Non-Stochastic Run and Tumble of Particle-Like Structures

Eduardo A. Droguett-Mora[1], Marcel G. Clerc[1]
[1]Departamento de Física, FCFM, Universidad de Chile, Casilla 487-3, Santiago, Chile

Running and tumbling is a motion widely observed in the dynamics of bacteria and self-propelled particles, characterized by a ballistic behavior over short timescales, transitioning to a diffusive regime at long times. This is attributed to forces with stochastic fluctuations. Here, we investigate the dynamics of particle-like solutions in a two-dimensional dissipative bistable system with a spatiotemporal forcing, the driven Turing-Swift-Hohenberg model. Unexpectedly, statistical analysis shows that despite the absence of stochastic fluctuations, these particles exhibit a running and tumbling motion. To reveal the underlying mechanism, a phenomenological one-dimensional model is proposed, the driven ϕ^4-model, which presents the same motion for kinks. A Newton-type description of these particle-like solutions highlights that the emergence of chaos in the system is closely tied to the observed behavior. These results introduce a new theoretical framework for self-propelled motion, suggesting that inertial localized structures can be interpreted as deterministic run-and-tumble particles, thereby bridging the gap between biological behavior and physical systems.

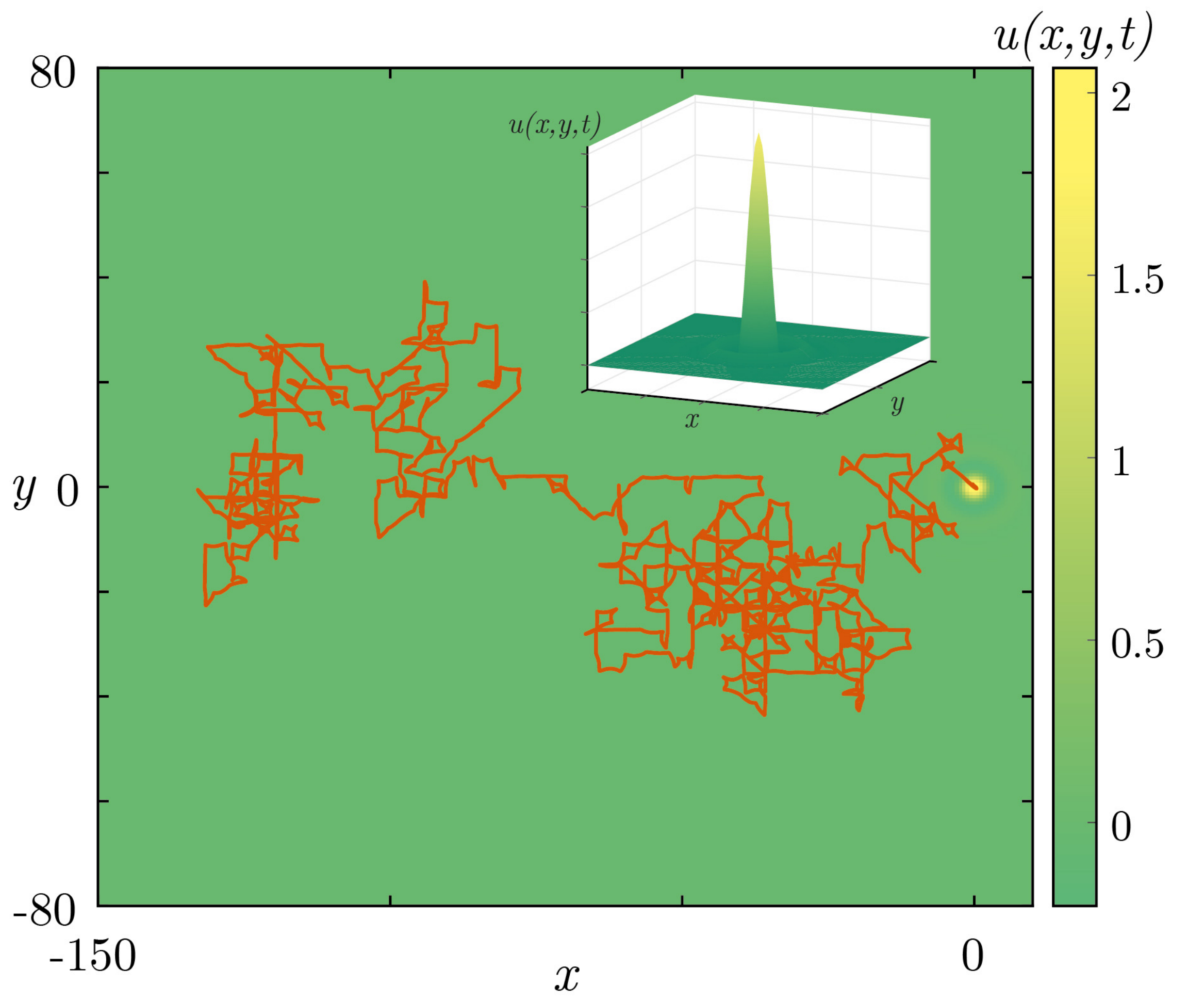

Figure 1: Deterministic run and tumble of particle-like solution in model Eq. (1), for $\varepsilon = -1.5$, $\beta = 2.2$, $\nu = 2$, $\mu = 0.185$, $\gamma = 0.1$, $\omega = 0.1$, and $k = 0.75$. Colormap of the scalar field $u(x,y,t)$. The orange line represents the trajectory of the localized structure. Inset: 3D profile of the localized structure.

Introduction

A widely studied model that presents localized structures is the Turing-Swift-Hohenberg equation [1]. Let us add inertia to this system, and a spatiotemporal forcing that corresponds to a stationary wave, obtaining the following model (driven Turing-Swift-Hohenberg equation):

$$\partial_{tt}u = \varepsilon u + \beta u^2 - u^3 - \nu\nabla^2 u - \nabla^4 u - \mu\partial_t u + \gamma\cos[\omega t]\sin[kx]\sin[ky]u, \quad (1)$$

where $u(x,y,t)$ is a scalar order parameter that may account for a bistable physical quantity, such as chemical concentrations, light intensity, population of a certain species, average molecular orientations, among others.

Statistical analysis

The Mean-Squared Displacement (MSD) of the trajectories follows a power law $\sim \tau^2$ (indicating a ballistic behavior) for short times that transitions to a power law $\sim \tau$ (indicating a diffusive behavior) for long times, signature of the running and tumbling motion [2].

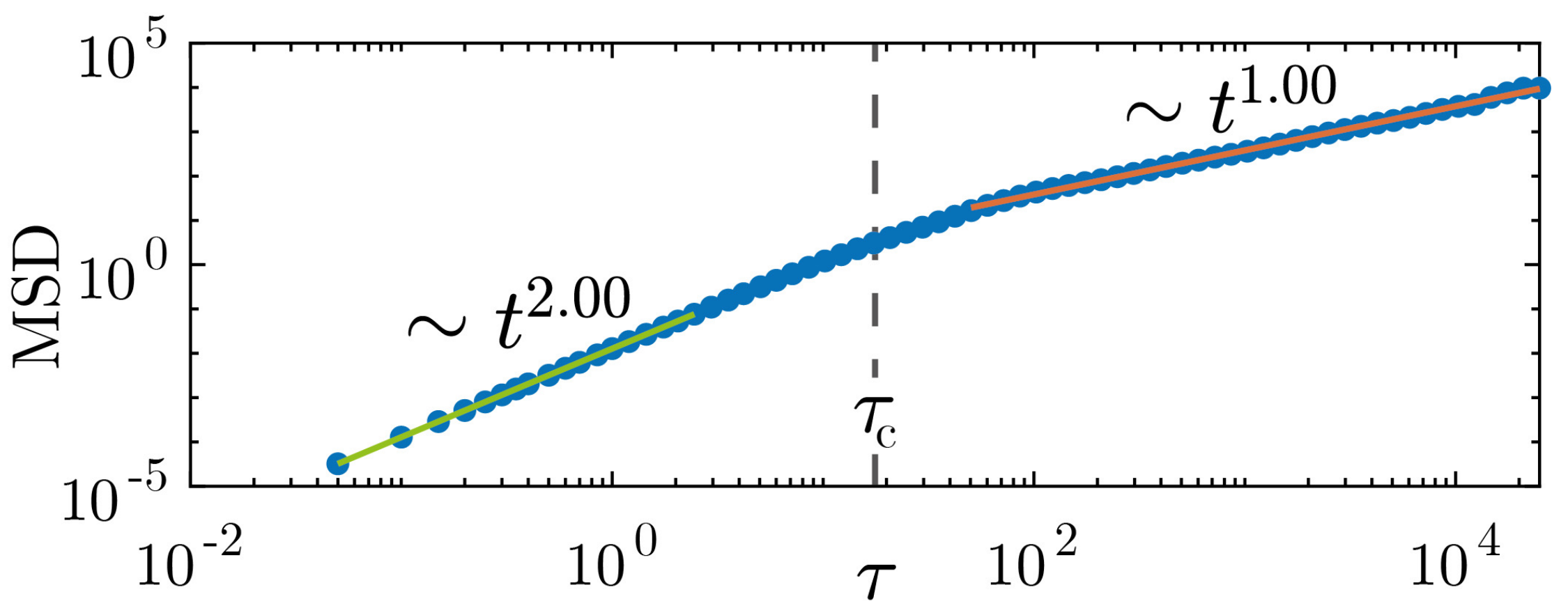

Figure 2: Mean-squared displacement of the localized structures using model Eq. (1), as a function of the time lag τ. The blue dots account for the computed mean squared displacement. The green (orange) line accounts for the power law fit for short (long) times. The vertical dashed line accounts for the crossover time τ_c.

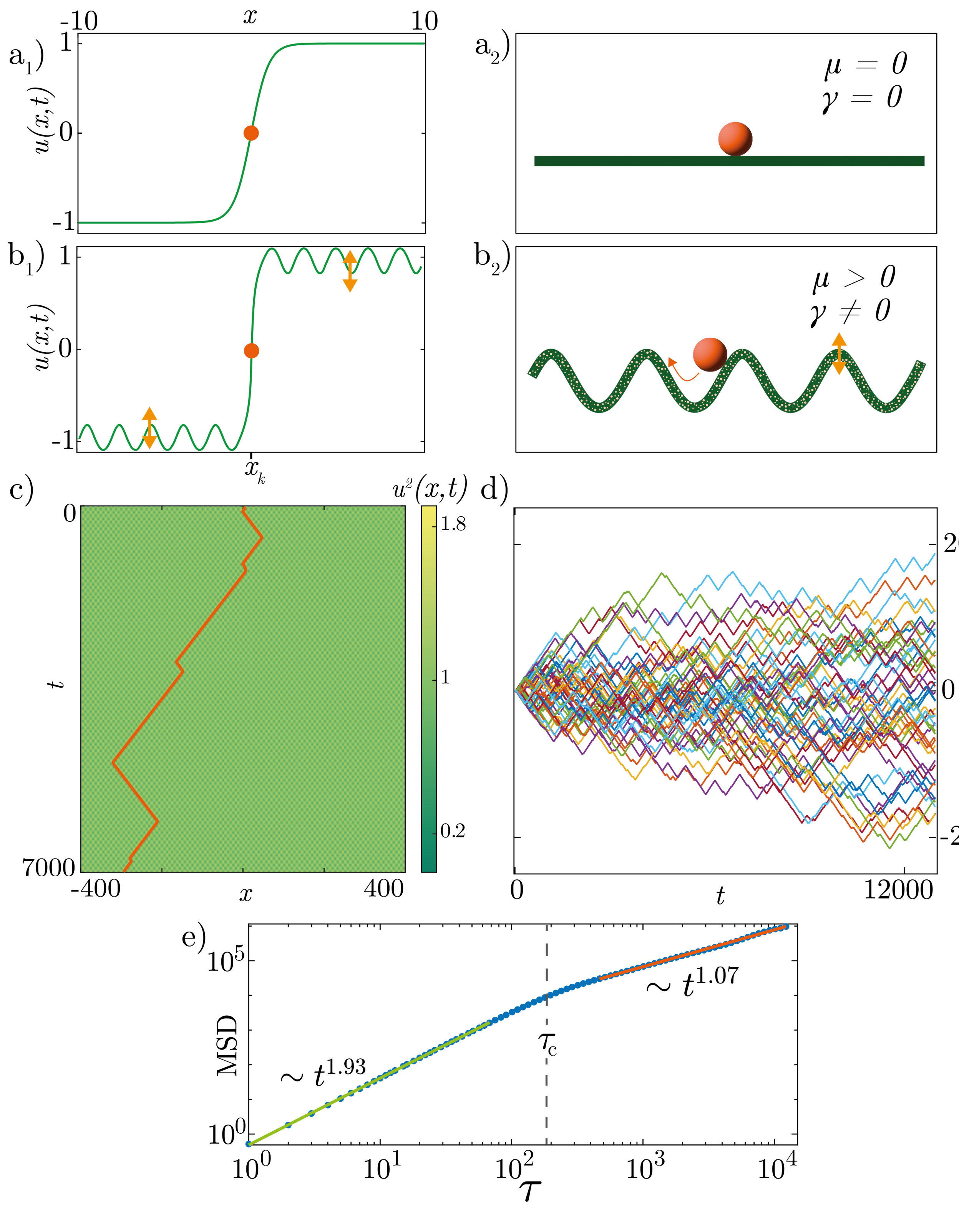

Figure 3: Deterministic run and tumble in the driven ϕ^4-model with, $\omega = 0.34$ and $k = 0.5$. Solutions with $(\gamma, \mu) =$ a) (0,0) and b) (0.3,0.2). 1) Shows an instant of the system. Green line accounts for the solution $u(x,t)$. Orange dot accounts for the kink position. Yellow arrows account for the forcing oscillations. 2) Schematic representation of the core as a particle (orange circles) in a surface (green lines). c) Spatiotemporal diagram of the solution $u(x,t)$ squared. Orange line represents the particle trajectory. d) Ensemble of trajectories of the particle-like solutions. e) Mean squared displacement of the particle-like solutions. Blue dots account for the computed mean squared displacement. Green (orange) line accounts for the power law fit for short (long) times. Vertical dashed line accounts for the crossover time.

One-dimensional phenomenological model

The minimal system that presents particle-like solutions is the ϕ^4-model [3]. Let us add dissipation and a similar spatiotemporal forcing studied in the previous section to this system, obtaining the driven ϕ^4-model, that reads as

$$\partial_{tt}\phi = \phi - \phi^3 + \partial_{xx}\phi - \mu\partial_t\phi + \gamma\cos[\omega t]\sin[kx]. \quad (2)$$

-The running motion emerges due to the inertial nature of the particle.

-The tumbling and redirection is caused by the combination of the damping term and the spatiotemporal forcing.

-The dispersion of particles arise from the chaotic nature of the system.

Conclusions

When given inertia, damping, and an adequate spatiotemporal forcing, deterministic particle-like solutions exhibit running and tumbling. Particle dispersion caused by the chaotic nature of the system. These findings pave the way for further studies on deterministic descriptions of other non-Brownian movements.

---

[1] Kozyreff, G., & Tlidi, M. (2007). Nonvariational real Swift-Hohenberg equation for biological, chemical, and optical systems. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *17*(3).
[2] Wu, X. L., & Libchaber, A. (2000). Particle diffusion in a quasi-two-dimensional bacterial bath. *Physical review letters*, *84*(13), 3017.
[3] Manton, N., & Sutcliffe, P. (2004). *Topological solitons*. Cambridge University Press.

The authors acknowledge financial support from ANID-Millennium Science Initiative Program ICN17_012 (MIRO).