Fuzzy Set Theory: a Playground for Mathematicians

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Abstract

When Zadeh first introduced the notion of a fuzzy set, it was laughed away by the ruling caste of 'serious' mathematicians and logicians. Driven by ignorance, the main reason for it was probably strategic: as fuzzy set theory touches upon the very building block of mathematics, it might be better to deny it than to risk being taken out of business. Even the ample practical realizations had no effect on them.

Unfortunately, the fuzzy mathematics community has offered plenty of stones to be thrown at them. One by one, the core mathematical theories were subjected to fuzzification: topology, algebra, analysis, etc. The literature has witnessed various waves of these activities, not in the least the 'replace min by a triangular norm' wave. The majority of these works are sheer mathematical exercises and have no reason to appeal to the established mathematical community.

Indeed, instead of trying to generalize things on a micro-level, the potential interest of fuzzy set theory lies on the macro-level, offering a global view and additional insight into existing mathematical theories. Fortunately, more and more works of this kind are appearing, and are, finally, attracting the interest of non-fuzzy mathematicians. A nice work of the latter kind is the far-reaching monograph "Metamathematics of fuzzy logic" by Hajek on fuzzy logic in 'narrow sense', making its way into the estabished community of many-valued logicians. Another example is the book "Triangular norms" of Klement, Mesiar and Pap, dealing with the omnipresent concept of a triangular norm, originating from the theory of probabilistic metric spaces, but elegantly further explored by this trio of fine 'fuzzy' mathematicians.

The purpose of this lecture is not to bore the listeners with a historic account of the contributions that fuzzy set theorists did make to the development of mathematics, but to offer the audience a digestible selection of mathematical appetizers illustrating what fuzzy set theory has to offer. One thing is clear: it takes a multi-skilled mathematician to succeed. Without the purpose of being pretentious, the author will discuss a number of developments he was privileged to be involved in. Possible topics include uninorms (ranging from semi-group aspects to applications in expert systems), fuzzy preference structures (ranging from functional equations to applications in decision support systems) and similarity measures (ranging from inequalities in quantum-mechanics to applications in numerical taxonomy).