OPTIMAL CONTROL APPLICATIONS AND METHODS
Optim. Control Appl. Meth. 2005; **26**:169–198
Published online 1 April 2005 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/oca.757

Design of a supervisory predictive controller and its application to thermal power plants

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SUMMARY

The optimization at supervisory level of thermal power plant controls is investigated. The design of a predictive supervisory controller that optimizes an objective function in order to determine the set-points for a given regulatory level is described. The objective function includes both an economic criterion and a regulatory criterion. The proposed supervisory controller is applied to the gas turbine of a thermal power plant and it is compared with the control strategy for constant optimum set-points. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: supervisory control; predictive control; economic optimization; gas turbine

1. INTRODUCTION

In an increasingly competitive electrical supply market, the power sector faces new challenges, such as improving productivity and reducing costs, whilst taking into account the process operation. As energy demand increases, especially in big cities, where the environmental concerns are very important and resources to produce energy are limited, the efficiency of operation of power plants becomes of paramount importance. In the following a method is proposed for improving the power plant efficiency, through the introduction of a supervisory controller, without modifying the lower regulatory level. The supervisory controller provides the

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Contract/grant sponsor: Universidad de Chile; contract/grant number: DI I2-03/14-2

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Received 14 May 2003
Revised 4 January 2005
Accepted 16 February 2005

regulatory level set-points, based on the objective function optimization. This objective function may represent the plant profit, operational costs, process energy consumption and other criteria [1], by including also regulatory objectives.

There are contributions that deal with set-point optimization based on steady-state models. For example, Ellis [2] presents the economic optimization of a fluidized catalytic cracking unit.

In addition, there are some papers that deal with dynamic models. For example, Becerra [3] proposes a multi-objective predictive formulation that includes both economic and regulatory objectives based on the use of state space models. Also, de Prada [4] proposes a predictive control strategy based on the optimization of an economic index. This strategy was applied to a chemical reactor.

Katebi [5] describes a decentralized control strategy, based on the optimization of a particular objective function, such as a GPC cost-index. In this work, a state space representation was used. The objective function had only regulatory objectives. The control strategy was applied to a thermal power plant simulator.

Bemporad [6] and Angeli [7] proposed a reference governor at the supervisory level. The objective function was given by the minimization of the reference trajectory error. The main goal was to satisfy certain constraints. The algorithms were developed using a state-space representation. A different approach for a reference governor with the same objective was proposed by Gilbert [8]. In this case, the reference governor was given by a non-linear pre-filter.

As the first stage of the supervisory optimal controller design, a supervisory level is considered that will provide the regulatory level set-points, based on the supervisory objective cost function optimization. This problem can be solved, either by numerical algorithms, where non-linear models are used, or by approximating the plant by a linear state-space representation. Predictive control theory for linear systems may then be applied [6,7]. This paper considers a particular case, where an explicit solution is derived, for a quadratic objective function and a linear process model. This explicit solution provides a simple implementation of the proposed supervisory controller.

The gas turbine system of a thermal power plant is first described, covering the gas turbine simulator and the regulatory level controls. Then, the general problem at the supervisory level that includes a quadratic objective function, the linear model of the process, and the model for the regulatory level is introduced. Next, the problem solution is formulated, including the derivation of the predictors of the process model and the regulatory level model. The explicit solution for minimizing the quadratic objective function is also derived. After that, the proposed supervisory controller is assessed using a gas turbine simulator and it is compared with a control strategy, where the set-points are constant and obtained from a static optimization. Finally, the conclusions are summarized.

2. GAS TURBINE OF A COMBINED CYCLE THERMAL POWER PLANT

2.1. Process description

Combined cycle power plants have high efficiencies and they require comparatively lower investment costs than other technologies. These plants consist of a gas turbine, a boiler and a steam turbine to generate electricity [9]. As shown in Figure 1, turbines are combined into just one cycle where the energy is transferred from one turbine to the other. The exhaust gases from

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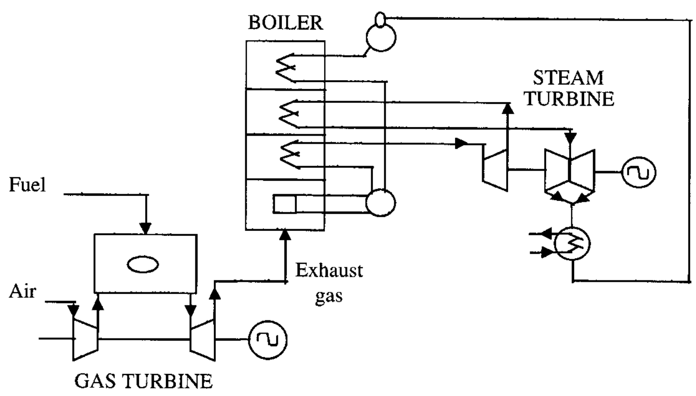

Figure 1. Combined cycle power plant.

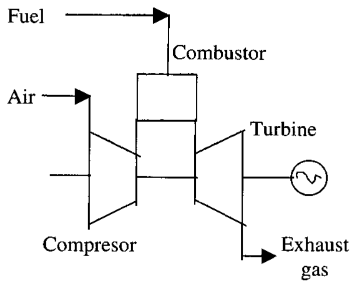

Figure 2. Gas turbine.

the gas turbine are used to provide the necessary heat for the boiler steam production. Finally, this steam is fed to the steam turbine.

Figure 2 shows the gas turbine configuration [9]. The gas turbine contains a combustion chamber, compressor and the turbine to convert the fuel energy into mechanical energy. Air at atmospheric pressure enters the compressor, where it is compressed and mixed with the fuel gas in the combustion chamber. The compressed products of combustion are then expanded through the turbine and finally exits to the atmosphere as escape gases. Part of the work generated in the turbine is used to drive the compressor, that is usually mounted on the same axis as the turbine.

To develop the gas turbine model, the following assumptions are made [9]:

- air and combustion products are treated as perfect gases;
- specific heats are assumed constant for combustion products, air and injected steam;
- an adiabatic uniform polytropic process describes flows through compressor and turbine;
- the energy storage and transport delay in the compressor, turbine and combustion chamber are relatively small, thus steady state equations can be applied;
- the kinetic energy of the inlet gas flows into the compressor and into the turbine are treated as negligible;

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- air mass flow through the compressor is controllable via inlet guide vanes;
- NO_x and CO formation is modelled as a function of the steam/fuel ratio.

2.2. *Gas turbine simulator*

The phenomenological simulator was developed for the gas turbine of a combined cycle power plant (50 MW). The gas turbine simulator is based on phenomenological equations and their parameters are obtained, and adapted, from Ordys' work [9]. The simulator consists of differential equations and non-linear algebraic equations that represent the non linear dynamic model of a real gas turbine.

To test different control strategies, the simulator was programmed using the S-function in Matlab-SIMULINK environment. In Figure 3, the gas turbine simulator based on the configuration shown in Figure 2, and programmed in Matlab-SIMULINK, is presented.

2.3. *Regulatory level*

In Figure 4, a gas turbine control strategy based on PI controllers is shown. The controlled variables are exhaust gas temperature ($T_{T\text{out}}$), the power of the gas turbine (P_g) and the NO_x concentration in the exhaust gases ($g_{c\text{NO}x}$). The manipulated variables are the air flow to the compressor (w_a), the fuel flow (F_d) and the flow of the steam injected into the combustion chamber (w_{is}). The regulatory control level includes a switching controller, that uses the minimum signal (Fcn) of two PI controllers in order to calculate the fuel flow control action. However, for the purpose of this paper, the PI that uses gas turbine power is assumed to be active all the time, therefore eliminating the need for switching.

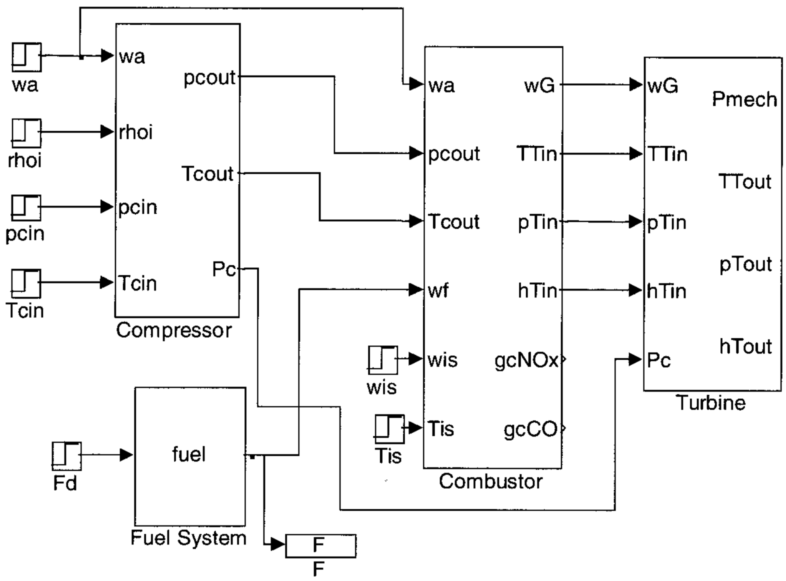

Figure 3. Gas turbine simulator.

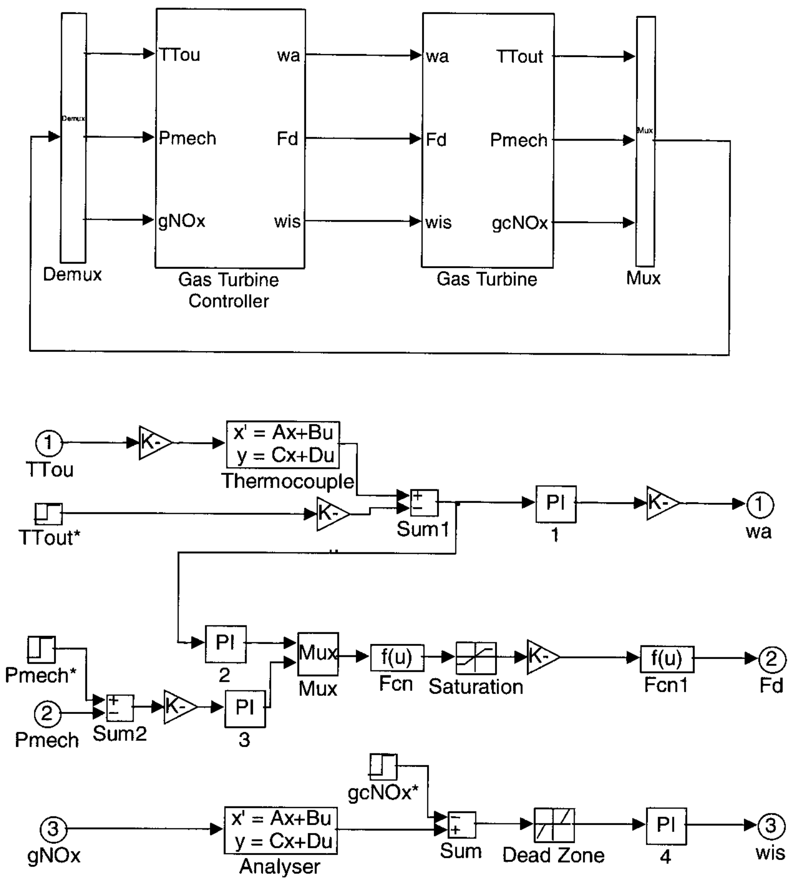

Figure 4. Control strategy for the gas turbine.

3. DESIGN OF THE SUPERVISORY PREDICTIVE CONTROLLER

A two level control diagram is shown in Figure 5. In this diagram, the regulatory level is given. The supervisory level will provide the regulatory level set-points (**r**) based on the optimization of the objective function (**J**), the trajectory of an external reference (**w**), the measurements of the controlled variables (**y**) and the measurements of the manipulated variables (**u**). The process outputs are influenced by unmeasurable disturbances (**e**).

The main elements used for the design of the proposed algorithm are now described. An explicit solution is then presented.

3.1. Elements of the algorithm

3.1.1. Process model. In order to design a supervisory controller, a controlled auto-regressive and moving-average (CARIMA) model [10] in which disturbances are non-stationary is used.

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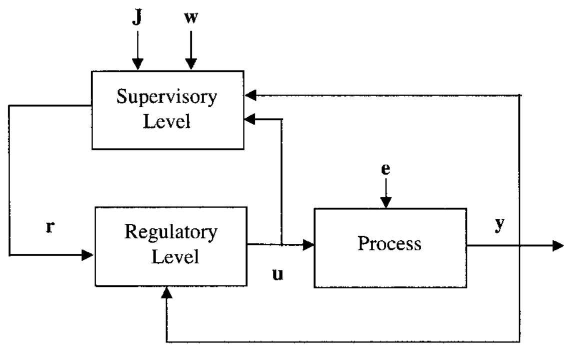

Figure 5. A two level control diagram.

A CARIMA SISO model is given by

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \frac{e(t)}{\Delta} \tag{1}$$

with $A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-na}, B(q^{-1}) = b_1 q^{-1} + \cdots + b_{nb} q^{-nb}$, $\Delta = 1 - q^{-1}$.

Also, let $y(t)$ denote the controlled variable and $u(t)$ the manipulated variable, respectively; $e(t)$ denotes a zero mean white noise signal and q^{-1} is the backward shift operator ($q^{-1}x(t) = x(t-1)$).

3.1.2. Regulatory level model. At the regulatory level, fixed controllers are considered and these may be represented by the following expression:

$$A_c(q^{-1})u(t) = B_{cr}(q^{-1})r(t) + B_{cy}(q^{-1})y(t) \tag{2}$$

where $r(t)$ denotes the set-point variable and $A_c(q^{-1}) = 1 + ac_1 q^{-1} + \cdots + ac_{nac} q^{-nac}$, $B_{cr}(q^{-1}) = b_{r0} + b_{r1} q^{-1} + \cdots + b_{rnb} q^{-rnb}$, $B_{cy}(q^{-1}) = b_{y0} + b_{y1} q^{-1} + \cdots + b_{ynb} q^{-ynb}$.

3.1.3. Supervisory level objective cost function. The objective function proposed at supervisory level includes an N step prediction horizon. That is

$$\begin{aligned} J = & \sum_{j=1}^{N} \Psi_y^j \hat{\mathbf{y}}^2(t+j) + \sum_{i=1}^{N} \Psi_u^i \mathbf{u}^2(t+i-1) + \sum_{i=1}^{N} \Psi_{\Delta u}^i \Delta\mathbf{u}^2(t+i-1) \\ & + \sum_{j=1}^{N} \sum_{i=1}^{N} \Psi_{yu}^{ji} \hat{\mathbf{y}}(t+j)\mathbf{u}(t+i-1) + \sum_{j=1}^{N} \xi_y^j \hat{\mathbf{y}}(t+j) + \sum_{i=1}^{N} \xi_u^i \mathbf{u}(t+i-1) \\ & + \sum_{i=1}^{N} \xi_{\Delta u}^i \Delta\mathbf{u}(t+i-1) \end{aligned} \tag{3}$$

where $\hat{\mathbf{y}}(t+j)$ are the j-step ahead prediction of the controlled variables on data up to time t, $\mathbf{u}(t+i-1)$ are the manipulated variables, $\Delta\mathbf{u}(t+i-1)$ is the manipulated variables increment;

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Ψ and ξ are weighting sequences; N is the prediction horizon. An external reference trajectory ($\mathbf{w}$) can also be included in the objective cost function.

As previously mentioned, this objective function may represent different optimization goals at the supervisory level. These may include the operational costs or the profit of a plant, or the process energy consumption. This objective function can also include a regulatory criterion.

3.2. *Solution for the supervisory controller*

An explicit solution for the supervisory controller is derived below. The optimization of the objective function, which is defined in Equation (3), is considered. The optimum solution is obtained using the theory of predictive control [10]. Also, in order to derive an analytic solution, this approach does not include inequality constraints. However, the inequality constraints could be included, and this would lead to numerically solving the optimization problem, using, for example, quadratic programming.

3.2.1. Prediction. As Equation (3) reveals, the objective function depends on the predictions of the controlled variables, manipulated variables and increment of manipulated variables. To obtain an explicit solution of the optimization problem, these variables are expressed as a function of the optimization variables (the set-points variables). The predictions of these variables are derived in Appendix A. The predictions of controlled variables, as a function of the manipulated variable increments, are obtained directly by using generalized predictive control theory [10] and solving the appropriate diophantine equations for the process model

$$\hat{\mathbf{y}} = \mathbf{G}_{\Delta u}\Delta\mathbf{u} + \mathbf{f}_{\Delta u} \tag{4}$$

where $\hat{\mathbf{y}} = [\hat{y}(t+1), \ldots, \hat{y}(t+N)]^{\mathrm{T}}$ and $\Delta\mathbf{u} = [\Delta u(t), \Delta u(t+1), \ldots, \Delta u(t+N-1)]^{\mathrm{T}}$.

The term $\mathbf{G}_{\Delta u}\Delta\mathbf{u}$ represents the unknown future signals (forced response) and $\mathbf{f}_{\Delta u}$ represent the known past signals (free response) (see Appendix A). The predictions of controlled variables, as a function of the manipulated variables, are also needed. Those are derived by using Equation (4) (see Appendix A)

$$\hat{\mathbf{y}} = \mathbf{G}_u\mathbf{u} + \mathbf{f}_u \tag{5}$$

with $\mathbf{u} = [u(t), u(t+1), \ldots, u(t+N-1)]^{\mathrm{T}}$. The matrices $\mathbf{G}_u$ and $\mathbf{f}_u$ are described in Appendix A. Also, $\mathbf{G}_u\mathbf{u}$ represents the unknown future signals and $\mathbf{f}_u$ represent the known past signals. Similarly, the predictions of manipulated variables are derived (see Appendix A) as

$$\mathbf{u} = \mathbf{G}_r\mathbf{r} + \mathbf{G}_y\hat{\mathbf{y}} + \mathbf{f}_{ryu} \tag{6}$$

with $\mathbf{r} = [r(t), \ldots, r(t+N)]^{\mathrm{T}}$.

The $\mathbf{G}_r$, $\mathbf{G}_y$ and $\mathbf{f}_{ryu}$ are defined by the solution of the diophantine equation of the regulatory level model (see Appendix A). $\mathbf{G}_r\mathbf{r} + \mathbf{G}_y\hat{\mathbf{y}}$ represents the unknown future signals and $\mathbf{f}_{ryu}$ represent the known past signals.

The predictions of manipulated variable increments are obtained by using Equation (6) (see Appendix A)

$$\Delta\mathbf{u} = \mathbf{G}_{\Delta r}\mathbf{r} + \mathbf{G}_{\Delta y}\hat{\mathbf{y}} + \mathbf{f}_{\Delta ryu} \tag{7}$$

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with $\mathbf{G}_{\Delta r}$, $\mathbf{G}_{\Delta y}$ and $\mathbf{f}_{\Delta ryu}$ defined in Appendix A. The signal $\mathbf{G}_{\Delta r}\mathbf{r} + \mathbf{G}_{\Delta y}\mathbf{y}$ represents the unknown future signals and $\mathbf{f}_{\Delta ryu}$ represent the known past signals.

Thus, the predictions of manipulated variables, as a function of the optimization variables (set-points), are obtained replacing $\hat{\mathbf{y}}$ given by Equation (5) in Equation (6)

$$\mathbf{u} = (\mathbf{I} - \mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{G}_r\mathbf{r} + \mathbf{f}_{ryu} + \mathbf{G}_y\mathbf{f}_u) \quad \text{or} \quad \mathbf{u} = (\mathbf{I} - \mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r\mathbf{r} + (\mathbf{I} - \mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu} + \mathbf{G}_y\mathbf{f}_u) \tag{8}$$

To obtain the predictions of manipulated variables increment, as a function of the optimization variables (set-points), the $\hat{\mathbf{y}}$ given by Equation (4) is replaced in Equation (7)

$$\Delta\mathbf{u} = (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{r} + (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu} + \mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}) \tag{9}$$

In Equation (5), the manipulated variables can be replaced by expression (6). Thus, the predictions of the controlled variables are given by

$$\hat{\mathbf{y}} = (\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}[\mathbf{G}_u\mathbf{G}_r\mathbf{r} + \mathbf{G}_u\mathbf{f}_{ryu} + \mathbf{f}_u]$$

or

$$\hat{\mathbf{y}} = (\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r\mathbf{r} + (\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}[\mathbf{G}_u\mathbf{f}_{ryu} + \mathbf{f}_u] \tag{10}$$

3.2.2. Optimization

In order to solve the optimization problem, the objective cost function defined in Equation (3) is expressed in matrix form

$$\mathbf{J} = \hat{\mathbf{y}}^{\mathrm{T}}\mathbf{\Psi}_y\hat{\mathbf{y}} + \mathbf{u}^{\mathrm{T}}\mathbf{\Psi}_u\mathbf{u} + \Delta\mathbf{u}^{\mathrm{T}}\mathbf{\Psi}_{\Delta u}\Delta\mathbf{u} + \hat{\mathbf{y}}^{\mathrm{T}}\mathbf{\Psi}_{yu}\mathbf{u} + \hat{\mathbf{y}}^{\mathrm{T}}\boldsymbol{\xi}_y + \mathbf{u}^{\mathrm{T}}\boldsymbol{\xi}_u + \Delta\mathbf{u}^{\mathrm{T}}\boldsymbol{\xi}_{\Delta u} \tag{11}$$

where the weighting matrices and vectors

$$\mathbf{\Psi}_y = \begin{bmatrix} \psi_y^1 & 0 & \cdots & 0 \\ 0 & \psi_y^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \psi_y^N \end{bmatrix}_{N\times N} \qquad \mathbf{\Psi}_{\Delta u} = \begin{bmatrix} \psi_{\Delta u}^1 & 0 & \cdots & 0 \\ 0 & \psi_{\Delta u}^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \psi_{\Delta u}^N \end{bmatrix}_{N\times N}$$

$$\mathbf{\Psi}_u = \begin{bmatrix} \psi_u^1 & 0 & \cdots & 0 \\ 0 & \psi_u^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \psi_u^N \end{bmatrix}_{N\times N} \qquad \mathbf{\Psi}_{yu} = \begin{bmatrix} \psi_{yu}^1 & 0 & \cdots & 0 \\ 0 & \psi_{yu}^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & \psi_{yu}^N \end{bmatrix}_{N\times N}$$

$$\boldsymbol{\xi}_y = [\xi_y^1 \ \xi_y^2 \ \cdots \ \xi_y^N]^{\mathrm{T}}_{1\times N}, \quad \boldsymbol{\xi}_u = [\xi_u^1 \ \xi_u^2 \ \cdots \ \xi_u^N]^{\mathrm{T}}_{1\times N} \quad \text{and} \quad \boldsymbol{\xi}_{\Delta u} = [\xi_{\Delta u}^1 \ \xi_{\Delta u}^2 \ \cdots \ \xi_{\Delta u}^N]^{\mathrm{T}}_{1\times N}$$

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Then, the predictions of manipulated variables, and their increments and controlled variables (Equations (8–10)) can be substituted into the objective function, defined by Equation (11)

$$\mathbf{J} = \tfrac{1}{2}\mathbf{r}^{\mathrm{T}}\mathbf{H}\mathbf{r} + \mathbf{r}^{\mathrm{T}}\mathbf{F}^{*} + \mathbf{D} \tag{12}$$

where the cost function terms

$$\begin{aligned}
\mathbf{H} = {} & 2((\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)^{\mathrm{T}}\psi_y((\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)\\
& + 2((\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r)^{\mathrm{T}}\psi_u((\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r)\\
& + 2((\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r})^{\mathrm{T}}\psi_{\Delta u}((\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r})\\
& + ((\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)^{\mathrm{T}}\psi_{yu}((\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r)\\
& + ((\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r)^{\mathrm{T}}\psi_{yu}((\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)
\end{aligned} \tag{13}$$

$$\begin{aligned}
\mathbf{F}^{*} = {} & 2[(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r]^{\mathrm{T}}\psi_y[(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)]\\
& + 2[(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r]^{\mathrm{T}}\psi_u[(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)]\\
& + 2[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{\mathrm{T}}\psi_{\Delta u}[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]\\
& + [(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r]^{\mathrm{T}}\psi_{yu}[(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)]\\
& + [(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r]^{\mathrm{T}}\psi_{yu}[(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)]\\
& + [(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r]^{\mathrm{T}}\xi_y + [(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}\mathbf{G}_r]^{\mathrm{T}}\xi_u + [(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{\mathrm{T}}\xi_{\Delta u}
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\mathbf{D} = {} & 2[\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u]^{\mathrm{T}}(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}\psi_y(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}[\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u]\\
& + 2[(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)]^{\mathrm{T}}\psi_u(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)\\
& + 2[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]^{\mathrm{T}}\psi_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})\\
& + [(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)]^{\mathrm{T}}\psi_{yu}[(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)]\\
& + [(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)]^{\mathrm{T}}\psi_{yu}[(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)]\\
& + [(\mathbf{I}-\mathbf{G}_u\mathbf{G}_y)^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)]^{\mathrm{T}}\xi_y + [(\mathbf{I}-\mathbf{G}_y\mathbf{G}_u)^{-1}(\mathbf{f}_{ryu}+\mathbf{G}_y\mathbf{f}_u)]^{\mathrm{T}}\xi_u\\
& + [(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]^{\mathrm{T}}\xi_{\Delta u}
\end{aligned} \tag{15}$$

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This expression may be minimized with respect to the optimization variables ($\mathbf{r}$) and results in

$$\mathbf{r}^* = \mathbf{H}^{-1}\mathbf{F} \tag{16}$$

where $\mathbf{r}^*$ represents the vector of the supervisory level optimum set-points: $\mathbf{r}^* = [r^*(t), \ldots, r^*(t+N)]^{\mathrm{T}}$ and $\mathbf{F} = -\mathbf{F}^*$. Notice that for Equation (16) to be valid, matrix $\mathbf{H}$ must be assumed non-singular. This matrix depends on the plant model, the low level controller model and on the weights in the supervisory level cost function.

As in the theory of generalized predictive control [10], the first element ($r^*(t)$) of vector $\mathbf{r}^*$ is applied to the process by using the receding horizon concept. Then, the optimum set-points at instant t, given by Equation (16), can be obtained as

$$A_s(q^{-1})r^*(t) = B_{sy}(q^{-1})y(t) + B_{su}(q^{-1})u(t) + C_s \tag{17}$$

In some cases, there may be cancellations of the terms forming (16). A special case is presented below where this cancellation occurs.

3.2.3. Stability analysis. To obtain the closed-loop relations, the optimal set-points at time instant t, $r^*(t)$ from Equation (17) are substituted for $r(t)$ in the regulatory level model (Equation (2)) and the following expression is obtained:

$$\begin{aligned}&(A_c(q^{-1})A_s(q^{-1}) - B_{cr}(q^{-1})B_{su}(q^{-1}))u(t)\\ &\quad = (B_{cr}(q^{-1})B_{sy}(q^{-1}) + B_{cy}(q^{-1})A_s(q^{-1}))y(t) + B_{cr}(q^{-1})C_s\end{aligned} \tag{18}$$

Next, Equation (18) is substituted into the process model (Equation (1))

$$\begin{aligned}&\Delta A(q^{-1})(A_c(q^{-1})A_s(q^{-1}) - B_{cr}(q^{-1})B_{su}(q^{-1}))y(t)\\ &\quad = \Delta B(q^{-1})(B_{cr}(q^{-1})B_{sy}(q^{-1}) + B_{cy}(q^{-1})A_s(q^{-1}))y(t)\\ &\qquad + (A_c(q^{-1})A_s(q^{-1}) - B_{cr}(q^{-1})B_{su}(q^{-1}))e(t) + B(q^{-1})B_{cr}(q^{-1})\Delta C_s\end{aligned} \tag{19}$$

Thus, the output $y(t)$ for closed-loop operation is given by:

$$\begin{aligned}&(A(q^{-1})(A_c(q^{-1})A_s(q^{-1}) - B_{cr}(q^{-1})B_{su}(q^{-1}))\\ &\quad - B(q^{-1})(B_{cr}(q^{-1})B_{sy}(q^{-1}) + B_{cy}(q^{-1})A_s(q^{-1})))y(t)\\ &\quad = (A_c(q^{-1})A_s(q^{-1}) - B_{cr}(q^{-1})B_{su}(q^{-1}))\frac{e(t)}{\Delta} + B(q^{-1})B_{cr}(q^{-1})C_s\end{aligned} \tag{20}$$

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Notice that C_s is a constant value, given by the following equation, which can be obtained from Equation (16)

$$C_s = -H^{-1}([(1 - G_uG_y)^{-1}G_uG_r]^{\mathrm{T}}\xi_y + [(1 - G_yG_u)^{-1}G_r]^{\mathrm{T}}\xi_u + [(1 - G_{\Delta y}G_{\Delta u})^{-1}G_{\Delta r}]^{\mathrm{T}}\xi_{\Delta u}) \quad (21)$$

and $e(t)$ is stochastic disturbance.

Therefore, the stability conditions are directly obtained from the pole location of the closed-loop output-error relation given by Equation (20).

3.2.4. Special case. The special case arises when the following generalized predictive control (GPC) objective function is considered at supervisory level. The cost-function may be written as

$$J = \sum_{j=1}^{N} (\hat{y}(t+j) - w)^2 + \lambda \sum_{i=1}^{N} \Delta u^2(t+i-1) \quad (22)$$

where $\psi_y^j = 1$, $\psi_{\Delta u}^i = \lambda$, $\xi_y^j = -2w$. In this case, from Equation (16), the optimum set-points are

$$\mathbf{r}^* = \mathbf{H}^{-1}\mathbf{F} \quad (23)$$

with

$$\mathbf{H} = ((\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)^{\mathrm{T}}((\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r) + ((\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r})^{\mathrm{T}}\boldsymbol{\lambda}((\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r})$$

and

$$\mathbf{F} = ((\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)^{\mathrm{T}}((\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}(-\mathbf{G}_u\mathbf{f}_{ryu} - \mathbf{f}_u))$$

$$+ ((\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r})^{\mathrm{T}}\boldsymbol{\lambda}((\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu} - \mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})) + ((\mathbf{I} - \mathbf{G}_u\mathbf{G}_y)^{-1}\mathbf{G}_u\mathbf{G}_r)^{\mathrm{T}}\mathbf{w}$$

Then, as shown in Appendix B, the control action at time instant t is given by

$$\Delta u(t) = \mathbf{g}(\mathbf{w} - \mathbf{f}_{\Delta u}) \quad (24)$$

where g is the first row of the matrix $(\mathbf{G}_{\Delta u}^{\mathrm{T}}\mathbf{G}_{\Delta u} + \boldsymbol{\lambda}\mathbf{I})^{-1}\mathbf{G}_{\Delta u}^{\mathrm{T}}$.

Thus, when a GPC objective function is considered at the supervisory level, the supervisory controller modifies the control action of the fixed regulatory controller. The result is that the system performs as if GPC optimal control action was generated for the regulatory level. This is, illustrated in Figure 6, where the GPC_1 controller at supervisory level and PI controller at regulatory level generate the same control action as GPC_2 controller at regulatory level. In both cases the same objective function is minimized.

It follows that the supervisory predictive controller can eliminate the need to optimally tune the low-level PID controller. However, certain assumptions are important in these considerations. For instance, it must be assumed that the process is controllable and the low level controller can provide a sufficient control signal, such that the process seen from the supervisory level is also controllable. This will form one of the conditions for non-singularity of the matrix **H**. Also, to avoid instability it must be assumed that in the formation of the total

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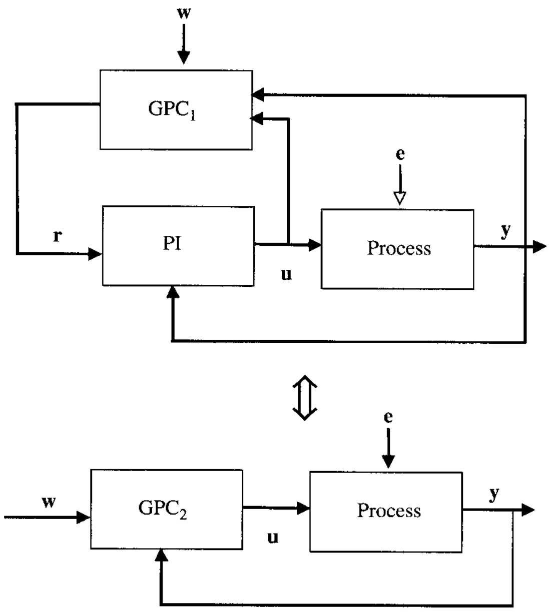

Figure 6. GPC supervisory controller.

cascaded controller structure no unstable hidden modes are created. For example, the supervisory control proposed cannot make up for poor lower loop if non-minimum phase controller is considered at regulatory level.

4. APPLICATION TO THE GAS TURBINE CONTROL SYSTEM

4.1. Statement of the supervisory level optimal control problem

The supervisory level, shown in Figure 7, is based on an economic optimizer that will provide the optimum set-points at the regulatory level. The optimization variable proposed is the set-points P_g^r because it depends directly on the main process supply F_d. The external set-point P_g^* for the gas turbine power is constant and previously fixed.

The set-points $g_{c\mathrm{NO}x}^r$ and T_{out}^r for the controlled variables $g_{c\mathrm{NO}x}$ and T_{out}, respectively, will be constant, because they do not affect the economic optimizer.

The proposed objective function (economic optimizer) considers both an economic and a regulatory level objective. That is, the maximization of the plant profit (J_{Cp}) and the minimization of the set-point trajectory error, together with the control action effort (J_{Cr}) are considered. Then, the total objective function to be optimized at supervisory level is given by

$$J = J_{Cp} - \eta J_{Cr} \tag{25}$$

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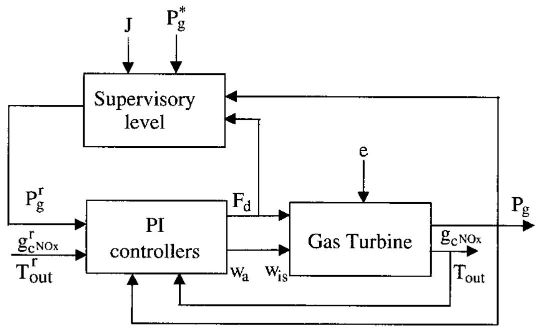

Figure 7. Proposed supervisory control strategy.

where η is a weighting factor. Also, considering the power plant as the main product and fuel flow F_d as the main process input, the economic objective function (J_{Cp}) is given by the following expression:

$$J_{Cf} = \sum_{i=1}^{N} C_p P_g(t+i-1) - \sum_{i=1}^{N} C_f F_d(t+i-1) + \mathrm{CF} \tag{26}$$

with $C_p = 10$ [\$\$/MW] is the power price, $C_f = 100$ [\$\$/kg] is the fuel cost, and CF fixed costs are given by the cost of operational technical personnel, etc. $N = 100$ is the prediction horizon. Also note that the regulatory level objective function (J_{Cr}) is given by

$$J_{Cr} = C_{rPg}\left(\sum_{j=1}^{N} (\hat{P}_g(t+j) - P_g^*)^2 + \lambda_{Fd} \sum_{i=1}^{N} \Delta F_d^2(t+i-1) \right) \tag{27}$$

where $\hat{P}_g(t+j)$ is the j-step ahead prediction for the gas turbine power, $C_{rPg} = 10^{-11}$ is the cost factors of the regulatory level and $\lambda_{Fd} = 10^{18}$ is the control weighting. The external set-point trajectory $P_g^* = 3.386 \times 10^7$ for the gas turbine power is constant and previously fixed.

For the general objective function, defined by Equation (11), the corresponding parameters of this economic optimizer (Equation (25)) are

$$\psi_{Pg} = \begin{bmatrix} -C_{rPg}\eta & 0 & \cdots & 0 \\ 0 & -C_{rPg}\eta & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & -C_{rPg}\eta \end{bmatrix}_{N\times N}$$

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$$\psi_{\Delta Fd} = \begin{bmatrix} -C_{rPg}\lambda_{Fd}\eta & 0 & \cdots & 0 \\ 0 & -C_{rPg}\lambda_{Fd}\eta & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & -C_{rPg}\lambda_{Fd}\eta \end{bmatrix}_{N\times N}$$

$$\xi_{Pg} = [C_p + 2C_{rPg}P_g^*\eta \;\; C_p + 2C_{rPg}P_g^*\eta \;\; \cdots \;\; C_p + 2C_{rPg}P_g^*\eta]^{\mathrm{T}}_{1\times N},$$

$$\xi_{wf} = [-C_f \;\; -C_f \;\; \cdots \;\; -C_f]^{\mathrm{T}}_{1\times N}$$

Also, $\Psi_u = 0$, $\Psi_{yu} = 0$, $\xi_{\Delta u} = 0$.

4.2. Gas turbine process models

The dynamics of the main variables of the gas turbine process were identified using CARIMA models at given operating points. The routines of the identification toolbox of Matlab with structure selection were used.

The CARIMA model for gas turbine (P_g [MW]), was obtained with 10 000 data generated by an excitation signal (F_d) being a discrete white noise. Sampling time $T_s = 1$ [s]. Thus, the CARIMA model is given by the following expression:

$$A(q^{-1})P_g(t) = B(q^{-1})F_d(t) + \frac{e(t)}{\Delta} \qquad (28)$$

with $A(q^{-1}) = 1 - 0.7661q^{-1}, B(q^{-1}) = 6.38 \times 10^6 q^{-1} + 5.149 \times 10^6 q^{-2}$.

Remark

The identification experiment described above was performed on the simulation model of the gas turbine. Therefore, it was easy to implement the white noise as the input signal. If the experiments were to be performed on the real plant, then a historical data on the F_d signal would have to provide enough excitation to enable accurate identification. This could be tested by taking the historical data from the gas turbine plant and feeding it into the simulation model.

4.3. PI controller models

As shown in Figure 4, the gas turbine system is regulated by proportional integral controller (PI). Then, the discrete model of the PI controller for fuel flow to the gas turbine is

$$A_c(q^{-1})F_d(t) = B_{cr}(q^{-1})P_g^r(t) + B_{cy}(q^{-1})P_g(t) \qquad (29)$$

with $A_c(q^{-1}) = 1 - q^{-1}$, $B_{cr}(q^{-1}) = 1.249 \times 10^{-9} - 1.022 \times 10^{-9} q^{-1}$, $B_{cy}(q^{-1}) = -1.249 \times 10^{-9} + 1.022 \times 10^{-9} q^{-1} K_p = 1.136 \times 10^{-009}$, $K_i = 2.272 \times 10^{-009}$.

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4.4. Supervisory level controller

The explicit solution obtained in Equation (16) can be used to optimize of the objective function defined by Equation (25). Therefore, the supervisory controller given by the optimum variable set-points for the gas turbine power is given by

$$A_s(q^{-1})P_g^*(t) = B_{sy}(q^{-1})P_g(t) + B_{su}(q^{-1})F_d(t) + C_s \tag{30}$$

with $A_s(q^{-1}) = 1 - 1.01q^{-1} + 0.157q^{-2}$, $B_{sy}(q^{-1}) = -2.167 + 1.374q^{-1} + 0.157q^{-2}$

$$B_{su}(q^{-1}) = (-1.698q^{-1} + 1.698q^{-2}) \times 10^8, \quad C_s = 2.68 \times 10^7$$

4.5. Simulation test

The proposed supervisory controller is compared with a control strategy, where the optimum set-points are constant and calculated from a static optimization of the objective function defined in Equation (25) by using a static model of the process. Then, the optimum static set-point for the gas turbine power is given by

$$P_g^r(t) = P_g^* - \frac{C_p K_{Pg} - C_f}{2C_{rPg}K_{Pg}} = 3.347 \times 10^7, \quad t \geqslant 0 \tag{31}$$

with $K_{Pg} = 4.48 \times 10^7$ is the static gain for the gas turbine power as function of fuel flow.

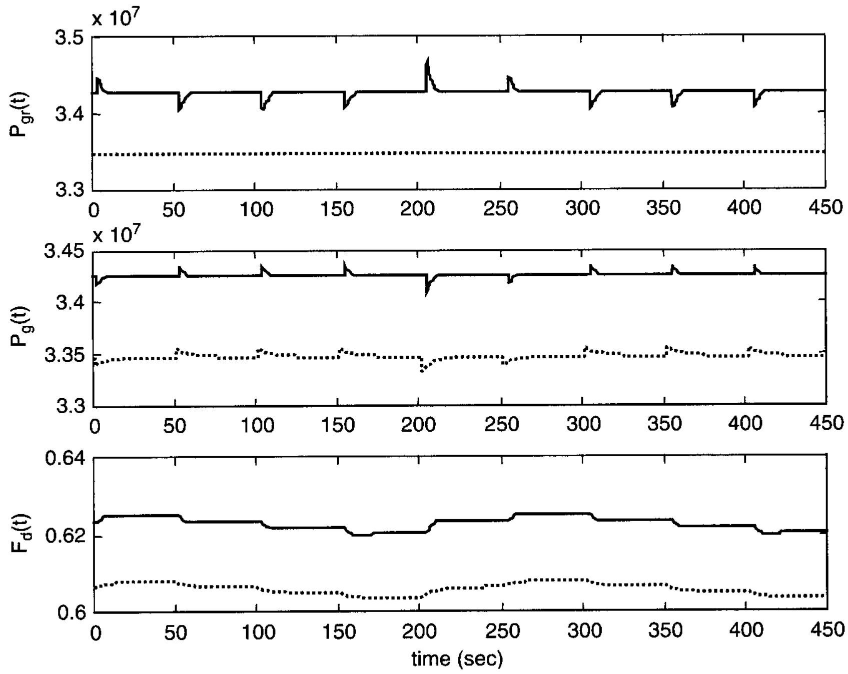

Figure 8. Closed-loop response ($\eta = 1$) with constant set-point (dashed line) and with the proposed supervisory level (solid line).

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On the other hand, for the comparative analysis, a disturbance is included, given by changes in the temperature of the air mass flow into the compressor in order to produce different operating conditions. The disturbance values (T_{cin}) move between 276 and 294 [K] for 450 s.

Figures 8 and 9 shows the closed-loop responses of the gas turbine system, with the constant set-point and with optimal supervisory controller for η, between 0.5 to 1, as defined in Equation (21). Notice that the figures show the steady state behaviour of the two systems. Therefore, the initial part of the response of the simulated systems was removed, so that the influence of the initial conditions does not obscure the picture. Initially, the set-points for both algorithms were starting from the same value but in the steady-state they converge to different values.

On Table I, the mean values of the objective functions (Equations (26) and (27)) are evaluated using the data presented in the previous figures. Also, the profit regarding the control strategy

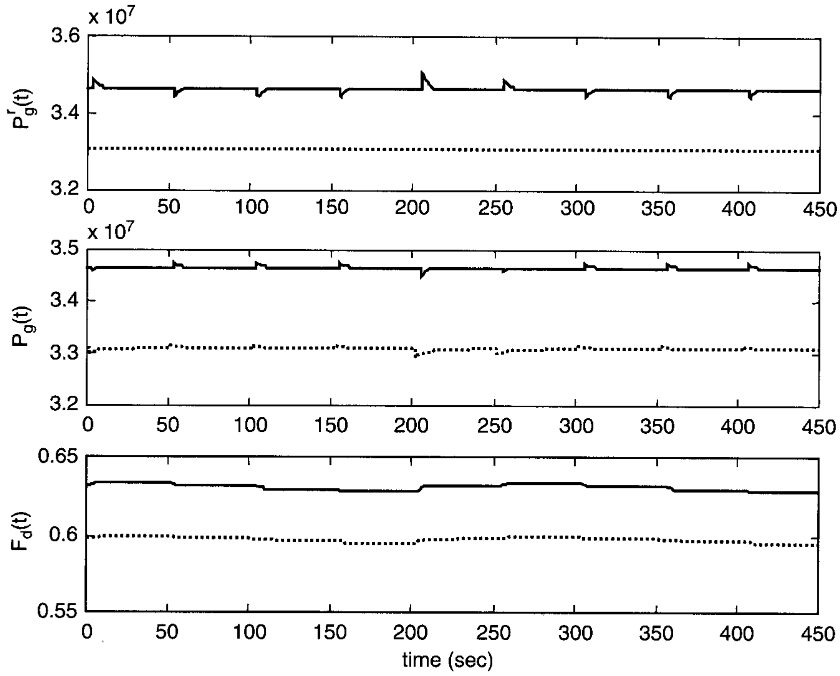

Figure 9. Closed-loop response ($\eta = 0.5$) with constant set-point (dashed line) and with the proposed supervisory level (solid line).

Table I. Mean values of the economic and regulatory objective functions.

η	Constant set-point		Supervisory level variable set-point		Profit % Benefits
	J_{Cp}	J_{Cr}	J_{Cp}	J_{Cr}	
1	27418	148.919	28034	160.109	2.19%
0.5	27112	598.847	28346	318.434	4.35%

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with constant set-point is defined by

$$\text{Profit} = \left(100 - 100\frac{J_{Cf} \text{ with supervisory level}}{J_{Cf} \text{ with constant set-points}}\right)\% \tag{32}$$

As shown in Table I, the power plant profit increases by introducing the supervisory level. The benefit for the proposed control strategy range from 2.4% to 4.4% for η, between 1 and 0.5.

5. CONCLUSIONS

A supervisory optimal controller, based on the theory of predictive control, was derived, providing a quick way of implementing this algorithm without numerical optimization. The proposed strategy provides the regulatory level set-points based on the objective function optimization without modifying the regulatory level.

The explicit solution obtained was applied to the optimization of the gas turbine operation of a thermal power plant. The supervisory controller includes an economic criterion and a regulatory criterion. This means that there is a trade-off between the criteria that can be handled depending on the aims of the plant operation. The control strategy that was designed at supervisory level was compared to a control strategy with optimum constant set-points. The benefits obtained for the gas turbine ranged from 2.4 to 4.4%.

Further work is in progress in order to analyze closed loop stability and robustness for this supervisory controller.

APPENDIX A: PREDICTION EQUATIONS

A.1. Prediction of the controlled variables

The prediction of the controlled variables, as a function of the increments of the manipulated variables, is obtained using generalized predictive control theory [10]

$$\hat{\mathbf{y}} = \mathbf{G}_{\Delta u}\Delta\mathbf{u} + \mathbf{f}_{\Delta u} \tag{A1}$$

The term $\mathbf{G}_{\Delta u}\Delta\mathbf{u}$ represents the unknown future signals (forced response) and $\mathbf{f}_{\Delta u}$ represents the known past signals (free response), and

$$\hat{\mathbf{y}} = [\hat{y}(t+1),\ldots,\hat{y}(t+N)]_N^{\mathrm{T}}$$

$$\Delta\mathbf{u} = [\Delta u(t), \Delta u(t+1),\ldots,\Delta u(t+N-1)]_N^{\mathrm{T}} \quad \text{and} \quad \mathbf{G}_{\Delta u} = \begin{bmatrix} g_{10} & 0 & \cdots & 0 \\ g_{21} & g_{20} & 0 & \\ \vdots & & & 0 \\ g_{NN-1} & g_{NN-2} & \cdots & g_{N0} \end{bmatrix}_{N\times N}$$

$$\mathbf{f}_{\Delta u} = [f_{\Delta u}(t+1),\ldots,f_{\Delta u}(t+N)]_N^{\mathrm{T}} \tag{A2}$$

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On the other hand, recall that

$$\Delta\mathbf{u}(t) = \begin{bmatrix} u(t) - u(t-1) \\ u(t+1) - u(t) \\ \vdots \\ u(t+N-1) - u(t+N-2) \end{bmatrix} \tag{A3}$$

and

$$\Delta\mathbf{u} = \mathbf{T}_1\mathbf{u} + \mathbf{T}_2, \quad \text{with } \mathbf{u} = [u(t), u(t+1), \ldots, u(t+N-1)]_N^{\mathrm{T}} \tag{A4}$$

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \\ 0 & & \ddots & 0 \\ 0 & & -1 & 1 \end{bmatrix}_{N\times N} \quad \text{and} \quad \mathbf{T}_2 = [-u(t-1)\ 0\ \cdots\ 0]_N^{\mathrm{T}} \tag{A5}$$

Then, the prediction of the controlled variables, as a function of the manipulated variables, is obtained replacing Equation (A4) in Equation (A1)

$$\hat{\mathbf{y}} = \mathbf{G}_u\mathbf{u} + \mathbf{f}_u, \quad \text{where } \mathbf{G}_u = \mathbf{G}_{\Delta u}\mathbf{T}_1, \ \mathbf{f}_u = \mathbf{G}_{\Delta u}\mathbf{T}_2 + \mathbf{f}_{\Delta u} \tag{A6}$$

Also, $\mathbf{G}_u\mathbf{u}$ represent the unknown future signals and $\mathbf{f}_u$ represents the known past signals.

A.2. Prediction of the manipulated variables. The model of the linear controller is represented by

$$A_c(q^{-1})u(t) = B_{cr}(q^{-1})r(t) + B_{cy}(q^{-1})y(t) \tag{A7}$$

where $A_c(q^{-1})$, $B_{cr}(q^{-1})$, $B_{cy}(q^{-1})$ are polynomials of degrees n_a, n_{br}, n_{by}, respectively. The following diophantine equation must be introduced:

$$1 = E_j^{ry}(q^{-1})A_c(q^{-1}) + q^{-j-1}F_j^{ry}(q^{-1}) \tag{A8}$$

with the degrees of polynomials E_j^{ry} and F_j^{ry} being j and na, respectively. Multiplying Equation (A4) by $E_j^{ry}(q^{-1})$, substituting the diophantine Equation (A8) and shifting the time index by j steps obtains

$$(1 - q^{-j-1}F_j^{ry}(q^{-1}))u(t+j) = E_j^{ry}(q^{-1})B_{cr}(q^{-1})r(t+j) + E_j^{ry}(q^{-1})B_{cy}(q^{-1})y(t+j) \tag{A9}$$

Then

$$u(t+j) = G_j^r(q^{-1})r(t+j) + G_j^y(q^{-1})y(t+j) + F_j^{ry}(q^{-1})u(t-1) \quad \text{(A10)}$$

with

$$G_j^r(q^{-1}) = E_j^{ry}(q^{-1})B_{cr}(q^{-1}),\ \ G_j^y(q^{-1}) = E_j^{ry}(q^{-1})B_{cy}(q^{-1}) \quad \text{(A11)}$$

$$G_j^r(q^{-1}) = g_{j0}^r + g_{j1}^r q^{-1} + \cdots + g_{jn_{rj}}^r q^{-n_{rj}} \quad \text{and} \quad G_j^y(q^{-1}) = g_{j0}^y + g_{j1}^y q^{-1} + \cdots + g_{jn_{yj}}^y q^{-n_{yj}}$$

Clearly, the degrees of the polynomials G_j^r and G_j^y are: $n_{rj} = j + n_{br},\ \ n_{yj} = j + n_{by}$. Substituting $j = 0, 1, 2, \ldots$.

$$u(t) = g_{00}^r r(t) + g_{01}^r r(t-1) + \cdots + g_{0n_{r0}}^r r(t - n_{r0} + 1)$$

$$+ g_{00}^y y(t) + g_{01}^y y(t-1) + \cdots + g_{0n_{y0}}^y y(t - n_{y0} + 1) + F_0^{ry}(q^{-1})u(t-1)$$

$$\vdots$$

$$u(t+j) = g_{j0}^r r(t+j) + g_{j1}^r r(t+j-1) + \cdots + g_{jj}^r r(t) + \cdots + g_{jn_{rj}}^r r(t - n_{rj} + 1)$$

$$+ g_{j0}^y y(t+j) + g_{11}^y y(t+j-1) + \cdots + g_{jj}^y y(t) + \cdots + g_{jn_{yj}}^y y(t - n_{yj} + 1)$$

$$+ F_j^{ry}(q^{-1})u(t-1) \quad \text{(A12)}$$

Let $f(t+j)$ be those components of $u(t+j)$, which at time t are the past signals, i.e. $y(t), y(t-1), \ldots, r(t-1), r(t-2), \ldots, u(t-1), u(t-2), \ldots$:

$$f(t) = [G_0^r(q^{-1}) - g_{00}^r]r(t) + G_0^y(q^{-1})y(t) + F_0^{ry}(q^{-1})u(t-1)$$

$$f(t+1) = [G_1^r(q^{-1}) - g_{10}^r - g_{11}^r q^{-1}]r(t+1) + [G_1^y(q^{-1}) - g_{10}^y]y(t+1) + F_1^{ry}(q^{-1})u(t-1)$$

$$\vdots \quad \text{(A13)}$$

The equations above may be written in the matrix form

$$\mathbf{u} = \mathbf{G}_r\mathbf{r} + \mathbf{G}_y\hat{\mathbf{y}} + \mathbf{f}_{ryu} \quad \text{(A14)}$$

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where $\mathbf{G}_r\mathbf{r} + \mathbf{G}_y\hat{\mathbf{y}}$ represents the unknown future signals and $\mathbf{f}_{ryu}$ represent the known past signals with

$$\mathbf{r} = [r(t), \ldots, r(t+N-1)]_N^{\mathrm{T}}$$

$$\mathbf{G}_r = \begin{bmatrix} g^r_{00} & 0 & \cdots & 0 \\ g^r_{11} & g^r_{10} & & \\ \vdots & \vdots & & 0 \\ g^r_{N-1,N-1} & g^r_{N-1,N-2} & \cdots & g^r_{N-1,0} \end{bmatrix}_{N\times N}, \quad \mathbf{G}_y = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ g^y_{10} & \ddots & & \\ g^y_{21} & g^y_{20} & \ddots & \vdots \\ \vdots & \vdots & \ddots \quad \ddots & 0 \\ g^y_{N-1,N-2} & g^y_{N,N-3} & \cdots \quad g^y_{N0} & 0 \end{bmatrix}_{N\times N} \tag{A15}$$

and

$$\mathbf{f}_{ryu} = \begin{bmatrix} f(t) \\ f(t+1) \\ f(t+N-1) \end{bmatrix}_N \tag{A16}$$

On the other hand, the prediction of the increment of the manipulated variables is obtained using Equation (A14), that is

$$\Delta\mathbf{u} = \mathbf{G}_r\Delta\mathbf{r} + \mathbf{G}_y\Delta\hat{\mathbf{y}} + \Delta(\mathbf{f}_{ryu}) \tag{A17}$$

where

$$\Delta\mathbf{r} = T_3\mathbf{r} + T_4 \quad \text{and} \quad \Delta\hat{\mathbf{y}} = T_3\mathbf{y} + T_5 \tag{A18}$$

with $\Delta\mathbf{r} = [\Delta r(t), \ldots, \Delta r(t+N-1)]_N^{\mathrm{T}}$

$$\mathbf{T}_3 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \\ 0 & & \ddots & 0 \\ 0 & & -1 & 1 \end{bmatrix}_{N\times N}, \quad \mathbf{T}_4 = [-r(t-1) \ 0 \ \cdots \ 0]_N^{\mathrm{T}}, \quad \mathbf{T}_5 = [-y(t) \ 0 \ \cdots \ 0]_N^{\mathrm{T}} \tag{A19}$$

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and

$$\Delta(\mathbf{f}_{ryu}) = \begin{bmatrix} f(t) - f(t-1) \\ f(t+1) - f(t) \\ \vdots \\ \vdots \\ f(t+N-1) - f(t+N-2) \end{bmatrix}_N \tag{A20}$$

Thus, the prediction of the increments of the manipulated variables is obtained by replacing Equation (A18) in Equation (A17)

$$\Delta\mathbf{u} = \mathbf{G}_{\Delta r}\mathbf{r} + \mathbf{G}_{\Delta y}\hat{\mathbf{y}} + \mathbf{f}_{\Delta ryu} \tag{A21}$$

The signal $\mathbf{G}_{\Delta r}\mathbf{r} + \mathbf{G}_{\Delta y}\mathbf{y}$ represents the unknown future signals and $\mathbf{f}_{\Delta ryu}$ represent the known past signals with

$$\mathbf{G}_{\Delta r} = \mathbf{G}_r\mathbf{T}_3 \quad \text{and} \quad \mathbf{G}_{\Delta y} = \mathbf{G}_y\mathbf{T}_3 \quad \text{and} \quad \mathbf{f}_{\Delta ryu} = \mathbf{G}_r\mathbf{T}_4 + \mathbf{G}_y\mathbf{T}_5 + \Delta(\mathbf{f}_{ryu}) \tag{A22}$$

APPENDIX B: SUPERVISORY LEVEL

B.1. GPC objective function at supervisory level

The following relation is formulated in Equation (A6):

$$\mathbf{G}_u = \mathbf{G}_{\Delta u}\mathbf{T}_1 \tag{B1}$$

with

$$\mathbf{G}_{\Delta u} = \begin{bmatrix} g_{10} & 0 & \cdots & 0 \\ g_{21} & g_{20} & 0 & \\ \vdots & & & 0 \\ g_{NN-1} & g_{NN-2} & \cdots & g_{N0} \end{bmatrix}_{N\times N}$$

Notice that

$$\mathbf{G}_{\Delta u} = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & 0 & \\ \vdots & & & 0 \\ g_{N-1} & g_{N-2} & \cdots & g_0 \end{bmatrix}_{N\times N} \tag{B2}$$

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because $g_{ij} = g_j$ for $j = 0, 1, 2, \ldots < i$ [10]. Also, in Equation (A6): $\mathbf{f}_u = \mathbf{G}_{\Delta u}\mathbf{T}_2 + \mathbf{f}_{\Delta u}$ and in Equation (A22):

$$\mathbf{G}_{\Delta r} = \mathbf{G}_r\mathbf{T}_3$$

with G_r defined by Equation (A15)

$$\mathbf{G}_r = \begin{bmatrix} g^r_{00} & 0 & \cdots & 0 \\ g^r_{11} & g^r_{10} & & \\ \vdots & \vdots & & 0 \\ g^r_{N-1,N-1} & g^r_{N-1,N-2} & \cdots & g^r_{N-1,0} \end{bmatrix}_{N\times N} \quad \text{and} \quad \mathbf{G}_r = \begin{bmatrix} g^r_0 & 0 & \cdots & 0 \\ g^r_1 & g^r_{10} & & \\ \vdots & \vdots & & 0 \\ g^r_{N-1} & g^r_{N-2} & \cdots & g^r_0 \end{bmatrix}_{N\times N} \tag{B3}$$

similarly to Equation (B2), $g^r_{ij} = g^r_j$ for $j = 0, 1, 2, \ldots < i$ [10]. From Equation (A22): $\mathbf{G}_{\Delta y} = \mathbf{G}_y\mathbf{T}_3$ where

$$\mathbf{G}_y = \begin{bmatrix} 0 & 0 & \cdots & & 0 \\ g^y_{10} & \ddots & & & \\ g^y_{21} & g^y_{20} & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ g^y_{N-1,N-2} & g^y_{N,N-3} & \cdots & g^y_{N0} & 0 \end{bmatrix}_{N\times N} \quad \text{and} \quad \mathbf{G}_y = \begin{bmatrix} 0 & \cdots & & 0 & 0 \\ g^y_0 & 0 & & & \vdots \\ g^y_1 & g^y_0 & \ddots & & \vdots \\ \vdots & \vdots & \ddots & 0 & \\ g^y_{N-2} & g^y_{N-3} & \cdots & g^y_0 & 0 \end{bmatrix}_{N\times N} \tag{B4}$$

similarly to Equation (B2), $g^y_{ij} = g^y_j$ for $j = 0, 1, 2, \ldots, < i$ [10].

Also notice that $\mathbf{T}_1 = \mathbf{T}_3$ as shown in the Equations (A5) and (A19).

Lemma 1

For two lower triangular matrices with all elements on the diagonal being equal and also all elements on each of the sub-diagonals being equal, the commutation property holds, i.e.

$$AB = BA$$

where

$$A = \begin{bmatrix} a_0 & 0 & \cdots & & 0 \\ a_1 & a_0 & \ddots & & \vdots \\ a_2 & a_1 & a_0 & & \\ & & & \ddots & 0 \\ a_n & a_{n-1} & \cdots & & a_0 \end{bmatrix}, \quad B = \begin{bmatrix} b_0 & 0 & \cdots & & 0 \\ b_1 & b_0 & \ddots & & \vdots \\ b_2 & b_1 & b_0 & & \\ & & & \ddots & 0 \\ b_n & b_{n-1} & \cdots & & b_0 \end{bmatrix}$$

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Proof
By expansion and simplifications. Thus, as G_u and G_r are lower triangular matrices with all elements on the diagonal equal and also all elements on each of the sub-diagonals being equal, using Lemma 1,

$$\mathbf{G}_u\mathbf{G}_r = \mathbf{G}_r\mathbf{G}_u \tag{B5}$$

Also, using (B1) and (A22)

$$\mathbf{G}_u\mathbf{G}_r = \mathbf{G}_{\Delta r}\mathbf{T}_1^{-1}\mathbf{G}_{\Delta u}\mathbf{T}_1 \tag{B6}$$

Also, $\mathbf{T}_1$ is a lower triangular matrix with all elements on each of the diagonals equal. Therefore, using Lemma 1,

$$\mathbf{G}_{\Delta u}\mathbf{T}_1 = \mathbf{T}_1\mathbf{G}_{\Delta u} \tag{B7}$$

Then, substituting (B7) into Equation (B6) we obtain

$$\mathbf{G}_u\mathbf{G}_r = \mathbf{G}_{\Delta r}\mathbf{T}_1^{-1}\mathbf{T}_1\mathbf{G}_{\Delta u} \quad \text{and} \quad \mathbf{G}_u\mathbf{G}_r = \mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u} \tag{B8}$$

where $\mathbf{G}_{\Delta u}$, $\mathbf{G}_{\Delta r}$, $\mathbf{T}_1$, $\mathbf{T}_1^{-1}$ are lower triangular matrixes. Similarly, using Lemma 1, the following identity is obtained:

$$\mathbf{G}_u\mathbf{G}_y = \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u} \tag{B9}$$

Recalling Equation (16) for the optimal value of the reference signal, and multiplying both sides of that equation by the matrix $\mathbf{H}$ which is assumed non-singular (see Section 3.2) we obtain:

$$\mathbf{H}\mathbf{r}^* = \mathbf{F} \tag{B10}$$

Then, substituting the relations (B8) and (B9) in Equations (13) and (14), which define the matrices $\mathbf{H}$ and $\mathbf{F}$, we obtain

$$\begin{aligned}\mathbf{H} &= [(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}]^{\mathrm{T}}[(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}] \\ &\quad + [(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{\mathrm{T}}\boldsymbol{\lambda}[(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]\end{aligned} \tag{B11}$$

and

$$\begin{aligned}\mathbf{F} &= [(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}]^{\mathrm{T}}[\mathbf{w} - (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu} + \mathbf{f}_u)] \\ &\quad + [(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{\mathrm{T}}\boldsymbol{\lambda}[(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu} - \mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]\end{aligned} \tag{B12}$$

Both sides of Equation (B10) may now be multiplied by

$$\{[(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}]^{\mathrm{T}}\}^{-1} = \{\mathbf{G}_{\Delta u}^{\mathrm{T}}\mathbf{G}_{\Delta r}^{\mathrm{T}}[(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}]^{\mathrm{T}}\}^{-1} = (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{\mathrm{T}}[\mathbf{G}_{\Delta r}^{\mathrm{T}}]^{-1}[\mathbf{G}_{\Delta u}^{\mathrm{T}}]^{-1}$$

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It therefore follows that

$$\{[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}]^{\mathrm{T}}\}^{-1}\mathbf{H}=(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}+[(\tilde{\mathbf{G}}_{\Delta u})^{\mathrm{T}}]^{-1}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r} \quad \text{(B13)}$$

and

$$\{[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}]^{\mathrm{T}}\}^{-1}\mathbf{F}=\mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)$$
$$+[(\tilde{\mathbf{G}}_{\Delta u})^{\mathrm{T}}]^{-1}\boldsymbol{\lambda}(I-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu}-\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}) \quad \text{(B14)}$$

where

$$[(\tilde{\mathbf{G}}_{\Delta u})^{\mathrm{T}}]^{-1}=(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{\mathrm{T}}[\mathbf{G}_{\Delta r}^{\mathrm{T}}]^{-1}[\mathbf{G}_{\Delta u}^{\mathrm{T}}]^{-1}\mathbf{G}_{\Delta r}^{\mathrm{T}}[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}]^{\mathrm{T}} \quad \text{(B15)}$$

It is easy to notice that Lemma 1 also applies to upper triangular matrices. Also notice that, if $\mathbf{X}$ is a lower triangular matrix then $\mathbf{X}^{-1}$ is also a lower triangular matrix. Then it follows that

$$[(\tilde{\mathbf{G}}_{\Delta u})^{\mathrm{T}}]^{-1}=[(\mathbf{G}_{\Delta u})^{\mathrm{T}}]^{-1}=\mathbf{G}_{\Delta u}^{-\mathrm{T}} \quad \text{(B16)}$$

Substituting Equations (B1), (A6) and (B9) in (B11) and (B12) we obtain

$$\mathbf{r}=\mathbf{H}^{-1}\mathbf{F}=[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{-1}$$
$$\times[\mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu}-\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})] \quad \text{(B17)}$$

On the other hand, the predictions of increments of the manipulated variable as a function of the variable set-points are given by Equation (9)

$$\Delta\mathbf{u}=(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{r}+(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}) \quad \text{(B18)}$$

Substituting Equation (B17) in Equation (B18) we obtain

$$\Delta\mathbf{u}=(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}[(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{-1}$$
$$\times[\mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu}-\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]$$
$$+(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}) \quad \text{(B19)}$$

Lemma 2

$$(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}\mathbf{G}_{\Delta u}=\mathbf{G}_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}$$

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Proof
Using Lemma 1, as $(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}$; $\mathbf{G}_{\Delta r}$ and $\mathbf{G}_{\Delta u}$ are lower triangular matrices with all elements on each of the sub-diagonals equal. Then

$$\begin{aligned}\Delta\mathbf{u} = {} & (\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}[(\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda})(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta r}]^{-1}\\ & \times[\mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu}-\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]\\ & +(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})\end{aligned} \tag{B20}$$

and

$$\begin{aligned}(\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda})\Delta\mathbf{u} = {} & [\mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)\\ & +(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{f}_{\Delta ryu}-\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})]\\ & +(\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda})(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u})\end{aligned} \tag{B21}$$

then

$$(\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}}\boldsymbol{\lambda})\Delta\mathbf{u}=\mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_u\mathbf{f}_{ryu}+\mathbf{f}_u)+\mathbf{G}_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{f}_{\Delta ryu}+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}) \tag{B22}$$

Assumption (*Lemma 3*)

$$(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta u}\mathbf{T}_1=\mathbf{G}_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{T}_1$$

Proof
Using Lemma 1, as $(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}$; $\mathbf{T}_1$ and $\mathbf{G}_{\Delta u}$ are lower triangular matrices with all elements on each of the subdiagonal equal. □

Substituting (B1), (A6) and (A22) to (B23) we obtain

$$\begin{aligned}(\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}})\Delta\mathbf{u} = {} & \mathbf{w}-(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{G}_{\Delta u}\mathbf{T}_1\mathbf{f}_{ryu}+\mathbf{G}_{\Delta u}\mathbf{T}_2+\mathbf{f}_{\Delta u})\\ & +\mathbf{G}_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}[\mathbf{G}_r\mathbf{T}_4+\mathbf{G}_y\mathbf{T}_5+\Delta(\mathbf{f}_{ryu})+\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}]\end{aligned} \tag{B23}$$

$$\begin{aligned}(\mathbf{G}_{\Delta u}+(\mathbf{G}_{\Delta u})^{-\mathrm{T}})\Delta\mathbf{u} = {} & \mathbf{w}-\mathbf{G}_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(-\mathbf{G}_{\Delta u}\mathbf{G}_{\Delta y}\mathbf{f}_{\Delta u}+\mathbf{f}_{\Delta u})\\ & +\mathbf{G}_{\Delta u}(\mathbf{I}-\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}[-\mathbf{T}_1\mathbf{f}_{ryu}-\mathbf{T}_2+\mathbf{G}_r\mathbf{T}_4+\mathbf{G}_y\mathbf{T}_5+\Delta(\mathbf{f}_{ryu})]\end{aligned} \tag{B24}$$

Lemma 4

$$[-\mathbf{T}_1\mathbf{f}_{ryu}-\mathbf{T}_2+\mathbf{G}_r\mathbf{T}_4+\mathbf{G}_y\mathbf{T}_5+\Delta(\mathbf{f}_{ryu})]=\mathbf{0} \tag{B25}$$

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Proof
From Equation (A13), the vector $\mathbf{f}_{ryu}$, as defined in Equation (A16) can be expressed as

$$\mathbf{f}_{ryu} = \tilde{\mathbf{G}}_r(q^{-1})\mathbf{r}(t) + \tilde{\mathbf{G}}_y(q^{-1})\hat{\mathbf{y}}(t) + \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} y(t) + \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} u(t-1) \tag{B26}$$

with

$$\tilde{\mathbf{G}}_r(q^{-1}) = \begin{bmatrix} G_0^r(q^{-1}) - g_0^r & 0 & & 0 \\ -g_1^r & G_1^r(q^{-1}) - g_0^r & 0 & \vdots \\ -g_2^r & -g_1^r & G_2^r(q^{-1}) - g_0^r & 0 \\ \vdots & & & \ddots \end{bmatrix} \tag{B27}$$

$$\tilde{\mathbf{G}}_y(q^{-1}) = \begin{bmatrix} 0 & & \cdots & 0 \\ G_1^y(q^{-1}) - g_0^y & & & \\ -g_1^y & G_2^y(q^{-1}) - g_0^y & & \\ \vdots & \vdots & \ddots & 0 \\ & & \cdots & \end{bmatrix} \tag{B28}$$

Consequently, the vector $\Delta(\mathbf{f}_r + \mathbf{f}_y)$, as defined in Equation (A20) is given by

$$\begin{aligned} \Delta(\mathbf{f}_{ryu}) &= \tilde{\mathbf{G}}_r(q^{-1})\Delta\mathbf{r} + \tilde{\mathbf{G}}_y(q^{-1})\Delta\mathbf{y} + \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} \Delta y(t) + \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} \Delta u(t-1) \\ &= \tilde{\mathbf{G}}_r(q^{-1})(\mathbf{T}_3\mathbf{r} + \mathbf{T}_4) + \tilde{\mathbf{G}}_y(q^{-1})(\mathbf{T}_3\mathbf{y} + \mathbf{T}_5) \\ &\quad + \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} y(t) - \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} y(t-1) + \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} u(t-1) \\ &\quad - \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} u(t-2) \end{aligned} \tag{B29}$$

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Then, Equation (B25) becomes

$$[-\mathbf{T}_1\mathbf{f}_{ryu} - \mathbf{T}_2 + \mathbf{G}_r\mathbf{T}_4 + \mathbf{G}_y\mathbf{T}_5 + \Delta(\mathbf{f}_{ryu})]$$

$$= -\mathbf{T}_1\left(\tilde{\mathbf{G}}_r(q^{-1})\mathbf{r} + \tilde{\mathbf{G}}_y(q^{-1})\mathbf{y} + \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} y(t) + \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} u(t-1)\right)$$

$$- \begin{bmatrix} -u(t-1) \\ 0 \\ \vdots \end{bmatrix} + \mathbf{G}_r\mathbf{T}_4 + \mathbf{G}_y\mathbf{T}_5 + \tilde{\mathbf{G}}_r(q^{-1})(\mathbf{T}_3\mathbf{r} + \mathbf{T}_4) + \tilde{\mathbf{G}}_y(q^{-1})(\mathbf{T}_3\mathbf{y} + \mathbf{T}_5)$$

$$+ \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} y(t) - \begin{bmatrix} G_0^y(q^{-1}) \\ 0 \\ \vdots \end{bmatrix} y(t-1) + \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} u(t-1)$$

$$- \begin{bmatrix} F_0^{ry}(q^{-1}) \\ F_1^{ry}(q^{-1}) \\ \vdots \end{bmatrix} u(t-2) \tag{B30}$$

$$[-\mathbf{T}_1\mathbf{f}_{ryu} + \mathbf{G}_r\mathbf{T}_4 + \mathbf{G}_y\mathbf{T}_5 + \Delta(\mathbf{f}_{ryu})]$$

$$= -\mathbf{T}_1\tilde{\mathbf{G}}_r\mathbf{r} + \tilde{\mathbf{G}}_r\mathbf{T}_3\mathbf{r} - \mathbf{T}_1\tilde{\mathbf{G}}_y\mathbf{y} + \tilde{\mathbf{G}}\mathbf{y}\mathbf{T}_3\mathbf{y} + (\mathbf{G}_r + \tilde{\mathbf{G}}_r)\mathbf{T}_4 + (\mathbf{G}_y + \tilde{\mathbf{G}}_y)\mathbf{T}_5$$

$$- \mathbf{T}_1\left(\begin{bmatrix} G_0^y \\ 0 \\ \vdots \end{bmatrix} y(t) + \begin{bmatrix} F_0^{ry} \\ F_1^{ry} \\ \vdots \end{bmatrix} u(t-1)\right) + \begin{bmatrix} G_0^y \\ 0 \\ \vdots \end{bmatrix} y(t) - \begin{bmatrix} G_0^y \\ 0 \\ \vdots \end{bmatrix} y(t-1)$$

$$+ \begin{bmatrix} F_0^{ry} \\ F_1^{ry} \\ \vdots \end{bmatrix} u(t-1) - \begin{bmatrix} F_0^{ry} \\ F_1^{ry} \\ \vdots \end{bmatrix} u(t-2) - \begin{bmatrix} -u(t-1) \\ 0 \\ \vdots \end{bmatrix}$$

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$$
= \begin{bmatrix} 0 & 0 & & 0 \\ G_0^r - G_1^r & 0 & 0 & \vdots \\ 0 & G_1^r - G_2^r & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} r(t) \\ r(t+1) \\ r(t+2) \\ \end{bmatrix} + \begin{bmatrix} 0 & 0 & & 0 \\ & 0 & 0 & \vdots \\ G_1^y - G_2^y & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} y(t+1) \\ y(t+2) \\ \vdots \end{bmatrix}
$$

$$
+ \begin{bmatrix} G_0^r & 0 & & 0 \\ 0 & G_1^r & 0 & \vdots \\ 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} -r(t-1) \\ 0 \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 & 0 & & 0 \\ G_1^y & 0 & 0 & \vdots \\ 0 & G_2^y & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} -y(t) \\ 0 \\ \vdots \end{bmatrix} - \begin{bmatrix} -u(t-1) \\ 0 \\ \vdots \end{bmatrix}
$$

$$
+ \begin{bmatrix} -G_0^y \\ G_0^y \\ 0 \\ \vdots \end{bmatrix} y(t) + \begin{bmatrix} -F_0^{ry} \\ F_0^{ry} - F_1^{ry} \\ F_1^{ry} - F_2^{ry} \\ \vdots \end{bmatrix} u(t-1) + \begin{bmatrix} G_0^y \\ 0 \\ \vdots \end{bmatrix} y(t) - \begin{bmatrix} G_0^y \\ 0 \\ \vdots \end{bmatrix} y(t-1)
$$

$$
+ \begin{bmatrix} F_0^{ry} \\ F_1^{ry} \\ \vdots \end{bmatrix} u(t-1) - \begin{bmatrix} F_0^{ry} \\ F_1^{ry} \\ \vdots \end{bmatrix} u(t-2)
$$

$$
= \begin{bmatrix} -G_0^r r(t-1) + u(t-1) - G_0^y y(t-1) - F_0^{ry} u(t-2) \\ F_0^{ry} u(t-1) - F_1^{ry} u(t-2) + (G_0^r - G_1^r) r(t) + (G_0^y - G_1^y) y(t) \\ F_1^{ry} u(t-1) - F_2^{ry} u(t-2) + (G_1^r - G_2^r) r(t+1) + (G_1^y - G_2^y) y(t+1) \\ \vdots \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \tag{B31}
$$

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because, from Equation (A10)

$$u(t) = G_0^r r(t) + G_0^y y(t) + F_0^{ry} u(t-1) \quad \text{then } u(t-1) = G_0^r r(t-1) + G_0^y y(t-1) + F_0^{ry} u(t-2)$$

$$u(t+1) = G_1^r r(t+1) + G_1^y y(t+1) + F_1^{ry} u(t-1) \quad \text{then } u(t) = G_1^r r(t) + G_1^y y(t) + F_1^{ry} u(t-2)$$

$$u(t+2) = G_2^r r(t+2) + G_2^y y(t+2) + F_2^{ry} u(t-1) \quad \text{then}$$

$$u(t+1) = G_2^r r(t+1) + G_2^y y(t+1) + F_2^{ry} u(t-2)$$

□

Lemma 5

$$\mathbf{G}_{\Delta u}(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta y} = (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta u}\mathbf{G}_{\Delta y}$$

Proof

Results directly from Lemma 1. □

Using the above in (B22) we obtain

$$(\mathbf{G}_{\Delta u} + [(\mathbf{G}_{\Delta u})^{\mathrm{T}}]^{-1}\lambda)\Delta\mathbf{u} = \mathbf{w} - (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{f}_{\Delta u} + (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}\mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u}\mathbf{f}_{\Delta u}$$

$$= \mathbf{w} - (\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})^{-1}(\mathbf{I} - \mathbf{G}_{\Delta y}\mathbf{G}_{\Delta u})\mathbf{f}_{\Delta u} = \mathbf{w} - \mathbf{f}_{\Delta u} \tag{B32}$$

Multiplying both sides of (B32) by $(\mathbf{G}_{\Delta u})^{\mathrm{T}}$ and then by $(\mathbf{G}_{\Delta u}^{\mathrm{T}}\mathbf{G}_{\Delta u} + \lambda\mathbf{I})^{-1}$ we obtain

$$\Delta\mathbf{u} = (\mathbf{G}_{\Delta u}^{\mathrm{T}}\mathbf{G}_{\Delta u} + \lambda\mathbf{I})^{-1}\mathbf{G}_{\Delta u}^{\mathrm{T}}(\mathbf{w} - \mathbf{f}_{\Delta u}) \tag{B33}$$

The last equation represents the standard GPC controller [10].

ACKNOWLEDGEMENTS

Doris Saez wishes to thank Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile for the support given to the project DI I2-03/14-2 'Design of supervisory optimal control strategies for multivariable non linear systems'.

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