Rough Set Theory – Fundamentals and an Overview of its Main Applications

Rough Set Theory (RST) can be approached as an extension of the Classical Set Theory, for use when representing incomplete knowledge. Rough sets can be considered sets with fuzzy boundaries – sets that cannot be precisely characterized using the available set of attributes. The basic concept of the RST is the notion of *approximation space*, which is an ordered pair *A=(U,R)*, where

- *U*: nonempty set of objects, called universe
- *R*: equivalence relation on *U*, called *indiscernibility relation*. If x, y $\in$ *U* and x*R*y then x and y are *indistinguishable* in *A*

Each equivalence class induced by *R*, ie, each element of the quotient set $\widetilde{R} = U/R$, is called an *elementary set* in A. An approximation space can be alternatively noted by $A=(U, \widetilde{R})$. It is assumed that the empty set is also elementary for every approximation space *A*. A *definable set* in *A* is any finite union of elementary sets in *A*. For x $\in$ *U* let $[x]_R$ denote the equivalence class of *R*, containing x. For each X $\subseteq$ *U*, X is characterized in *A* by a pair of sets – its *lowe*r and *upper approximation* in *A*, defined respectively as:

$$A_{low}(X) = \{x \in U \mid [x]_R \subseteq X\}$$
$$A_{upp}(X) = \{x \in U \mid [x]_R \cap X \neq \varnothing\}$$

A *rough set* in *A* is the family of all subsets of *U* having the same lower and upper approximations.

Many different problems can be addressed by RST. During the last few years this formalism has been approached as a tool used in connection with many different areas of research. There have been investigations of the relations between RST and the Dempster-Shafer Theory and between rough sets and fuzzy sets. RST has also provided the necessary formalism and ideas for the development of some propositional machine learning systems. It has also been used for, among many others, knowledge representation; data mining; dealing with imperfect data; reducing knowledge representation and for analysing attribute dependencies. The notions of rough relations and rough functions are based on RST and can be applied as a theoretical basis for rough controllers, among others.

This tutorial intends to present the main concepts involved in RST and to examine the contribution of this formalism to a few research areas mentioned above.

Topics (4 hours talk):

- Basic Concepts
- Knowledge Representation Systems
- Dependence and Independence of Attributes - Data Reduction
- Rough Sets and Fuzzy Sets
- Rough Sets and Machine Learning
- Reasoning about Knowledge – Rough Set Based Logic
- Rough Relations
- Appraisal

Target Audience: those who are interested in:

Knowledge Representation
Data Reduction
Generation of minimal sets of decision rules from data
Formal representation of indiscernibility
Becoming familiar with state-of-the-art in RST and its Applications