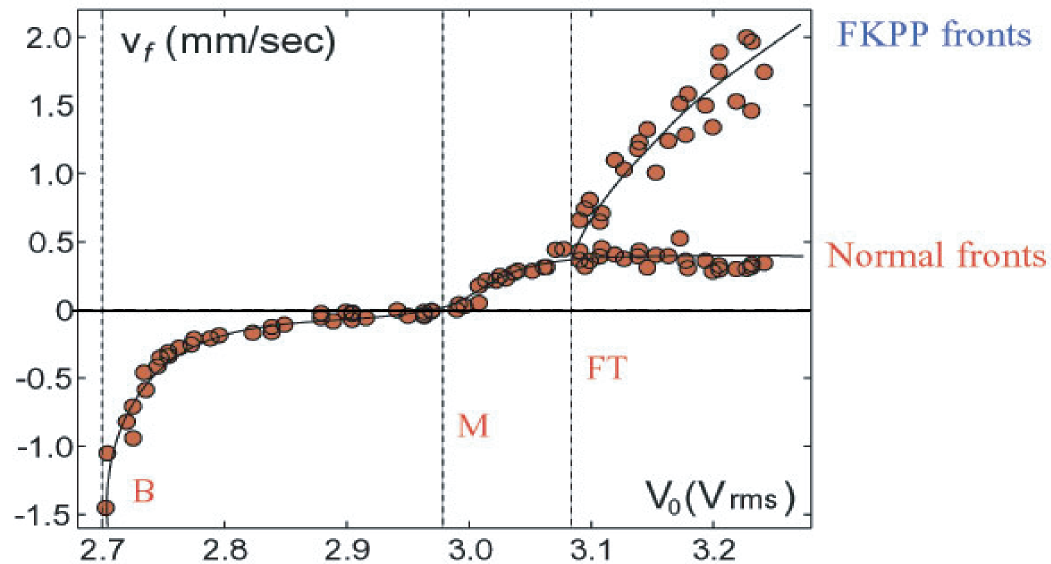
- Liquid crystal light valve with optical feedback (1-D experiment)









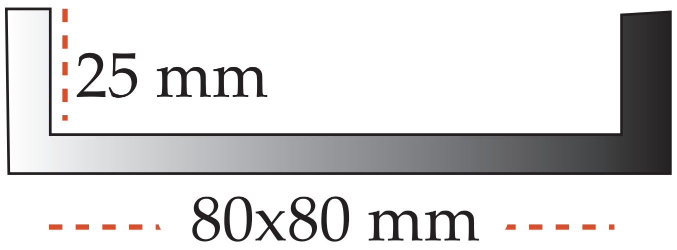


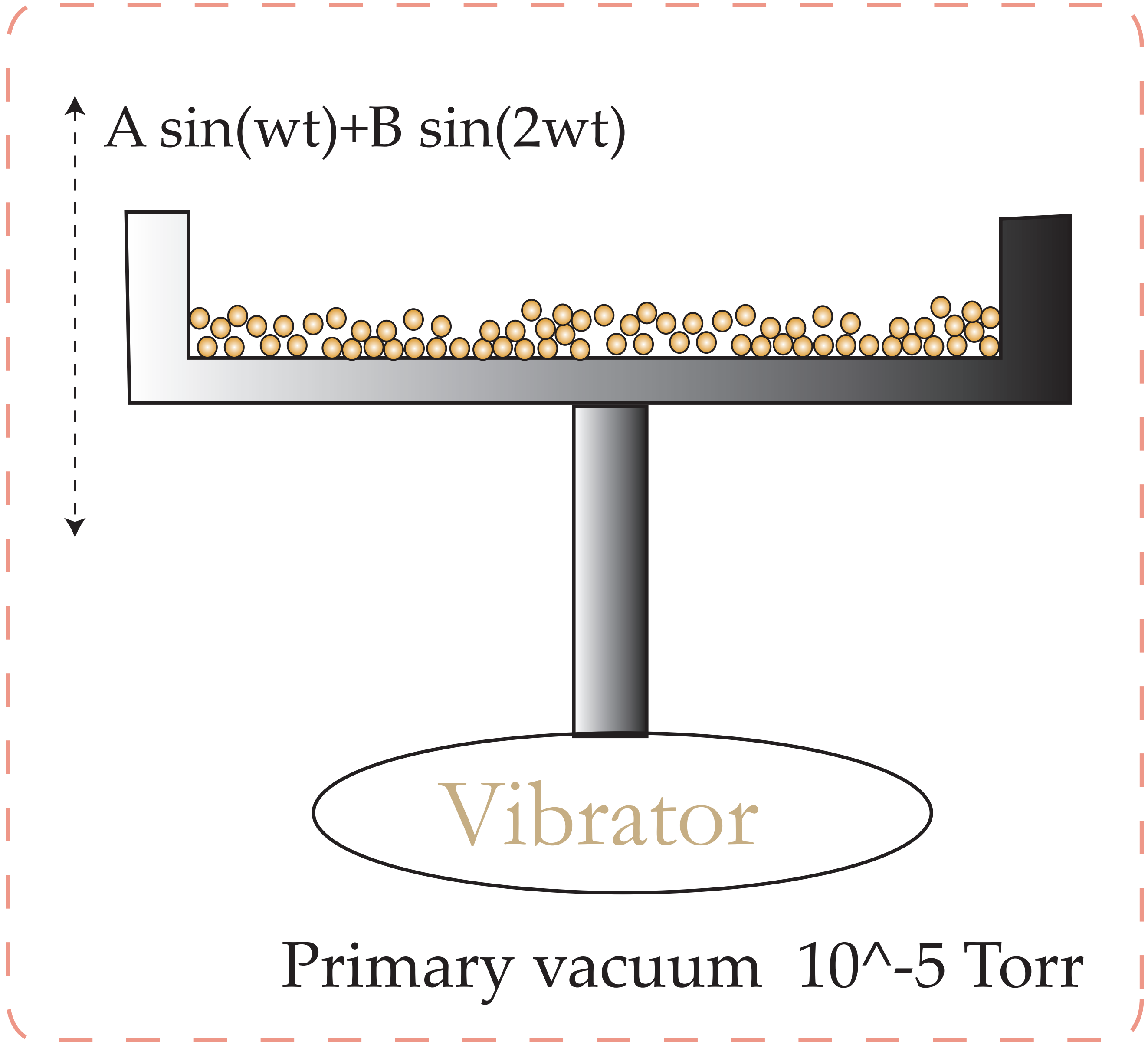
M.G. Clerc et al, Eur. Phys. J. D 28, 435 (2004).

Pattern and uniform states in fluidized granular system

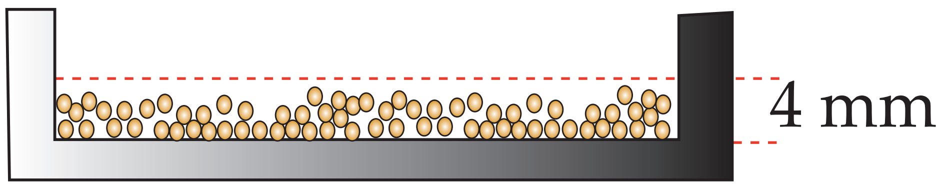
Experimental setup: container with broze grain


100 microns (grains)

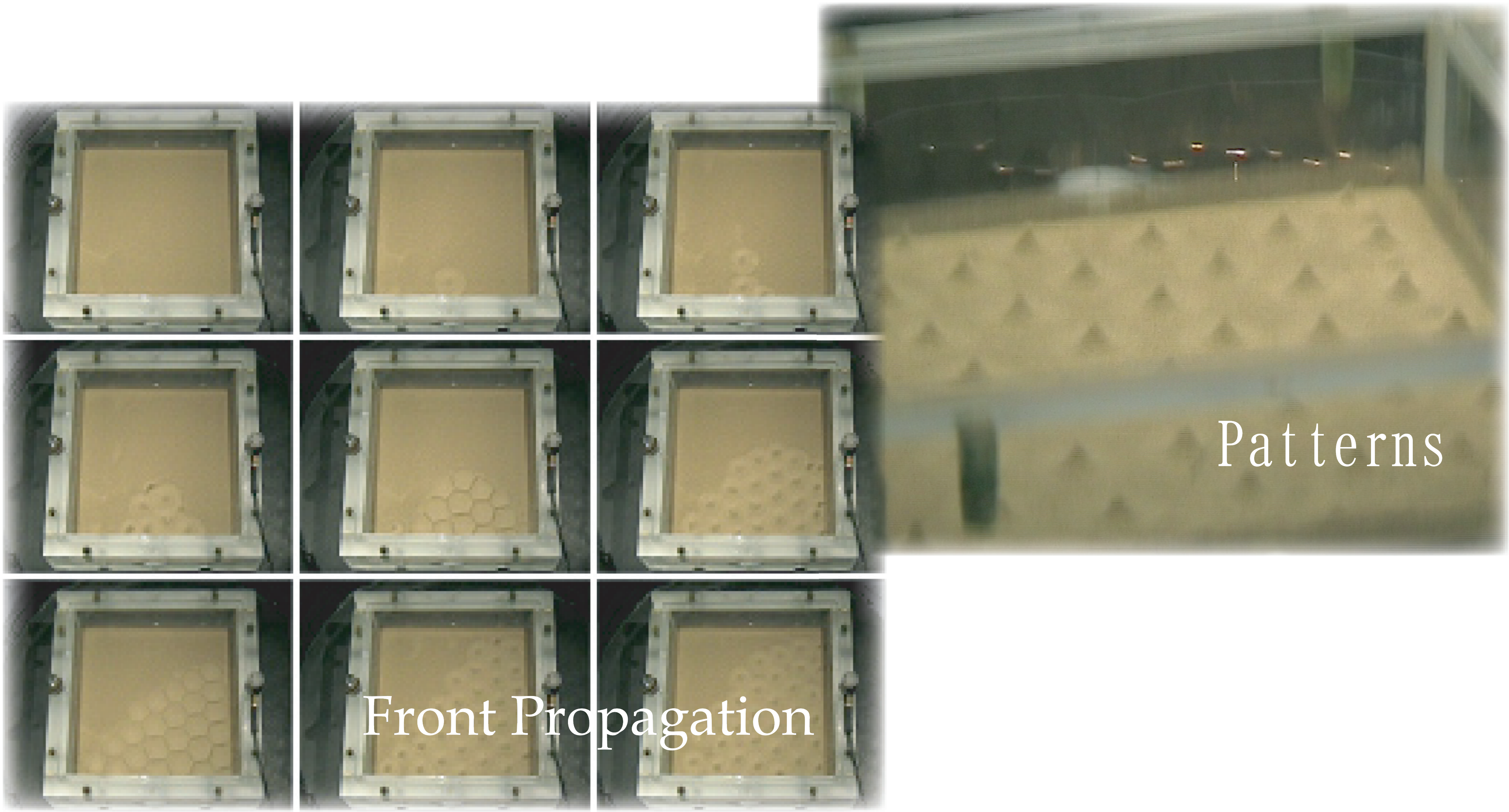




Initial conditions



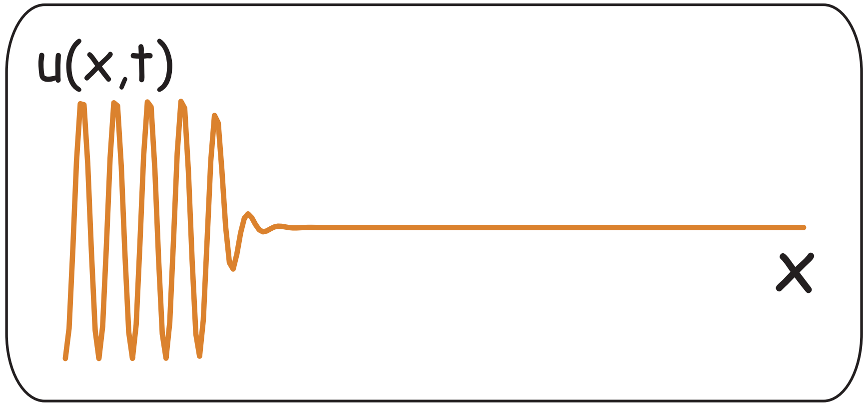
Front solutions



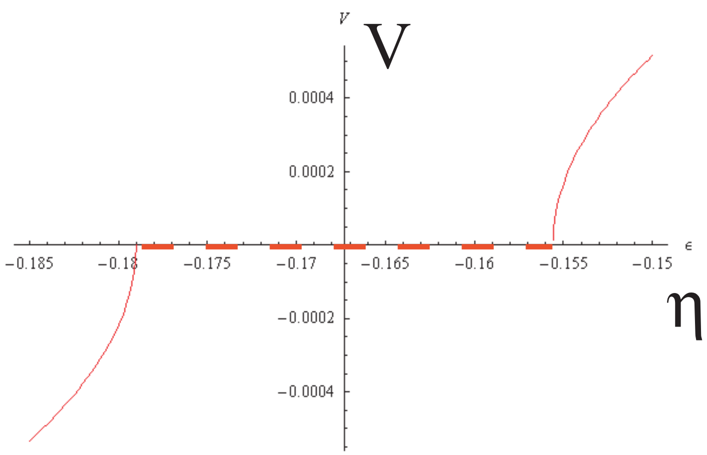
Courtesy of S. Residori, C. Falcon, U. Bortolozzo and M.G. Clerc (INLN)

Properties of front in one extended systems (1D)

- Locking phenomenon and pinning range

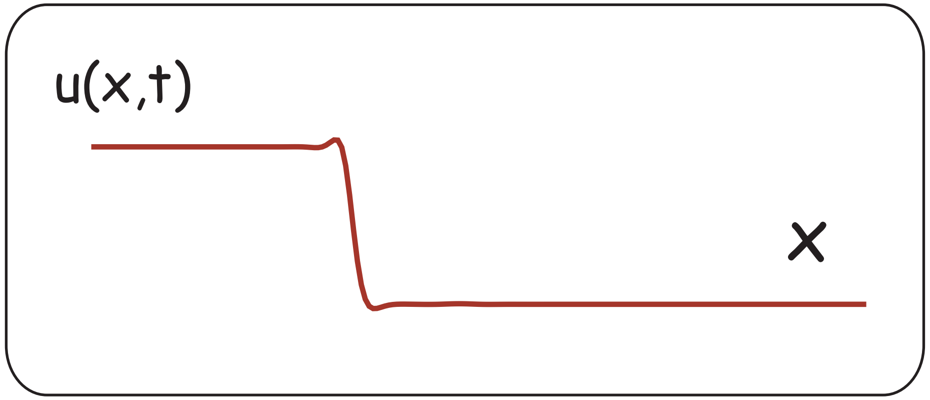


The front is stationary in a width range of parameters, pinning range

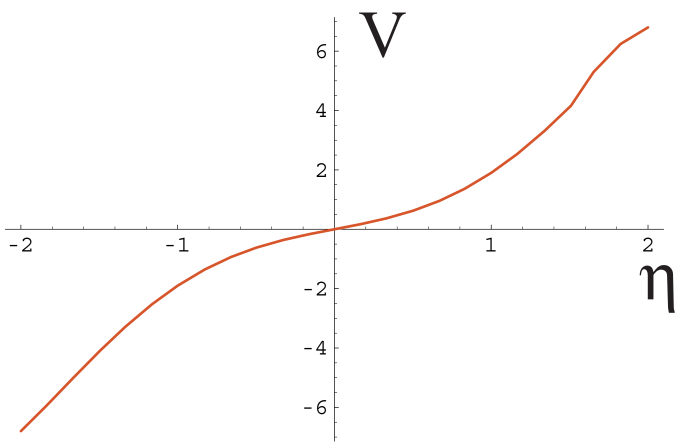


Y Pomeau, Physica D 23, 3 (1986)

- Normal form



The front is stationary in one point, Maxwell point.

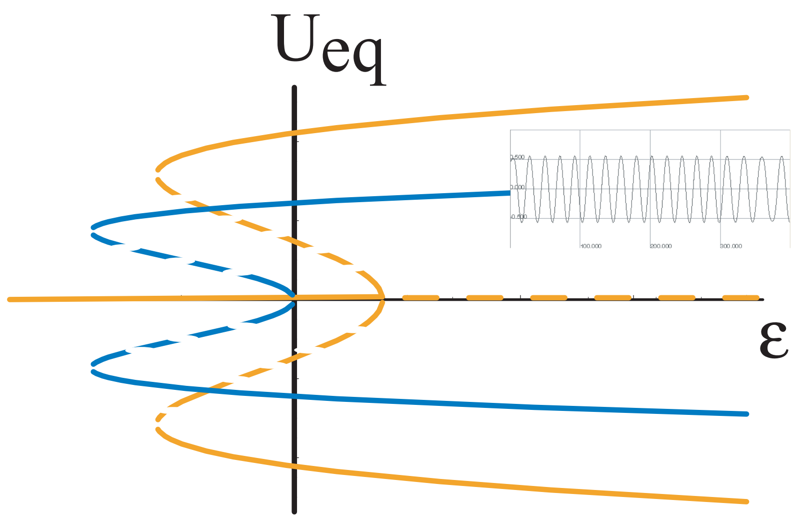


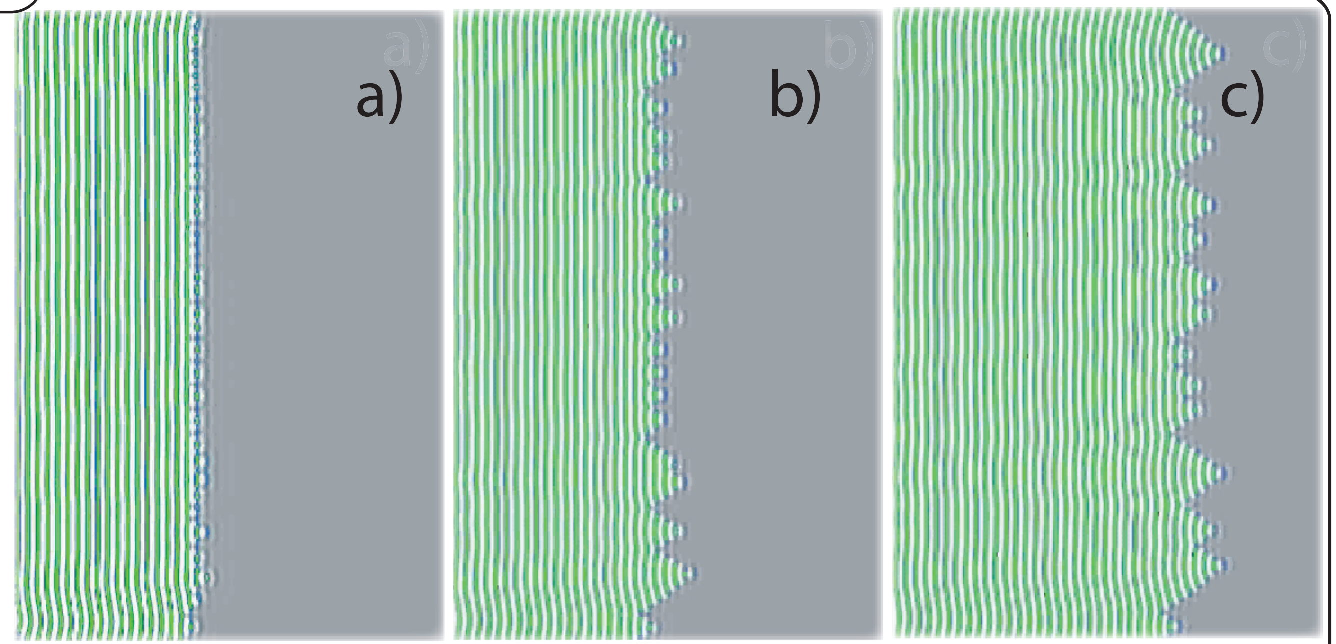
Which are the features of front in 2D?

- For the sake of simplicity, we consider front connecting a stripe pattern with a uniform state. Numerical simulations of the *isotropic Swift-Hohenberg equation* show a complex dynamics of front solution.

$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - \left(\vec{\nabla}^2 + q^2\right)^2 u,$$

Bifurcation Diagram

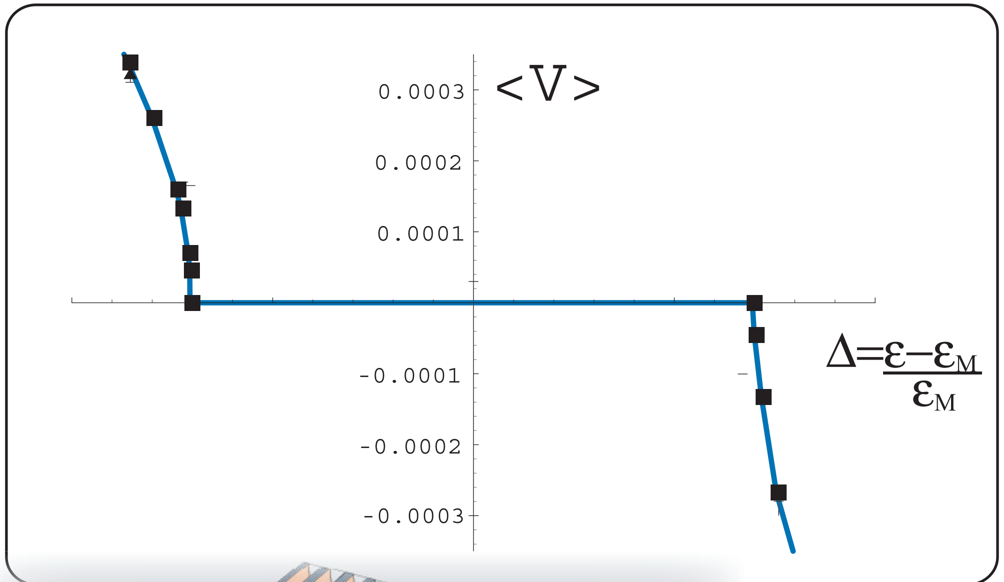


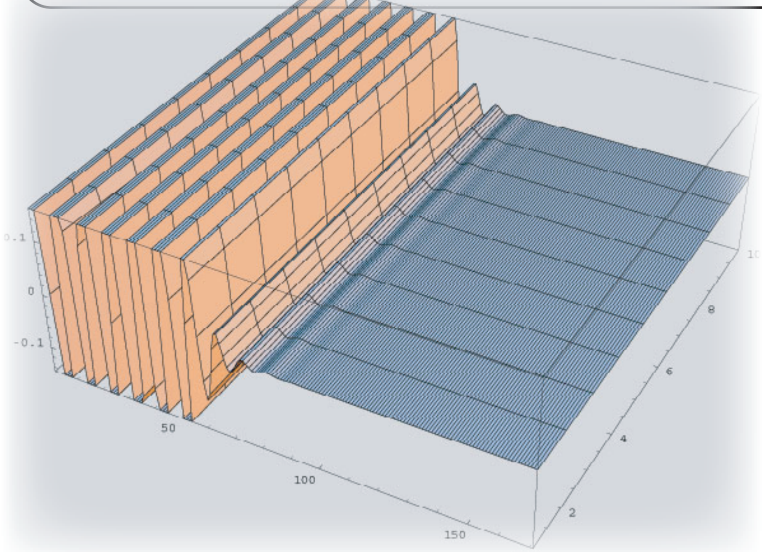


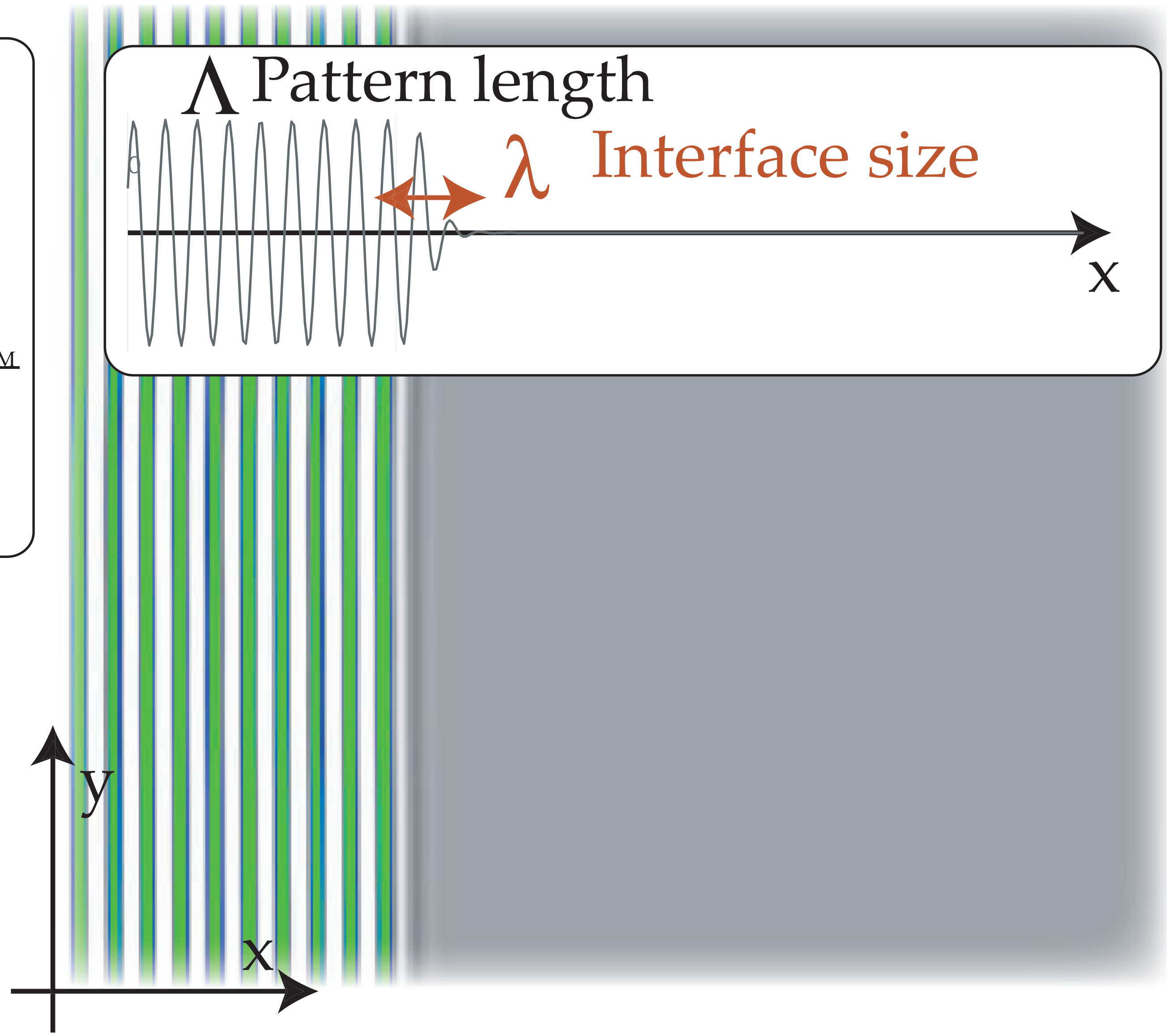
Numerical simulation

Locking phenomena

- The front speed is characterized by







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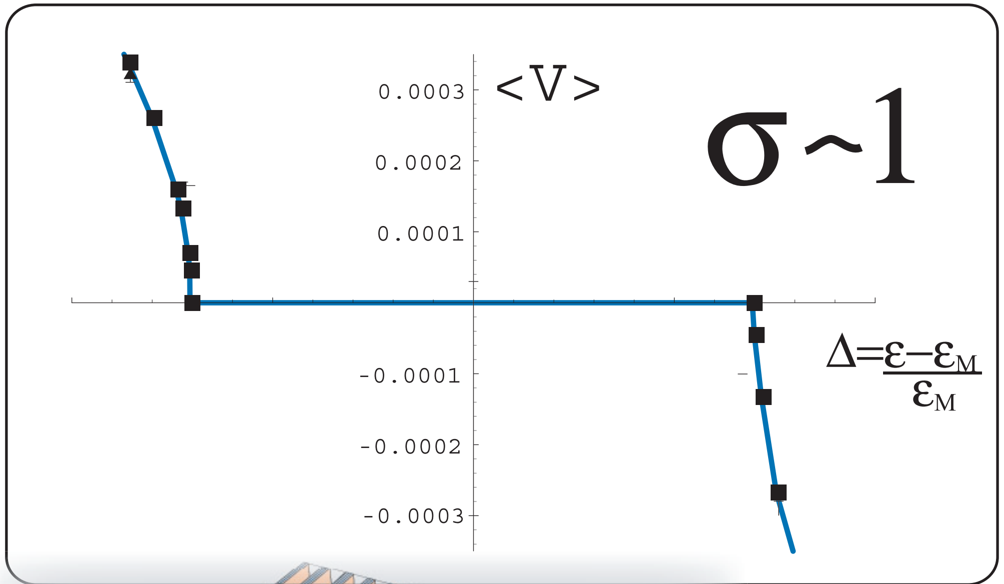
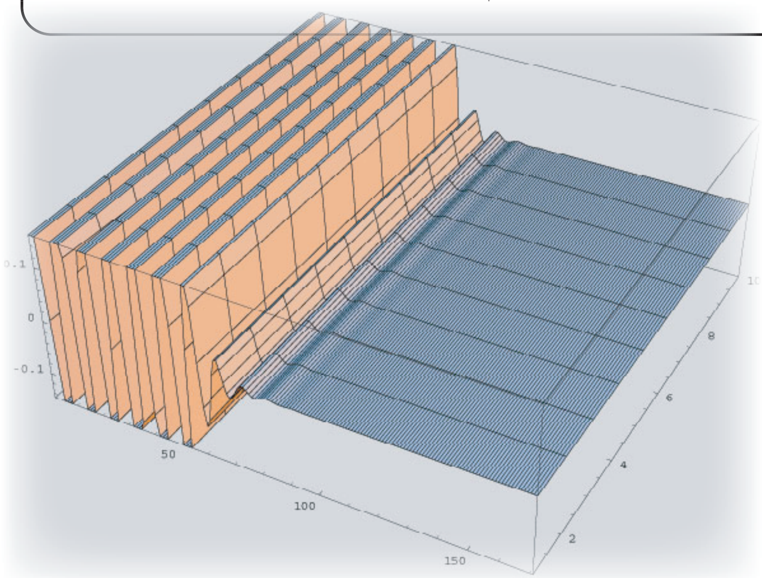
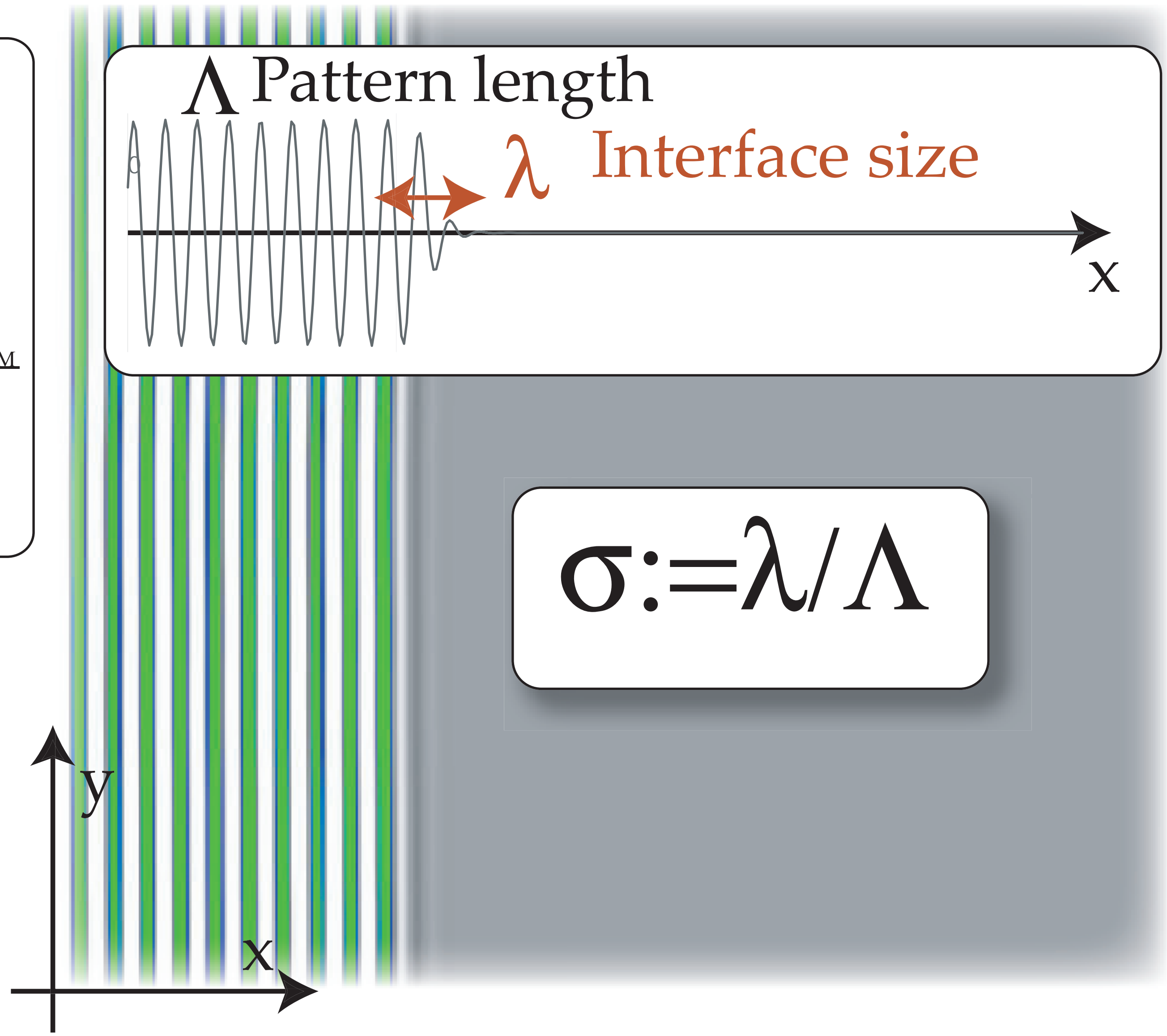
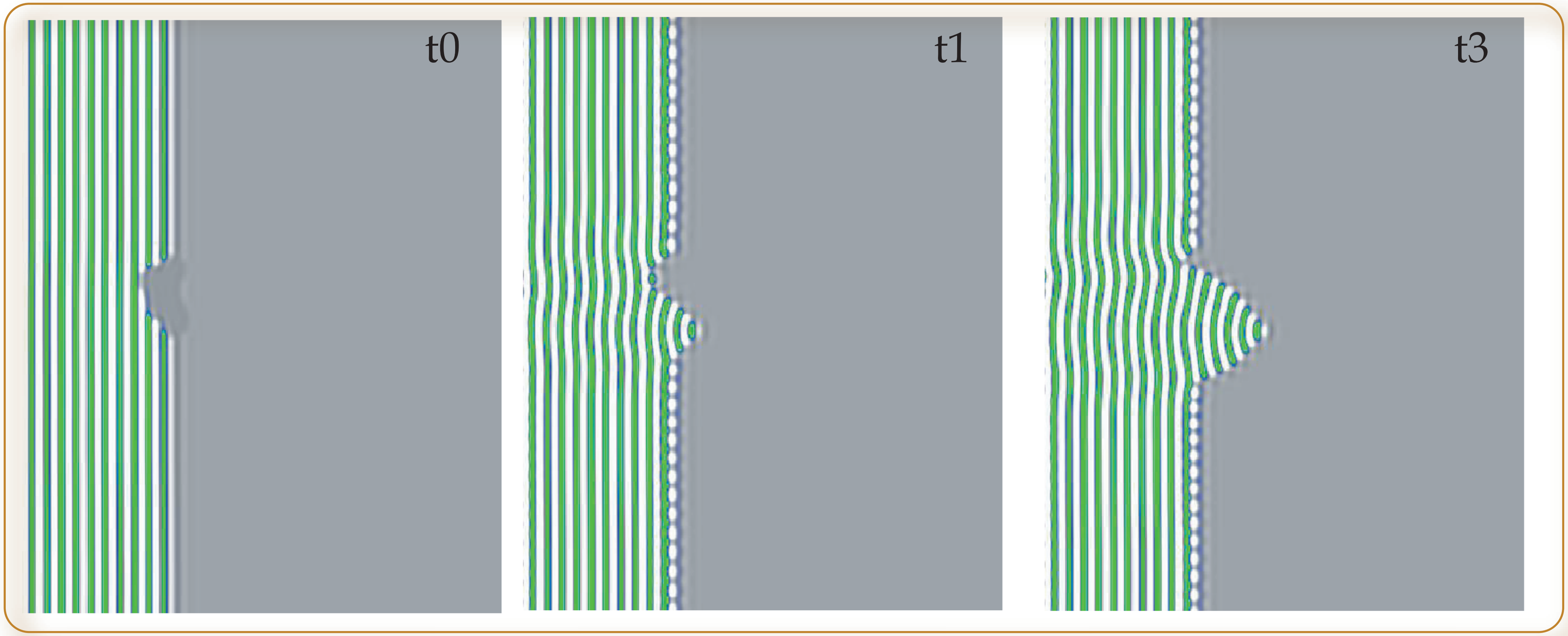






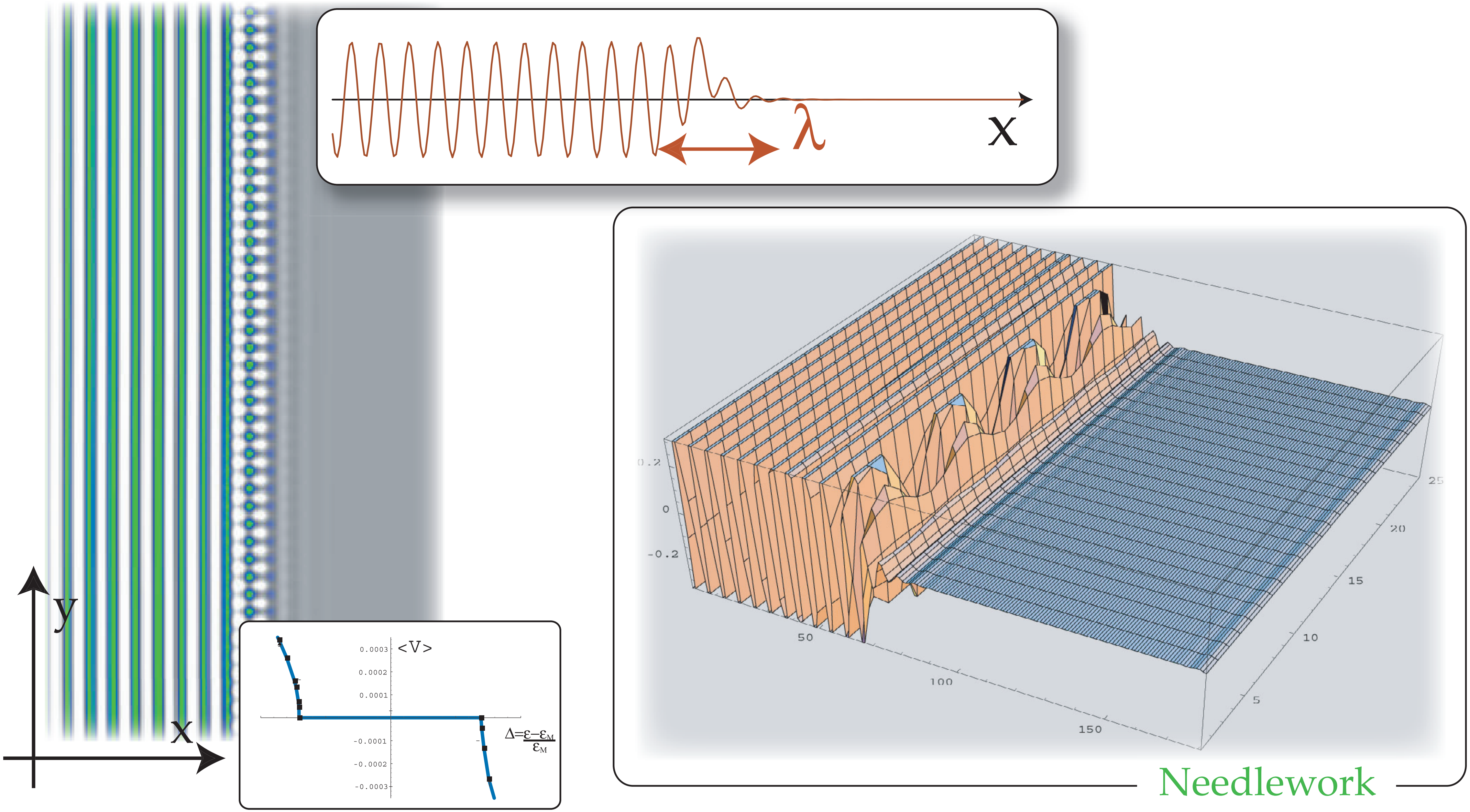
Zig-Zag dynamics

- For a finite perturbation, the interface exhibits zigzag dynamics, which is characterized by coarsening evolution



Needlework (Bordado, Broderie,..)

- For σ>1, the system exhibits a spatial interface instability



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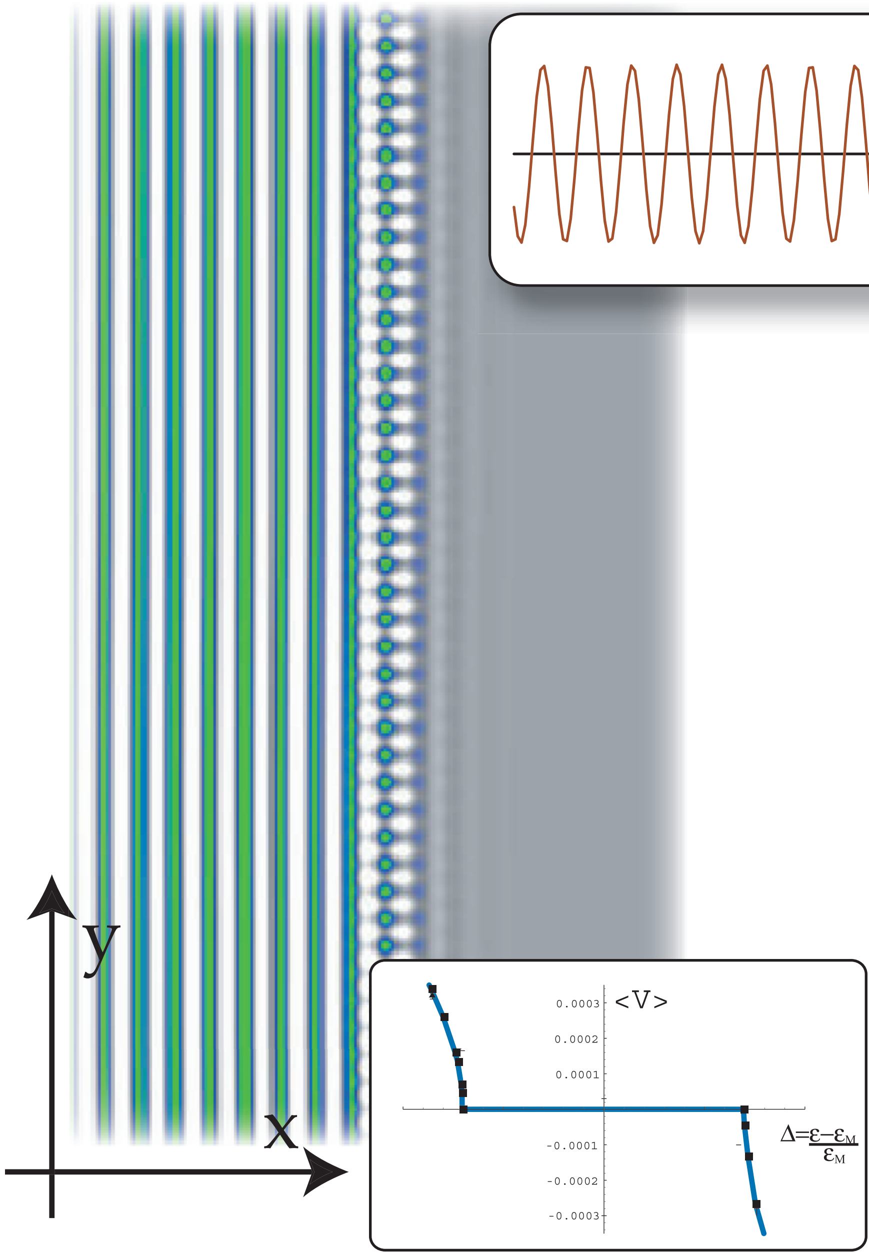
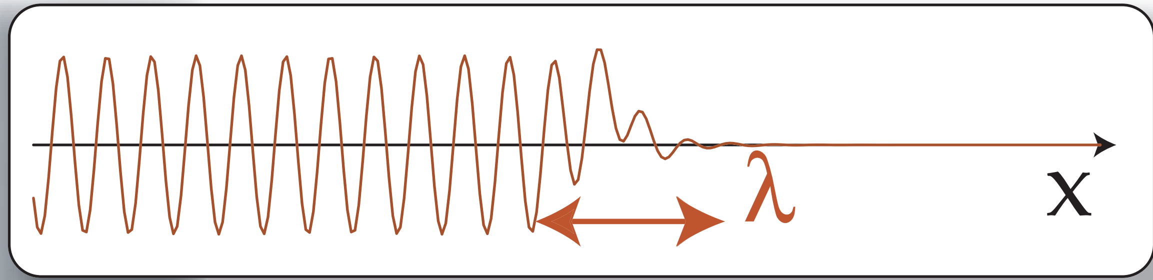
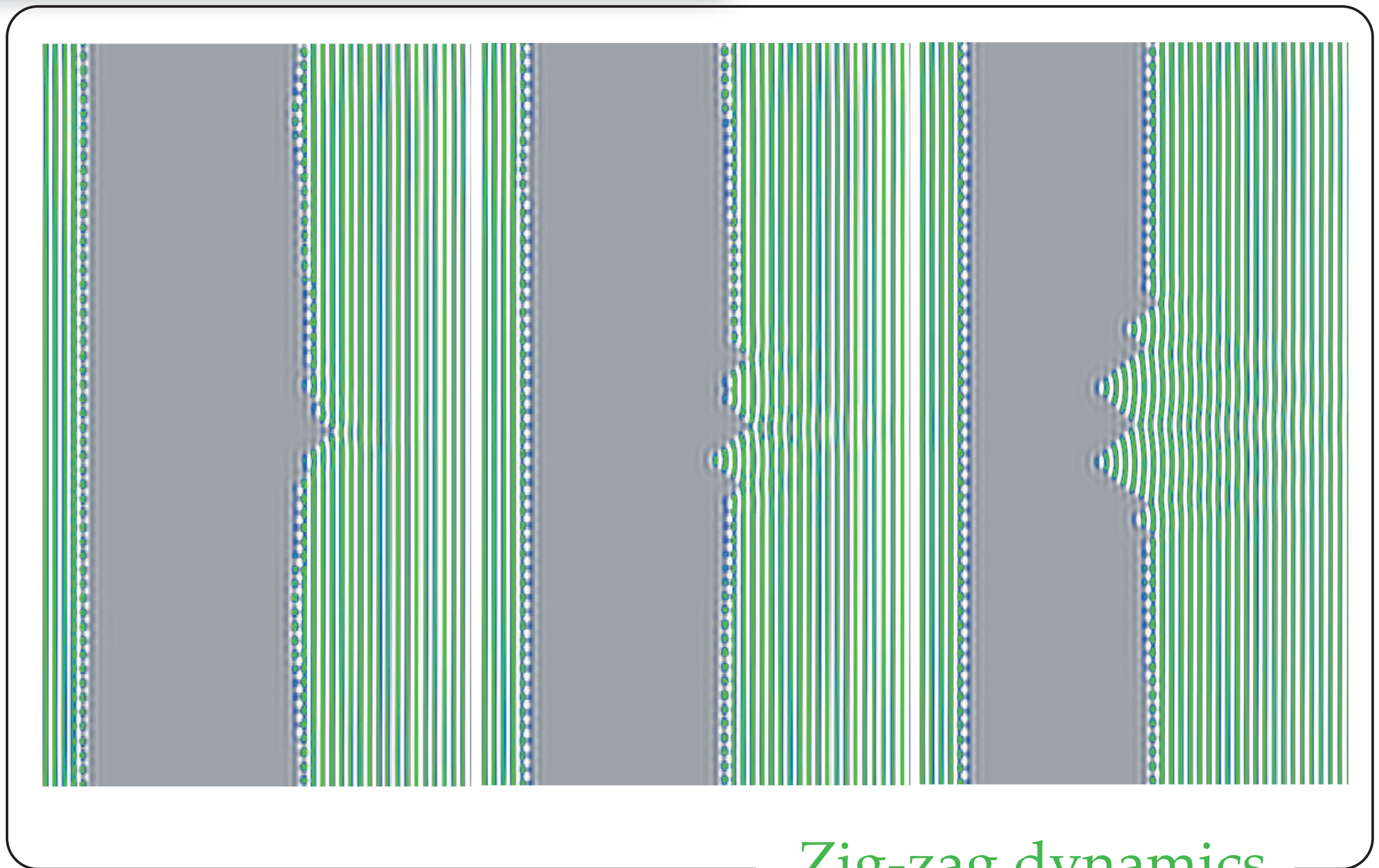


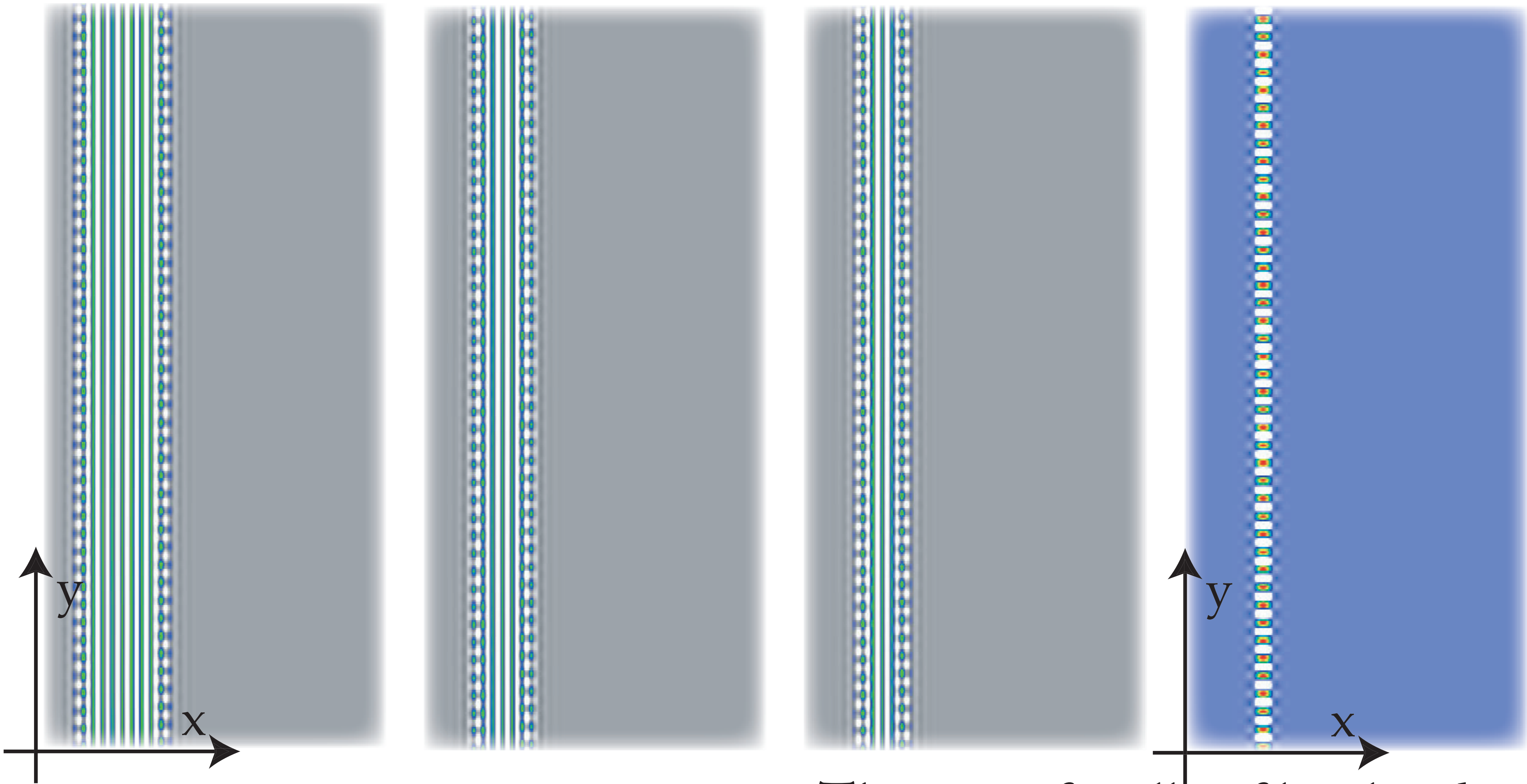




Zig-zag dynamics

Localized states

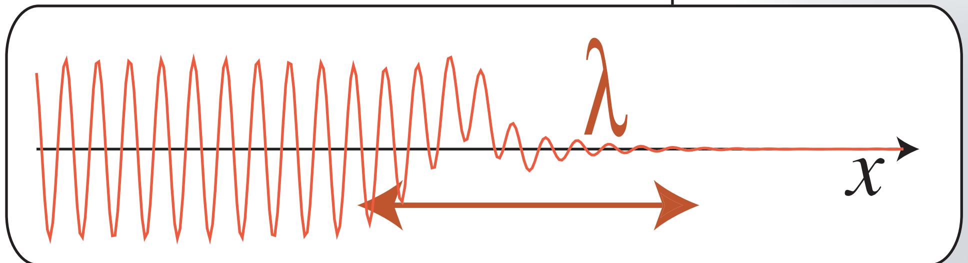
- As consequences of locking phenomenon:



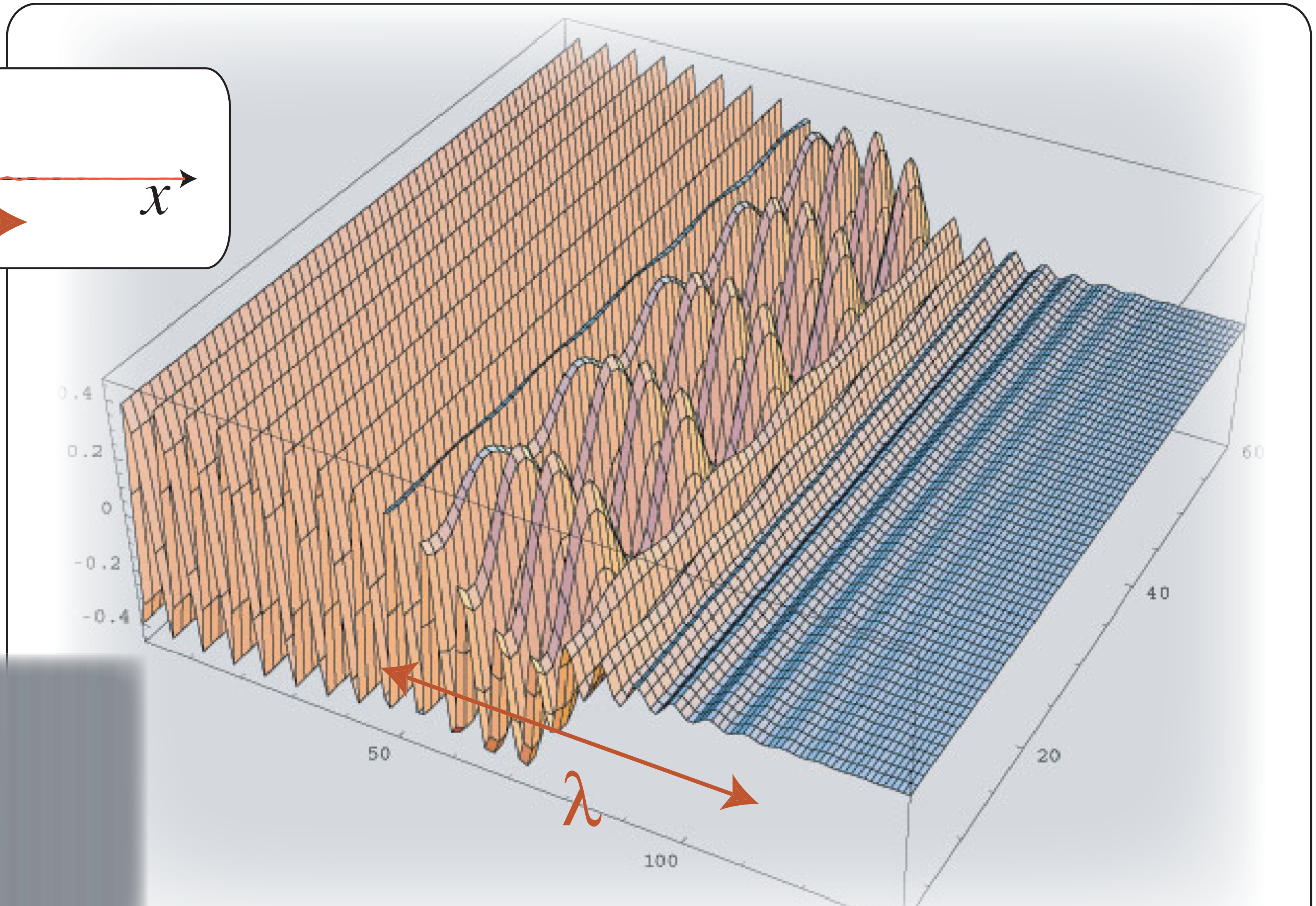
There is a familly of localized states

Needlework (Bordado, Broderie,..)

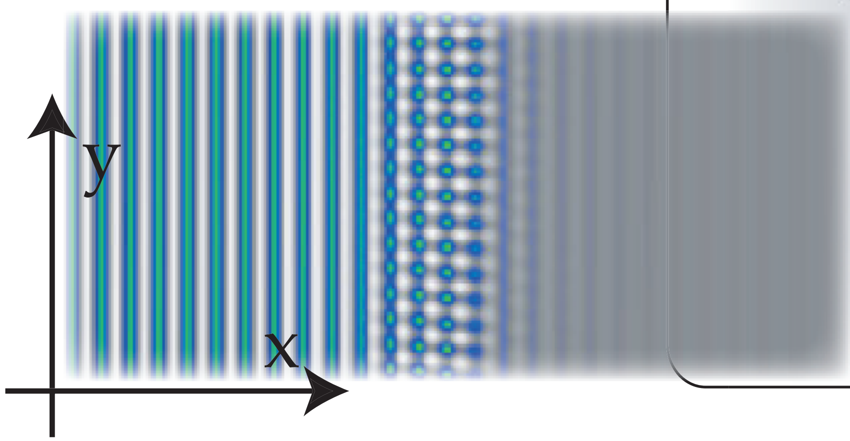
- *Increasing* $\sigma >> 1$ (*large* σ), *the needlework persists*



- *The system has* *pinning range* *and it exhibits* *zigzag dynamics*

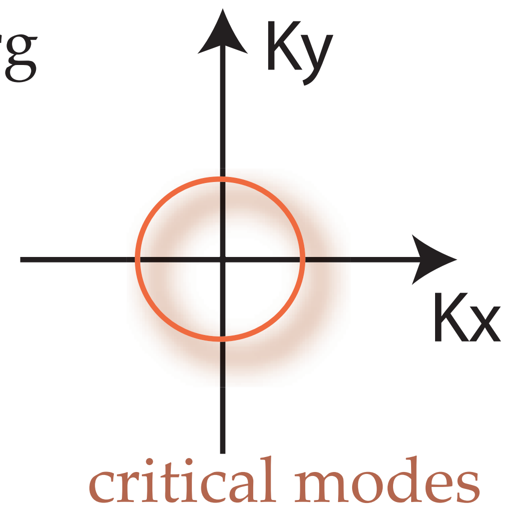


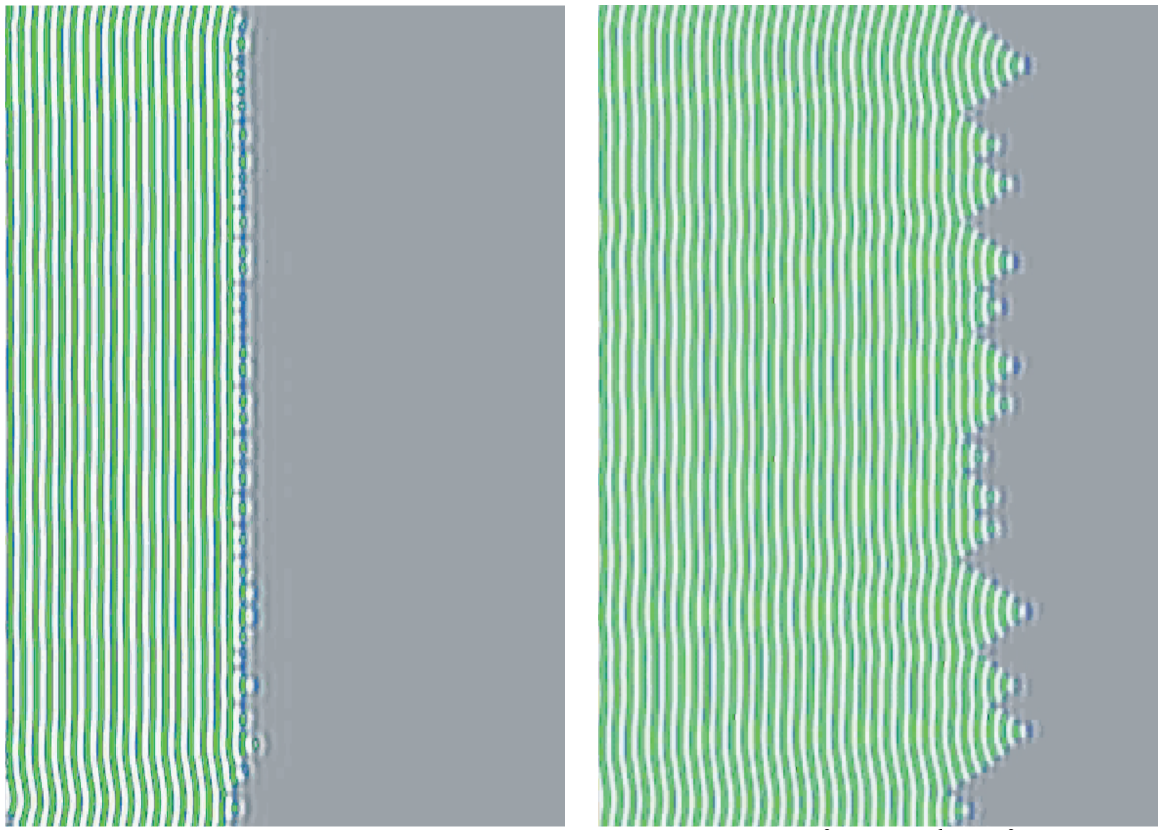
Needlework



Mechanism of zigzag dynamics

- The zigzag dynamics exhibited by model Swift-Hehenberg model is triggered by the isotropy features of stripe orientation, that is, the orientation of the stripe is arbitrary and it is determined by the initial condition, whose influence is enhanced in the wall.





Simulation

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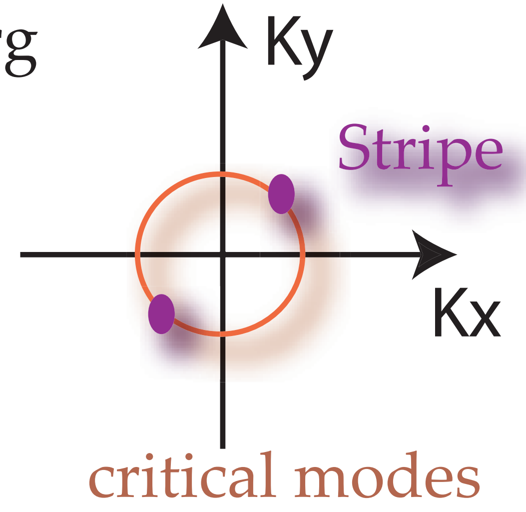
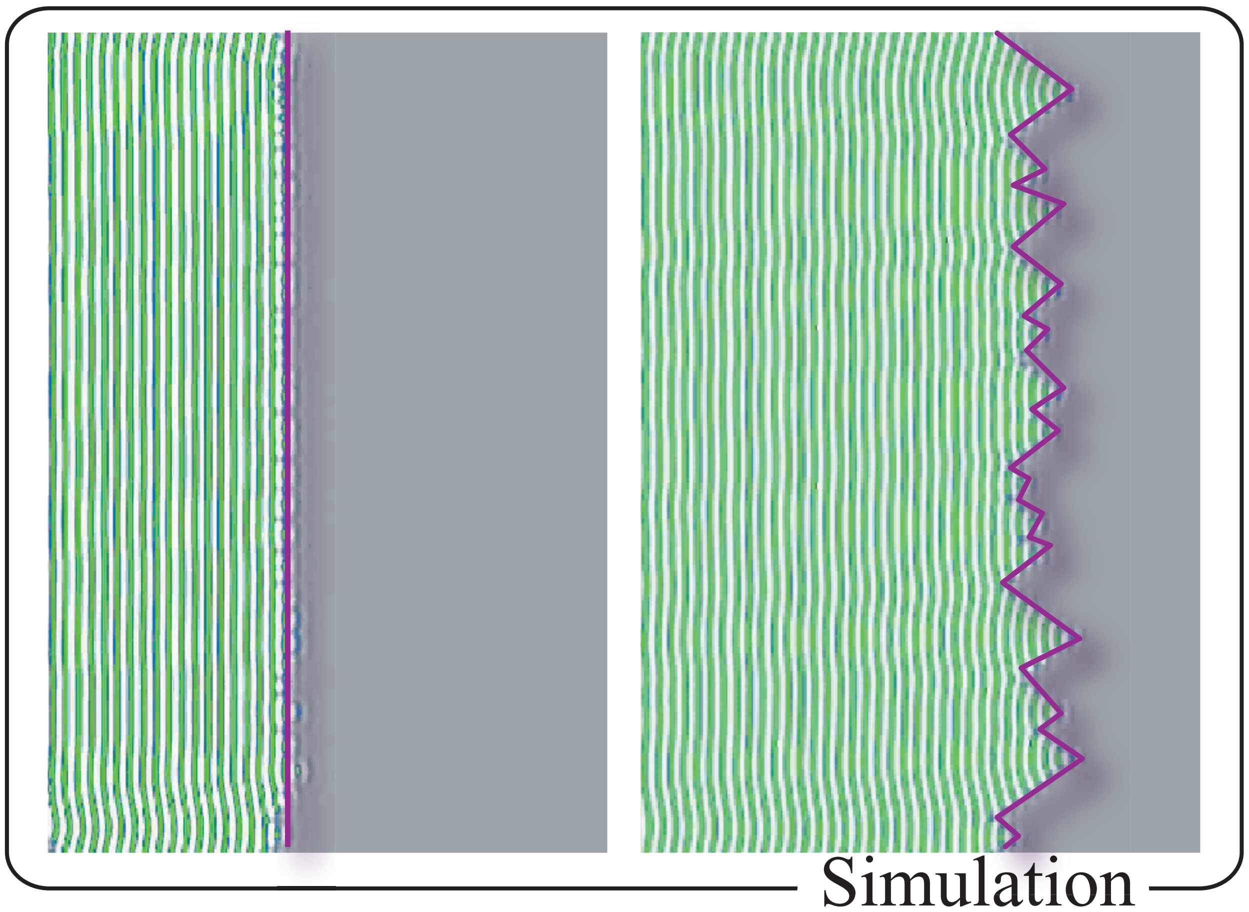




Zigzag dynamics and coarsening

Universal description

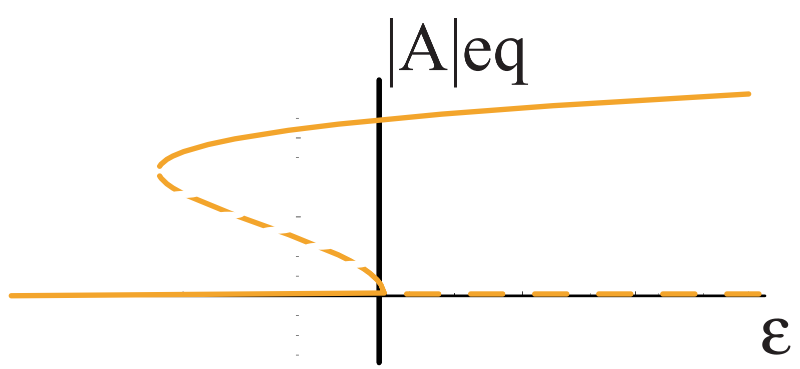
- For small ε and ν, we can introduce the ansatz

$$u = \sqrt{\frac{2\nu}{10}}\varepsilon^{1/4}\left\{A\left(X=\frac{x}{l_0},Y=\frac{y}{l_0},\tau=\frac{9\nu^2\left|\varepsilon\right|}{10}t\right)e^{iqx}\right.$$
$$\left.+w_1\left(x,y,\tau\right)+c.c\right\},$$

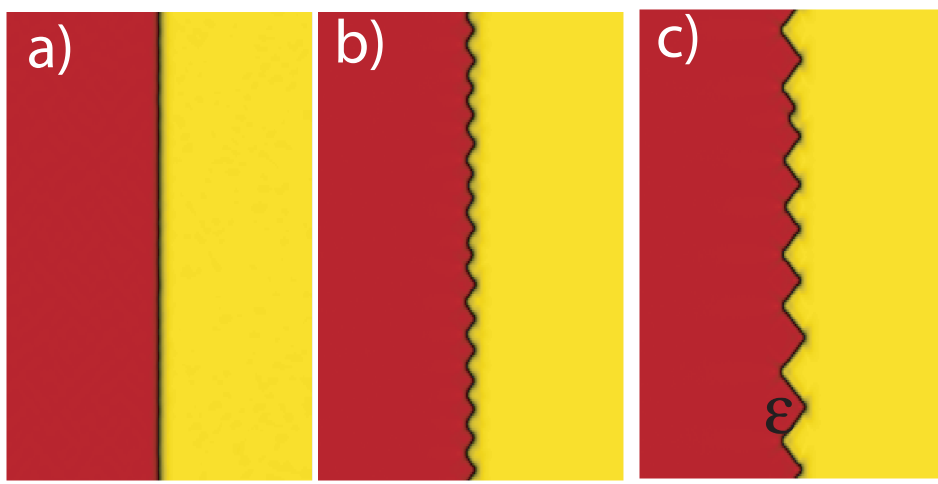
where the amplitud equation satisfies (Newell-Whitehead-Segel equation)

$$\partial_\tau A = \epsilon A + \left|A\right|^2 A - \left|A\right|^4 A + (\partial_x - i\frac{2}{q^2}\partial_{yy})^2 A$$

Bifurcation Diagram



Numerical simulation



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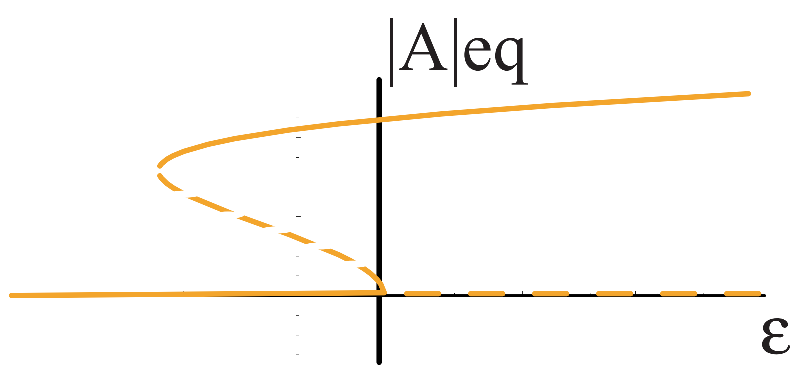

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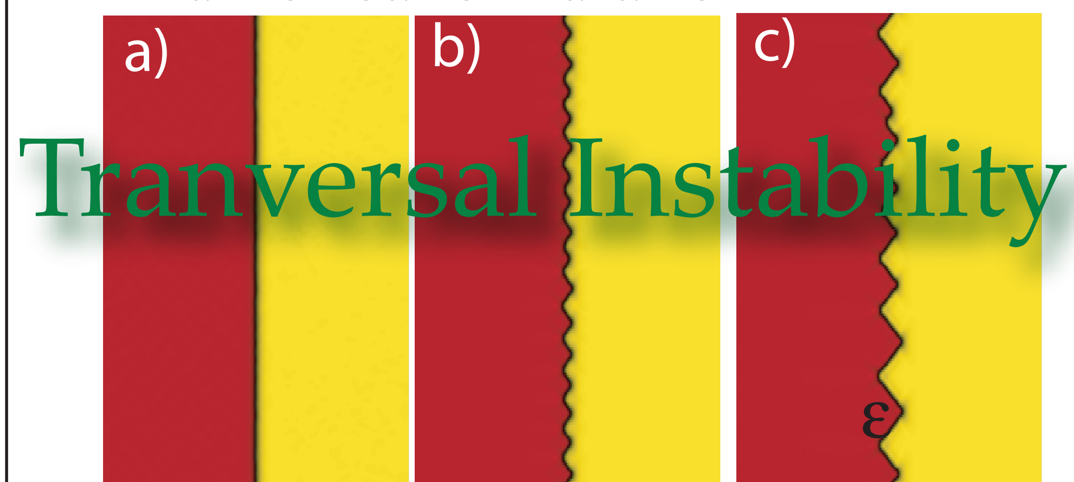
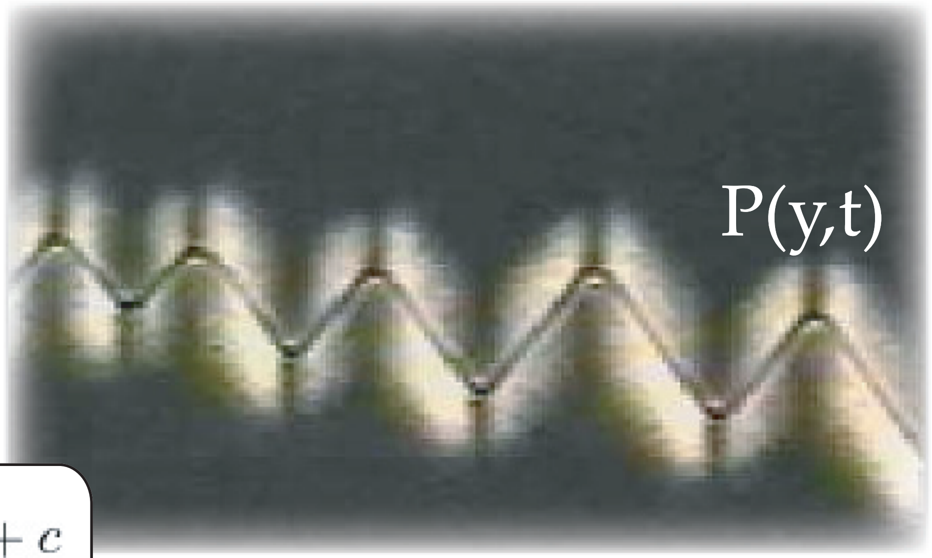


Interface equation

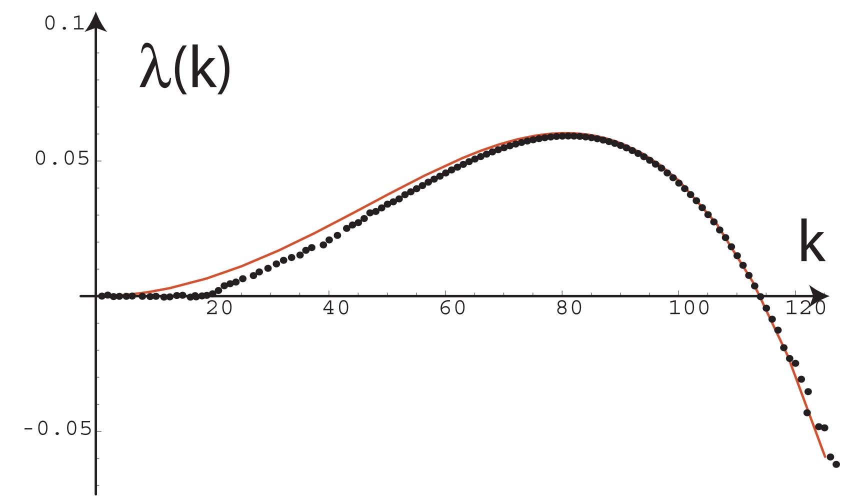
- A standard method to figure out the dynamics exhibited by the system is to derive an equation for the interface. The propagative zigzag interface is universally described by convective Cahn-Hilliard model



$$\partial_t P = \varepsilon P_{yy} + P_y^2 P_{yy} - P_{yyyy} + \alpha P_y^2 + b P_{yy}^2 + e P_y P_{yyy} + c$$

M. Clerc et al Eur. Phys. J. E 1, 179 (2000).

Spectrum



- Using this method, we have obtained a generalized Canh-Hilliard equation, which shows that the flat interface is marginal (linearly) and nonlinear stable. Hence, this method does not give account of the depinning effect.

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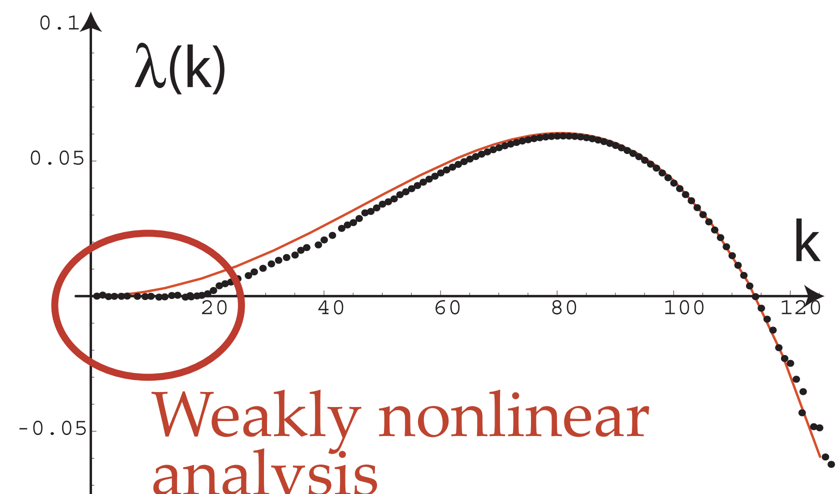
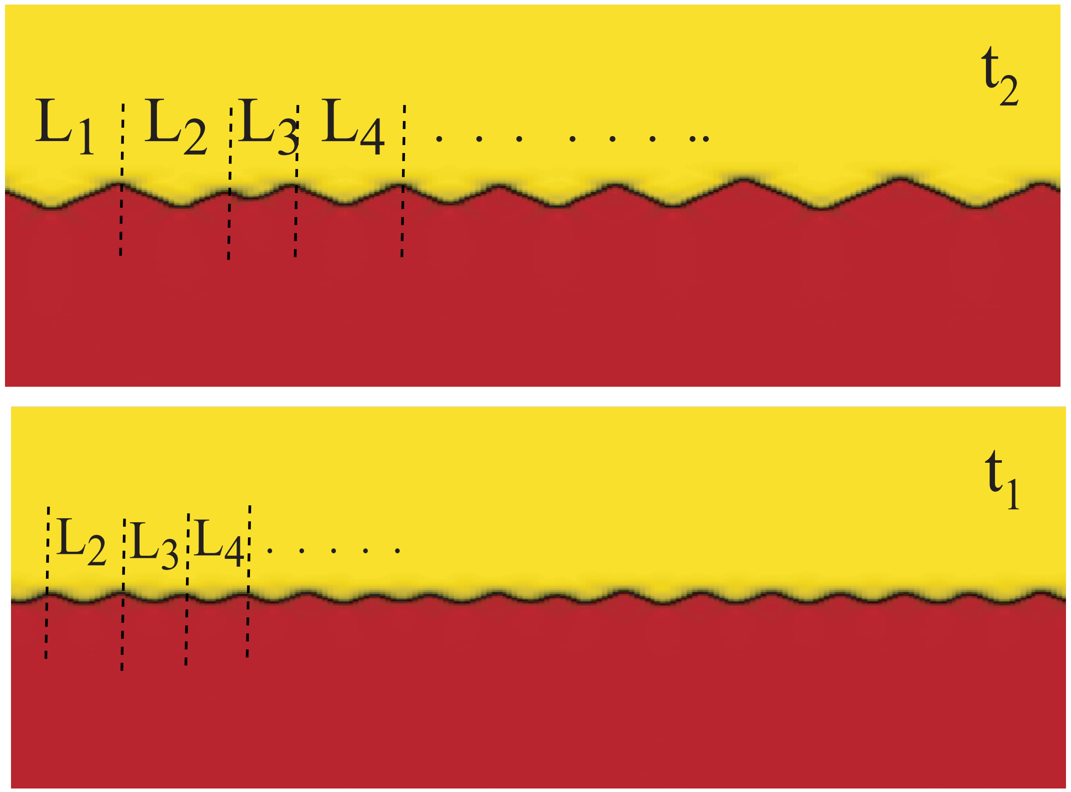
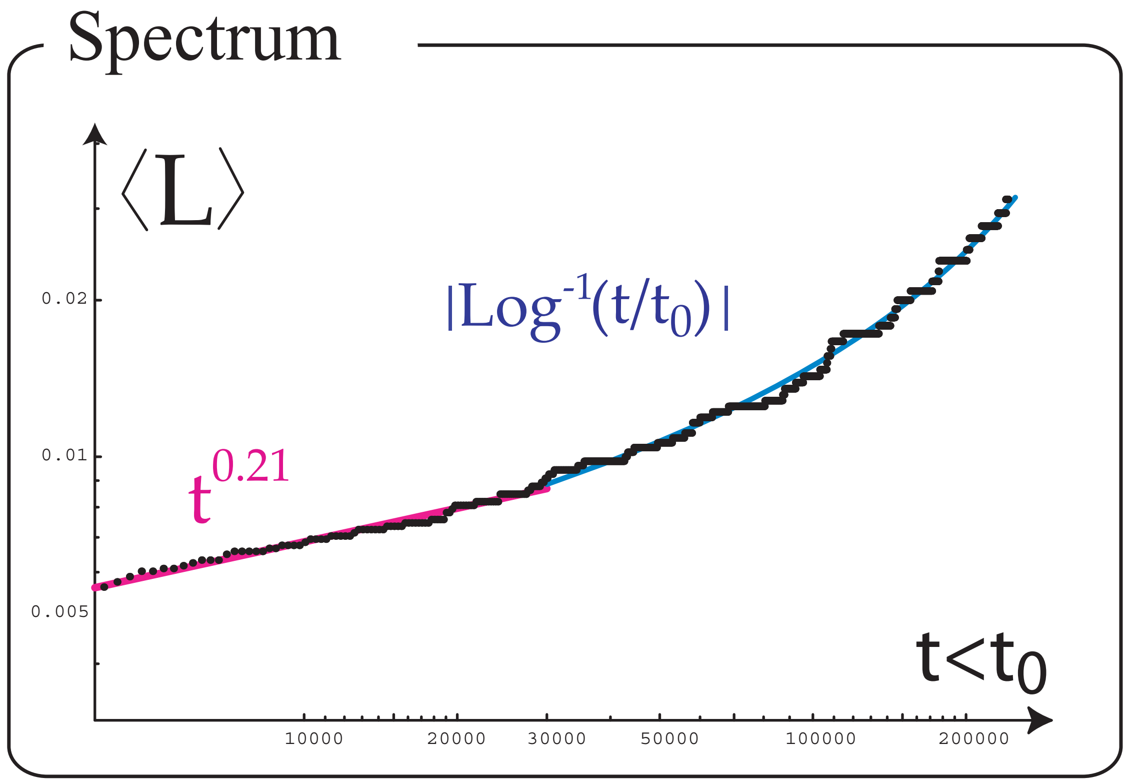


Coarsening

- Numerically, we compute the average length size <L(t)> between two successive extreme points of the interface of model





$$\langle L \rangle = \frac{1}{N} \sum_{i=1}^{N} L_i$$

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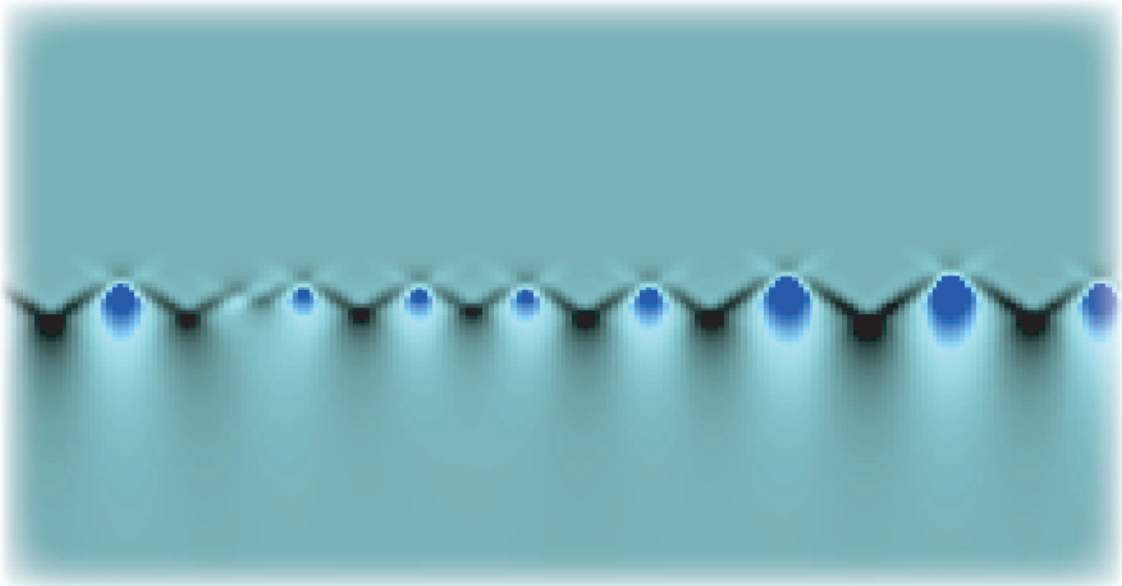
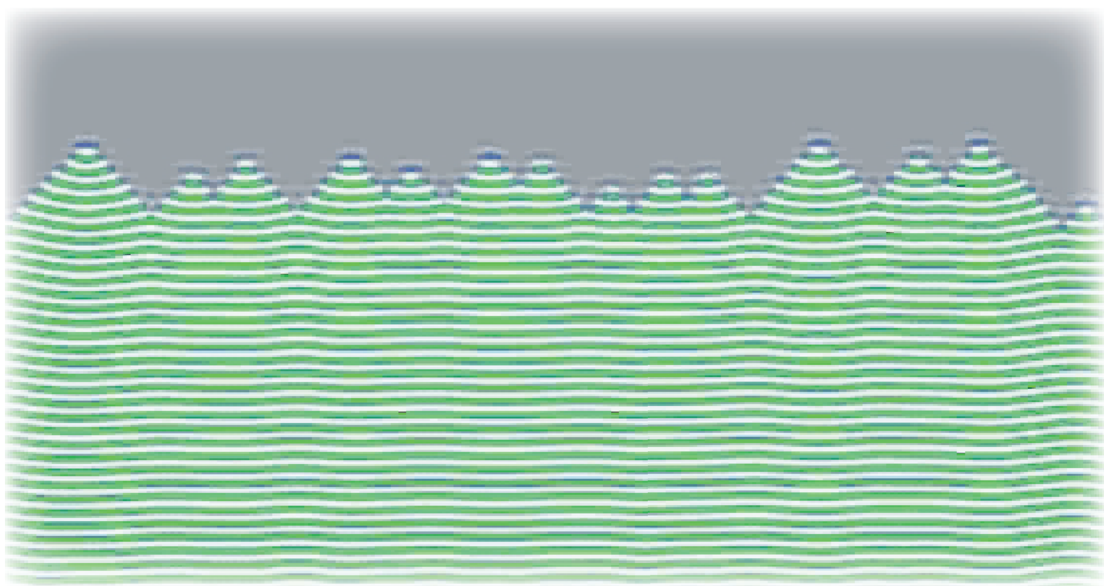
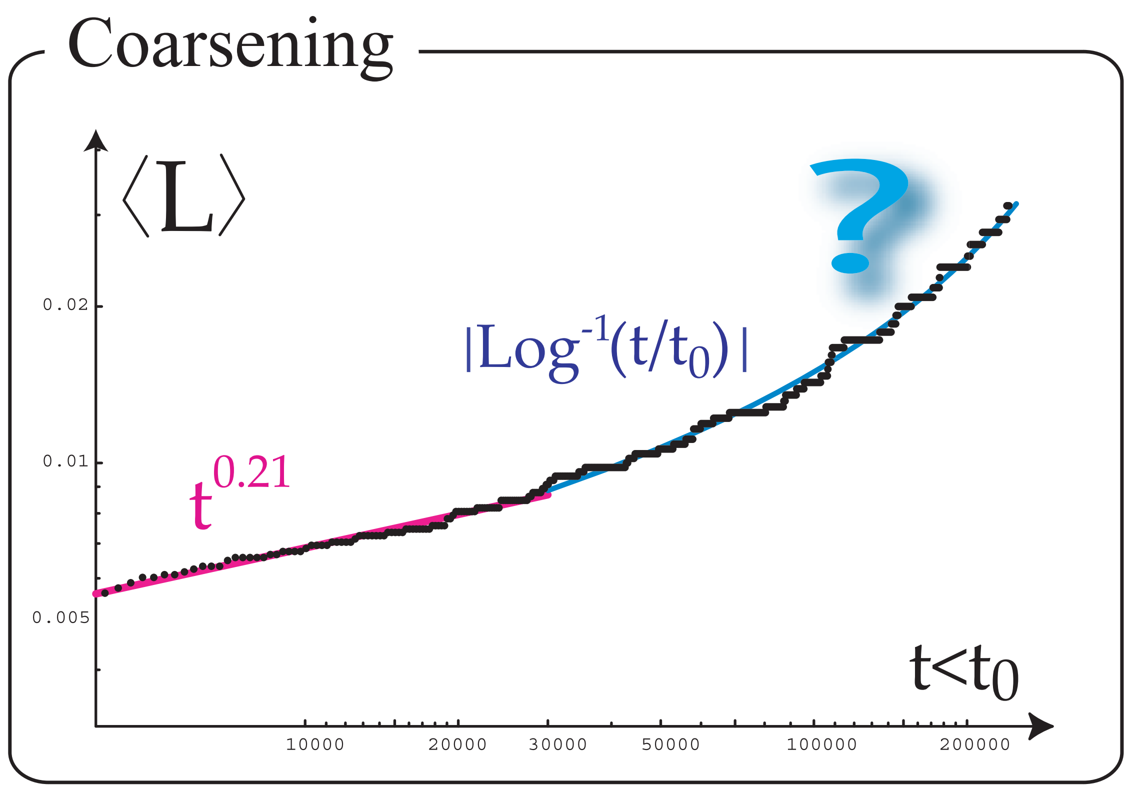






Interface-phase-interface interaction

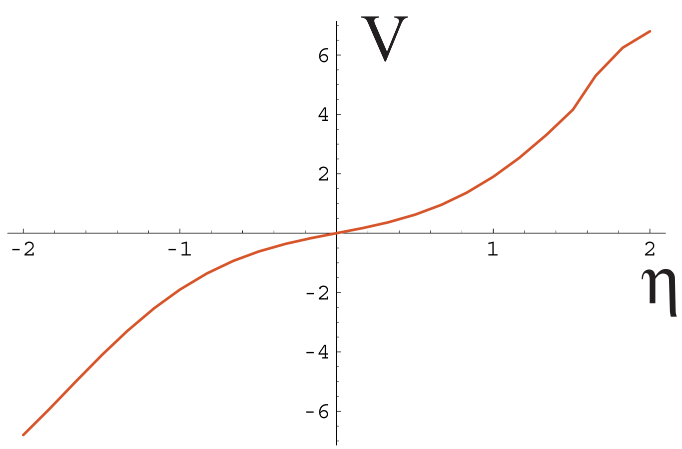
Amended amplitud equation

- To explain the appearance of pining range, needlework and complex dynamics, we consider

$$\partial_t A = \epsilon A + |A|^2 A - |A|^4 A + (\partial_x - \frac{i}{q}\partial_{yy})^2 A$$

Resonant terms

Front speed



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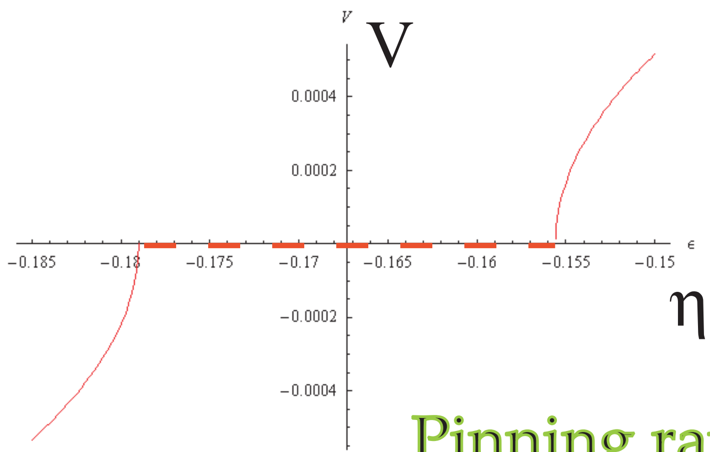
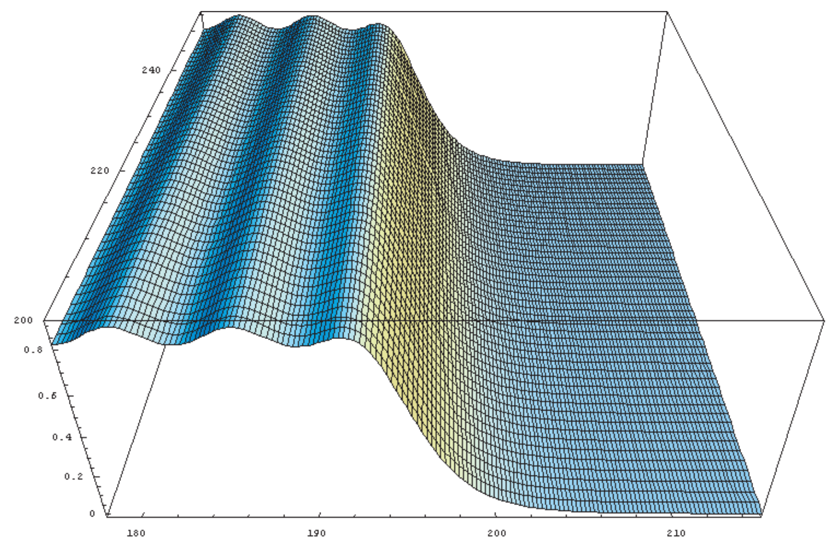
Non Resonant terms

Front speed





large η

D. Bensimon, B.I. Shraiman, and V. Croquette, Phys. Rev.A 38, 5461 (1988).
M.G. Clerc, C. Falcon, and E. Tirapegui, Phys. Rev. Lett. 94, 148302 (2005).

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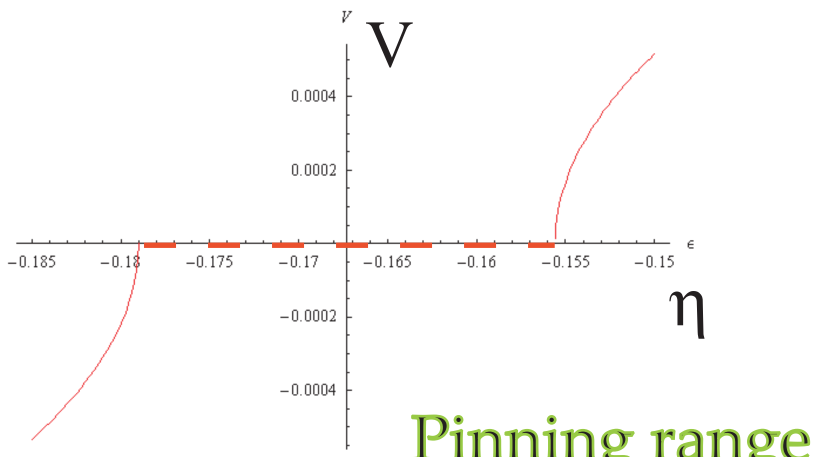
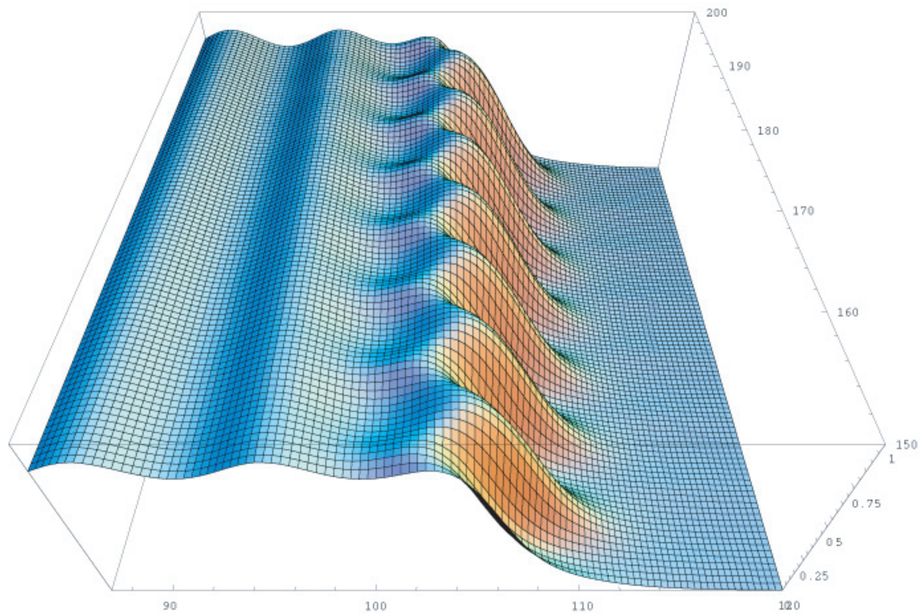
Non Resonant terms

Front speed





Smaller η

D. Bensimon, B.I. Shraiman, and V. Croquette, Phys. Rev.A 38, 5461 (1988).
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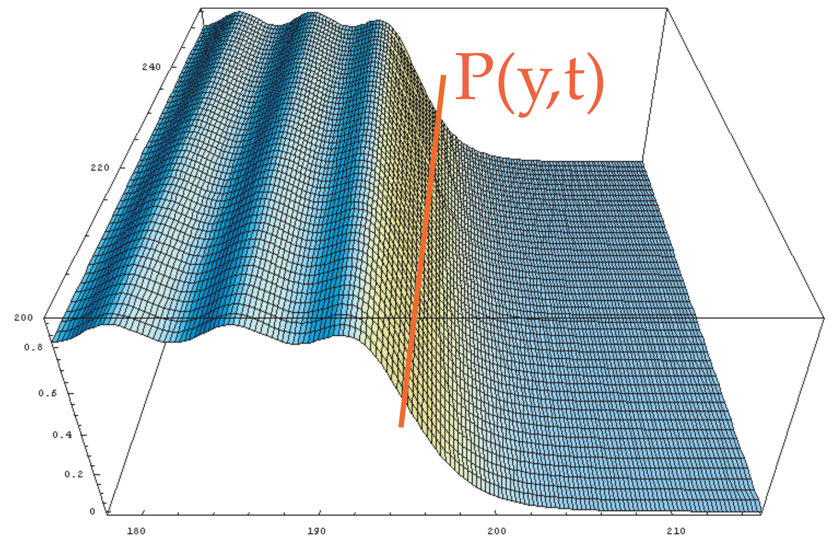
Simple prototype model

- P(y,t) stand for the interface bewteen the stripe pattern and uniform state

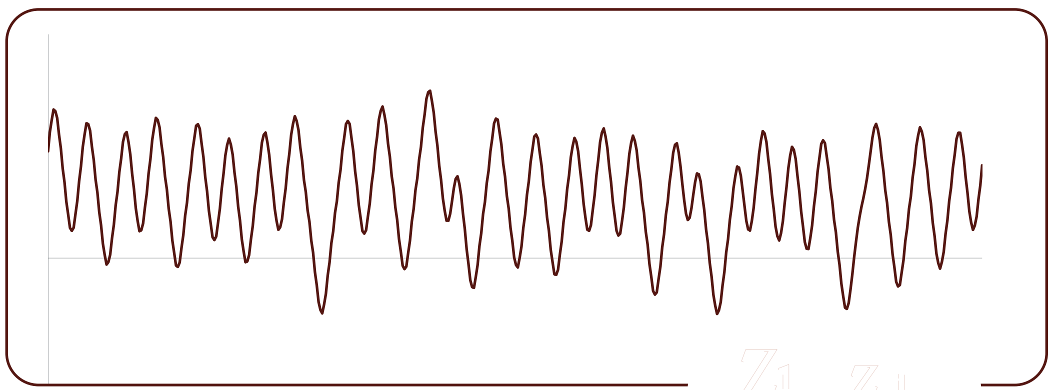


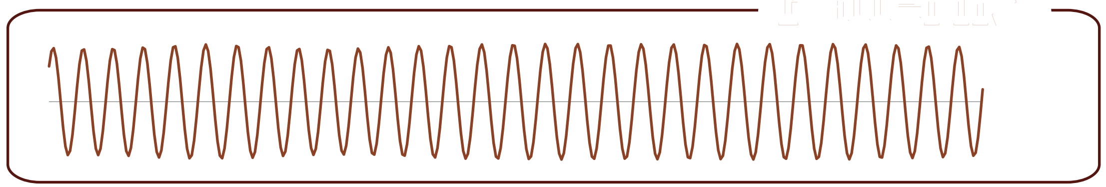
- Modified Cahn-Hilliard equation

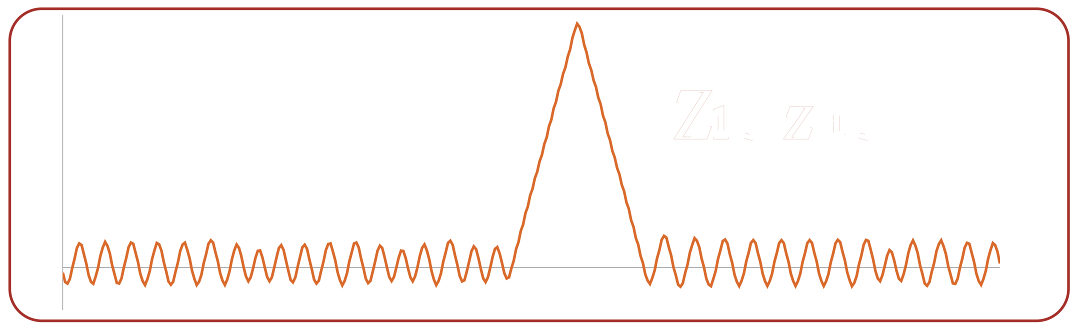
$$\partial_t P = \varepsilon P_{yy} + P_y^2 P_{yy} - P_{yyyy} + \alpha P_y^2 - \nu \sin(\kappa P)$$

Zig-zag Instability

Spatial forcing

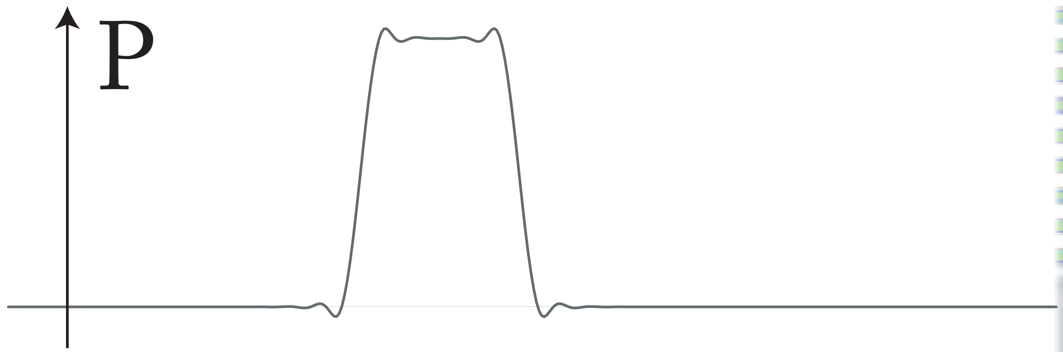


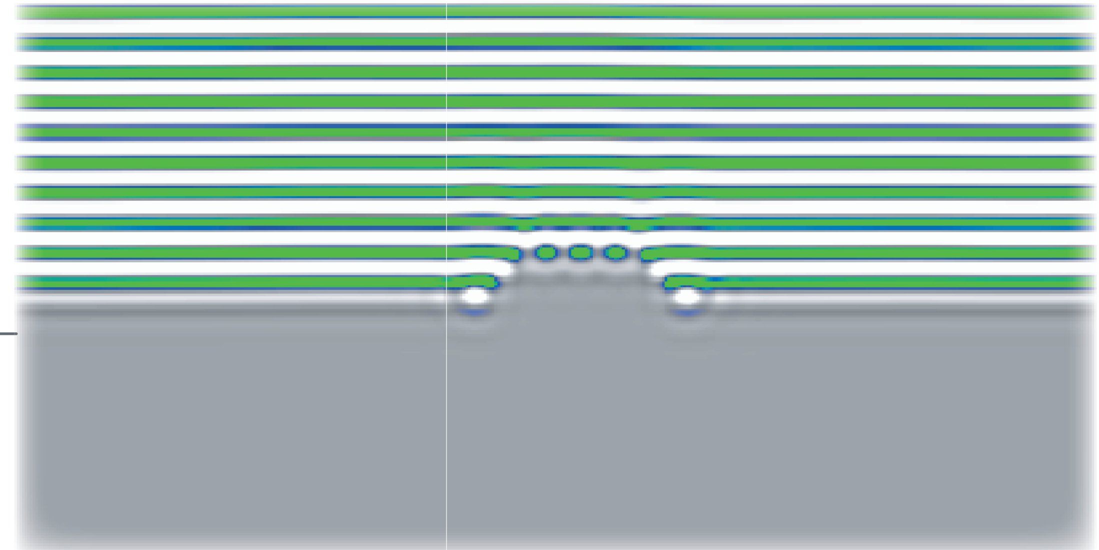


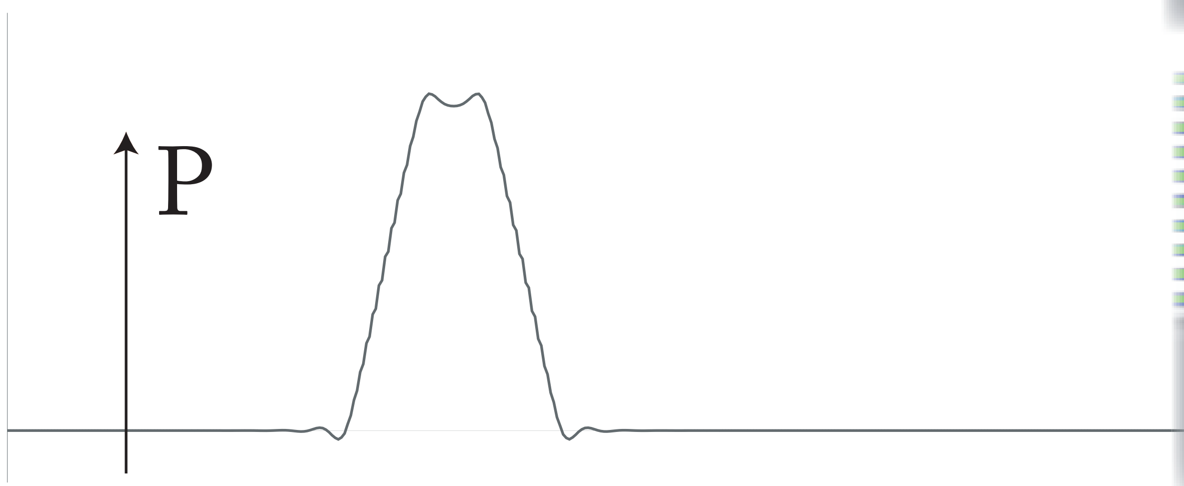


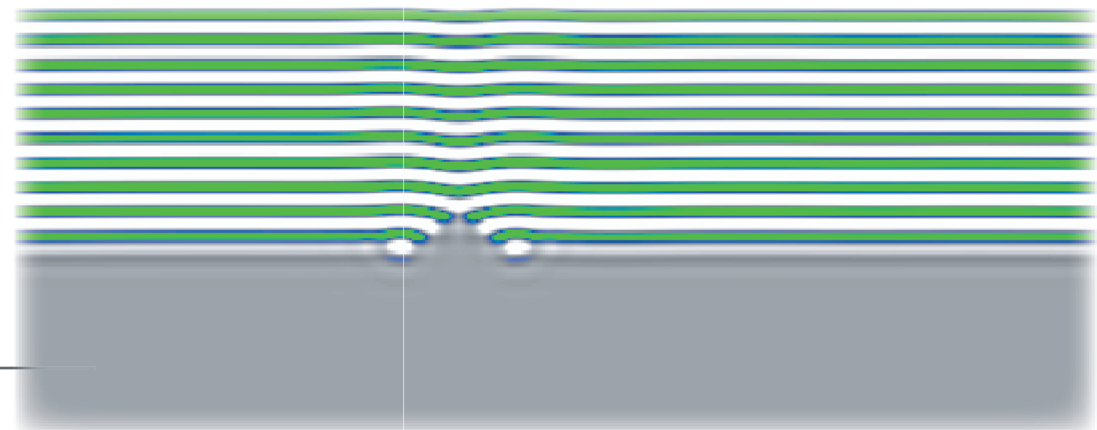
localized states in the interface

$$\partial_t P = \varepsilon P_{yy} + P_y^2 P_{yy} - P_{yyyy} + \alpha P_y^2 - \nu \sin(\kappa P)$$



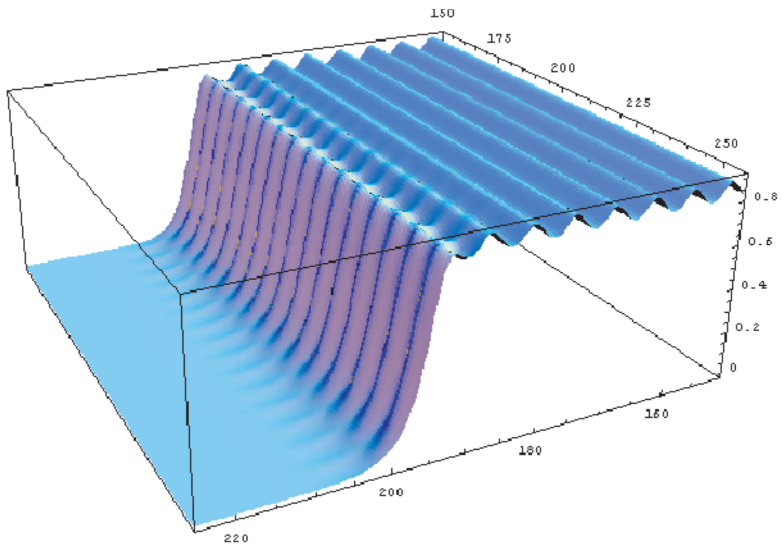


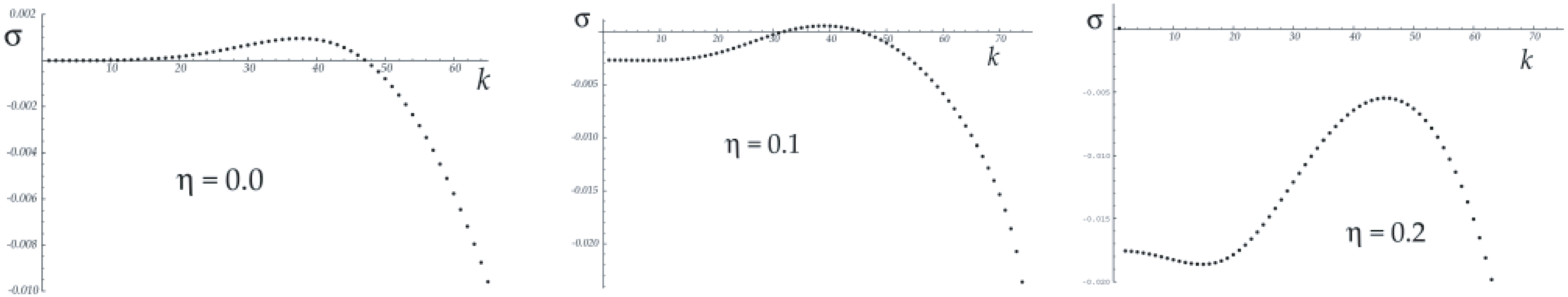




Amended amplitud equation

- The spectrum of the interface





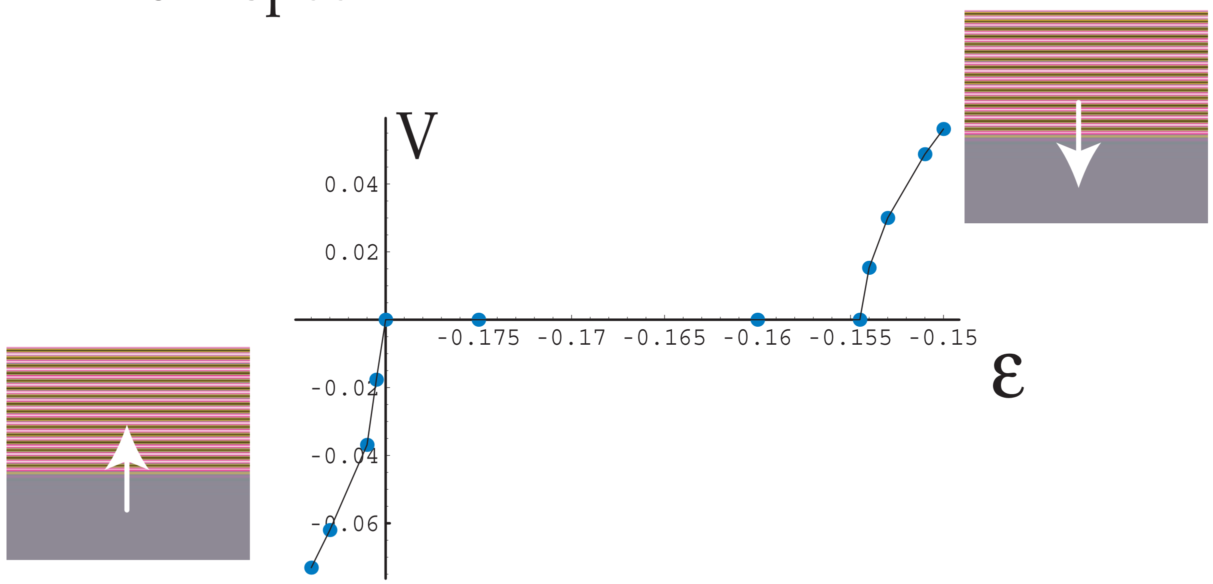
Hence, the needlework are consequence of the interaction of envelope variation with the small scale underlying the spatial periodic solution

Pinning effect in anisotropic system

- The origin of the depinning in NewellWhiteheadSegel equation is the anisotropic spatial coupling. We consider the following model

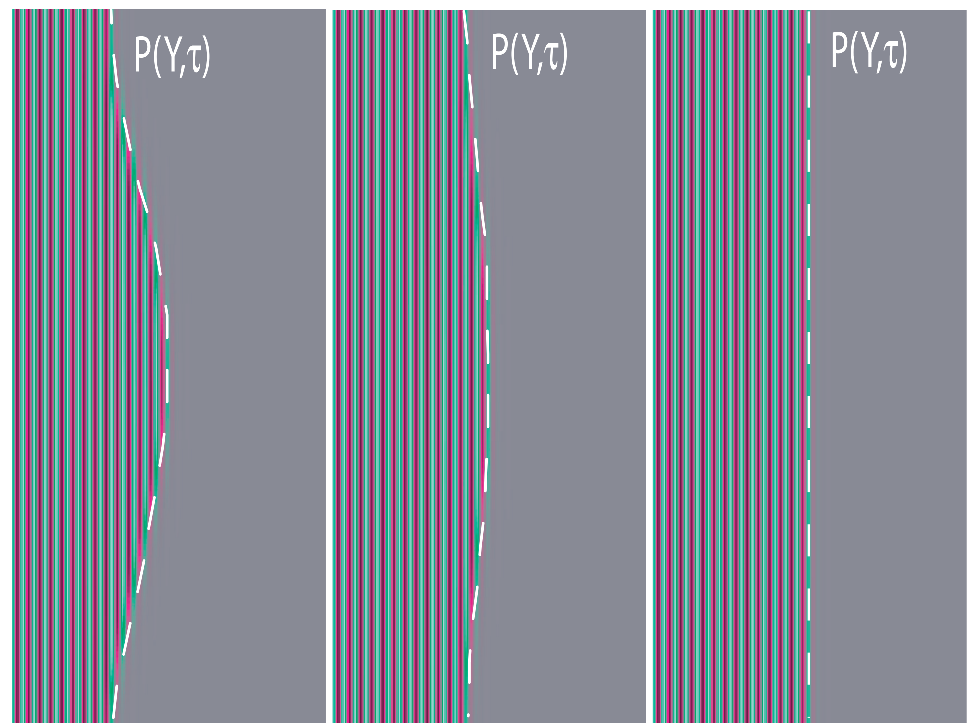
$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - \left(\partial_{xx} + q^2\right)^2 u + D\partial_{yy} u,$$

Front speed



$$\partial_\tau B = \sigma B + |B|^2 B - |B|^4 B + \vec{\nabla}^2 B$$

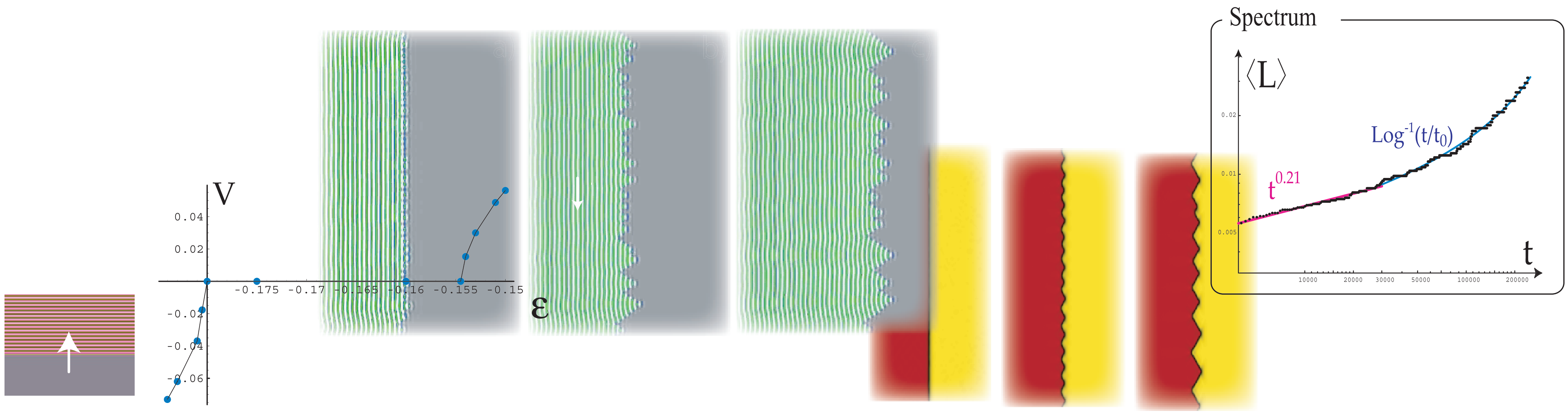
Dynamical behavior



Conclusions

Systems which have coexistence between stable stripe pattern and uniform states can exhibit interfaces connecting these states.

- The interface dynamics have complex behaviors: flat interface, periodic solutions, localized state, zigzag dynamics,....



Outlook

Noise induce front propagation

$$\partial_t u = \varepsilon u + \nu u^3 - u^5 - (\partial_{xx} + q^2)^2 u + D\partial_{yy} u, \text{+NOISE}$$