

Route to spatiotemporal complexity of two-dimensional patterns in a liquid crystal light valve with optical feedback

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ABSTRACT

Pattern formation and its spatiotemporal complexity have significantly been studied in different systems out of equilibrium, such as nonlinear optics, chemistry, ecology, and biology. However, the complex behavior of two-dimensional patterns is not yet fully understood. We use a Liquid Crystal Light Valve (LCLV) with Optical Feedback to characterize the transition, or route, to spatiotemporal complexity of the patterns formed in this system. A universal model of pattern formation, derived from the LCLV simultaneously close to the nascent of bistability and a spatial instability, Lifshitz point, is numerically integrated to obtain the theoretical route to spatiotemporal complexity. The studied complexity of the experimental and theoretical patterns is characterized via the computation of the Largest Lyapunov Exponent and Spatial Permutation Entropy.

Keywords: Pattern formation, Spatiotemporal complexity, Nonlinear Optics, Liquid Crystal Light Valve

1. INTRODUCTION

Periodic spatial structures, usually called *patterns*, can be found in various macroscopic systems under a permanent injection and dissipation of energy, as a result of the self-organization of its constituents.¹⁻⁴ Patterns are formed through a spatial spontaneous-breaking symmetry instability² from a homogeneous steady state, and exhibit an intrinsic wavelength. These structures can be found in various nonlinear out-of-equilibrium systems,⁵ including liquid crystals.⁶ In some cases, patterns present complex spatial distributions.^{2-5,7-9} The mechanism of pattern formation is well understood.¹⁻⁵ When an injection energy parameter is further increased, the initially stationary patterns acquire complex spatiotemporal dynamics. This has been observed in various fields, including electroconvection¹⁰ and nonlinear optics.¹¹ The route to spatiotemporal complexity has been studied theoretically in one-dimension,¹² but in two dimensions is still unknown.

To understand how the two-dimensional patterns acquire spatiotemporal complex dynamics, a characterization of this complexity is needed. Here, we use an experimental system that exhibits patterns, the Liquid Crystal Light Valve (LCLV) with optical feedback¹³⁻¹⁸ (see Fig. 1) to characterize the route to complexity. To understand the temporal aspect of the complexity, we computed the Largest Lyapunov Exponent of the solutions,¹⁹ and, to understand the spatial complexity of the solutions, we computed the Spatial Permutation Entropy^{20,21} of the intensity field. The initial homogeneous state exhibits non-chaotic dynamics with a Largest Lyapunov Exponent near 0, and high spatial permutation entropy due to the dominance of noise in the observed dynamics. Once the temporal dynamics appear, the solution becomes chaotic. The spatial permutation entropy is increased due to the chaotic dynamics, that produces irregular spatial distributions. To contrast the experimental characterization, we numerically integrated a universal pattern forming model, the Non-Variational Turing-Swift-Hohenberg equation, that can be derived from the phenomenological model of the LCLV close to a Lifshitz point.¹³ The numerical simulations of the Non-Variational Turing-Swift-Hohenberg are in great agreement with the experimental observations. This characterization is a step towards the understanding of the mechanism that leads to spatiotemporal complexity in two-dimensional pattern forming systems.

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2. METHODS

2.1 Experimental setup

To characterize the experimental route to complexity, we use a simple physical system that exhibits pattern formation with nonvariational behaviors, the Liquid Crystal Light Valve with Optical feedback. Figure 1(a) shows a schematic representation of the LCLV with Optical Feedback setup. Figure 1(b) shows a schematic representation of the LCLV. The Liquid Crystal Light Valve is composed by a nematic liquid crystal with negative dielectric anisotropy constant placed between two glass layers with planar anchoring. Transparent Indium Tin Oxide (ITO) electrodes and a photoconductive layer are deposited on the glasses to maintain the liquid crystal to a driven oscillatory voltage. A dielectric Bragg mirror is placed at the back of the liquid crystal cell. A detailed study of liquid crystal light valve with optical feedback and pattern formation is presented in the review article.¹⁴

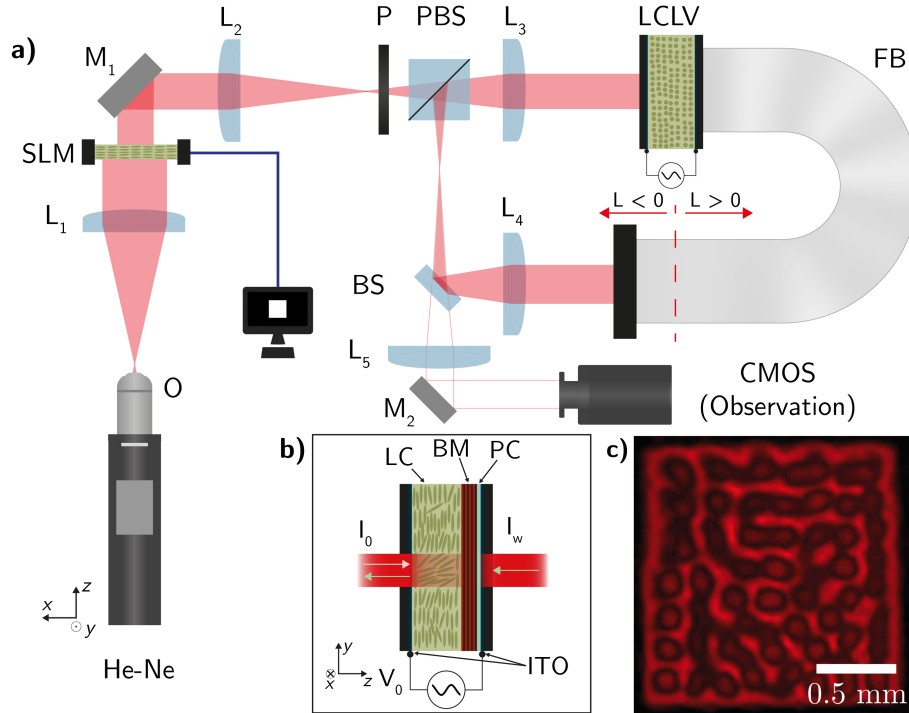


Figure 1. Liquid Crystal Light Valve (LCLV) with Optical Feedback. a) Schematic representation of the experimental setup. He-Ne accounts for a Helium-Neon laser light source. O is an optical objective, L_i stands for Lens ($i=\{1,2,3,4\}$), SLM is a spatial light modulator, P is a $\lambda/2$ polarizer, PBS corresponds to a polarizer beam splitter, BS is a beam splitter, FB is a high resolution optical fiber bundle, M stands for mirror, and CMOS accounts for a complementary metal-oxide-semiconductor camera. b) Schematic representation of the LCLV. LC accounts for the liquid crystal layer, BM is a Bragg mirror, PC is a photoconductive layer, ITO are transparent indium tin oxide electrodes, V_0 is the driving voltage, I_0 is the laser intensity and I_w is the writing intensity after passing through the LCLV. c) Snapshot of a recorded solution.

We have studied the solution exhibited by the system for different diffraction lengths $0 < -L < 10$ cm. When $-L \ll 1$ cm, the system presents a homogeneous solution. Then, by increasing the diffraction length, a spatial instability occur, and the solution changes to a static pattern. When further increasing $-L$, and since the spatial dependence of I_w is nonlocal, the dynamics are nonvariational. This causes the pattern to acquire permanent spatiotemporally complex dynamics.

A universal pattern forming model, the Non-Variational Turing-Swift-Hohenberg (NVTSH) equation, that can be derived from the phenomenological model of the LCLV close to a Lifshitz point.¹³ This model reads

$$\partial_t u = \eta + \mu u - u^3 + \nu \nabla^2 u - \nabla^4 u + \kappa u \nabla^2 u + c(\nabla u)^2, \quad (1)$$

where $u(\vec{r}, t)$ accounts for the biomass around a critical value b_c that exhibits a nascent of bistability.

The NVTSH Eq. (1) was also integrated to further understand this route to complexity. The parameters used were $\eta = 0.09$, $\mu = 0.4$, $\nu = -0.1$, $c = 15$ and different nonlinear diffusion parameters $2.5 < -b < 3.34$. A small multiplicative white noise was also added. For the lower bound of $-b$, the observed solution was the homogeneous. In a similar manner to the experimental observations, when increasing $-b$ a spatial instability occur, and the solution changes to a static pattern. Finally, for the upper bound, one can observe a pattern with permanent and complex spatiotemporal dynamics.

The recorded videos and the numerical simulations are analyzed in the next subsections in terms of different complexity parameters. Figure 2 shows snapshots of characteristic solutions of the system.

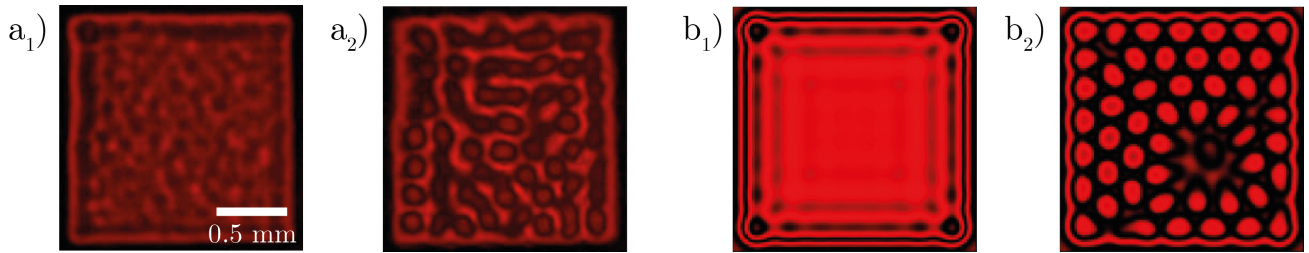


Figure 2. Characteristic solutions of the liquid crystal light valve with optical feedback. a) Snapshots of the recorded experimental data. $-L = a_1$) 1 cm, a_2) 6 cm respectively. b) Snapshots of the integrated numerical data with $-b = b_1 = 2.50$, and $-b = b_2 = 3.14$, respectively.

2.2 Temporal complexity

To characterize the dynamical aspect of the patterns, we computed the Largest Lyapunov Exponent. This gives information about the exponential sensitivity to initial conditions of the system. Because of the impossibility to prepare the system with infinitesimally separated initial conditions, to compute the experimental Largest Lyapunov Exponent, we have employed the strategy propose in Ref.¹⁸ Namely, two similar frames are searched along the recorded evolution.

Once a set of similar frame pairs $\{(t_1, t_2)\}$ are found, the Global Difference is tracked in time, and the Largest Lyapunov Exponent is defined by¹⁸

$$\lambda_1 = \lim_{t \rightarrow \infty} \lim_{\delta(t_1, t_2) \rightarrow 0} \frac{1}{t} \ln \left[\frac{\delta(t_1 + t, t_2 + t)}{\delta(t_1, t_2)} \right], \quad (2)$$

where $\delta(t_1, t_2) = \|u(x, y, t_1) - u(x, y, t_2)\|$ is a quantity that accounts for the global difference of the two states of the intensity field $u(x, y, t)$, where the symbol $\|f\| \equiv \iint f(x, y) dx dy / \Omega$ and Ω is the spatial domain.

To compute the Largest Lyapunov Exponent of the numerical simulations, we took the evolved solutions in the qualitative equilibrium, and added random noise of the order of 10^{-3} to the final state. We then evolved the solutions with and without the noise for 3500000 time steps. Finally, we computed the LLE as Eq. (2) indicates.

2.3 Spatial complexity

The *permutation entropy* (PE) is a complexity parameter characterized by its simplicity, extremely fast calculation and noise robustness.²⁰ To compute the permutation entropy, symbols of a certain length n are defined as the permutation of the relative values in n subsequent points in each frame. A frequentist probability of each symbol is defined. The permutation entropy is then defined as $H_{SP} = -\sum_m p(m) \ln[p(m)]$,²⁰ where $p(m)$ is the probability for a symbol m of length i . This can give information on the spatial distribution of the solution; a low (high) spatial permutation entropy (SPE) indicates that the system is ordered (disordered) in space. Note that the permutation entropy has an upper bound, given by $H_{sp} = \ln[n!]$. This bound indicates that the spatial distribution is fully random.

Because of the two-dimensional nature of the observed phenomena, the symbols can be defined in different shapes. The characteristic distance between the points was the pattern wavelength, and because of the square boundary conditions of the system, the chosen symbol shape was square. Finally, the Spatial Permutation Entropy is normalized and then averaged for every frame.

3. RESULTS

3.1 Temporal complexity

Figure 3a₁ shows the experimental LLE. For small diffraction length, the system has a LLE close to 0. When increasing this parameter, the system becomes chaotic. Figure 3a₂ shows the numerical LLE for varying nonlinear diffusion parameter. For small nonlinear diffusion, the system also has a LLE close to 0. When increasing this parameter, the system becomes intermittently chaotic.

3.2 Spatial complexity

Figure 3b₁ shows the SPE for the experimental data. Initially, the SPE is high due to the system spatial variations being mainly caused by noise. when the pattern is formed, the SPE is lowered. Finally, as the pattern becomes dynamic, the SPE is again increased due to the spatial irregularities introduced by the dynamics. Figure 3b₂ exhibits the SPE for the numerical data. Initially, the SPE is low, probably due to the presence of a pattern precursor. As the nonlinear diffusion κ is increased, the emergence of pattern defects increase the SPE. Finally, in a similar manner as the experimental data due to the dynamics of the pattern, the SPE reaches its maximum.

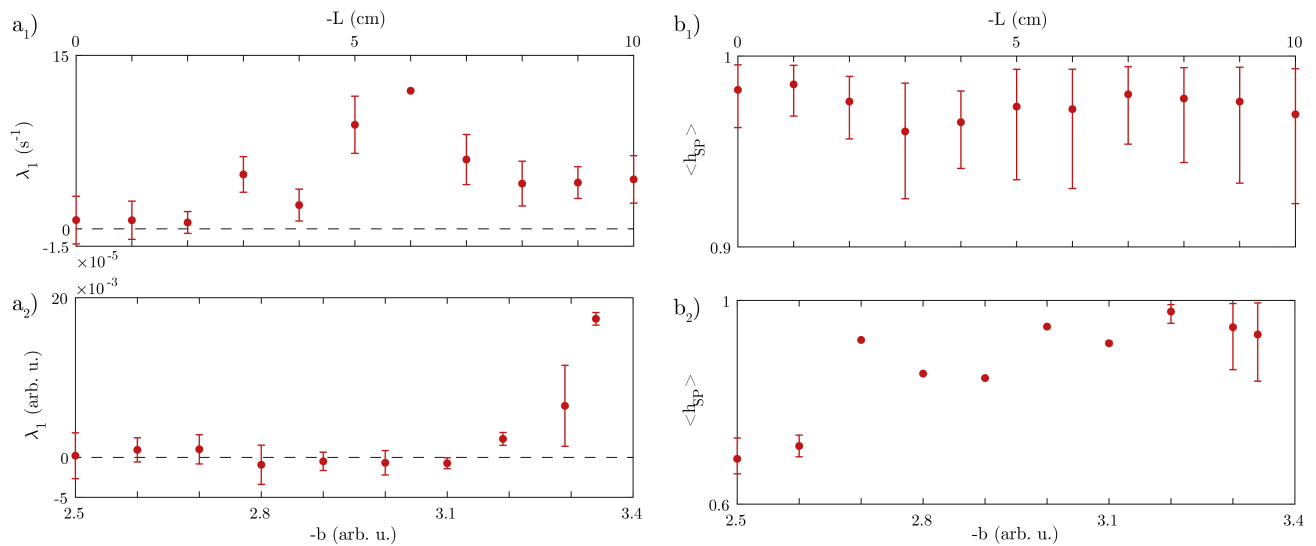


Figure 3. Complexity parameters. a) Largest Lyapunov Exponent λ_1 for varying diffraction length L and nonlinear diffusion b . a₁) Experimental data. a₂) Numerical data. The points correspond to the spatial average and the error bars account for the standard deviation of the data. b) Spatial Permutation Entropy $\langle h_{SP} \rangle$ for varying diffraction length L and nonlinear diffusion b . b₁) Experimental data. b₂) Numerical data. The points correspond to the spatial average, and the error bars account for the minimum and maximum value obtained.

4. CONCLUSIONS

In conclusion, when the system becomes non-variational, permanent complex dynamics appear. The emergence of the spatiotemporal complexity is characterized by chaotic dynamics associated with the increase of the Largest Lyapunov Exponent. Furthermore, this chaotic dynamics leads to irregular spatial distributions, characterized by a high spatial permutation entropy. These features of the complexity in two-dimensional optical patterns were also obtained by integrating a universal model of pattern formation derived from the phenomenological model of the experimental system. This indicates that these findings are intrinsic of two-dimensional patterns, and contribute to a deeper understanding of the mechanisms driving spatiotemporal complexity in two-dimensional patterns.

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