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Valuation of users' benefits in transport systems

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Transport projects involve costs and benefits. Benefits to users appear in the form of more and/or better trips. Once the neoclassical idea of demand is accepted, the variation of utility levels underlie the measurement of benefits. In the evaluation process, benefits have to be compared with costs, and this can be done converting utility into monetary units. This paper deals with the treatment of this problem, starting with the general relation among utility, demand and the various forms of consumers' surplus, to move further into the particular forms that these relations take in the transport field. The rule-of-a-half is followed from the intuitive initial justification to a strict (and general) analytical derivation. More rigorous forms of users' surplus variation are then presented for fairly general cases, including both aggregate and disaggregate transport demand models, emphasizing the manner in which welfare measures are derived in each case. Discussion is centred around the comparative advantages and limitations of available approaches, searching for improvements in demand formulation and benefits measurement.

1. Introduction

Other things being constant, cheaper, faster, safer or more comfortable forms of transport make people feel better off. This is a subjective perception which supposedly can show up in disguise, hidden behind people's attitudes. On the other hand, improving transport systems requires additional resources which could have been assigned to other activities. Benefits of better transport are behind the former phenomenon; costs are behind the latter. Happiness on one hand, resources on the other. This article deals with the various forms of assigning money values to utility, developed in the transport field as part of the process through which transport projects are evaluated. Proper valuation of users' benefits is a relevant task from at least two viewpoints. First, it has to be accounted for since the ultimate objective of projects is to improve people's satisfaction; as an example, transport services of low quality (minimum cost) would be the undisputed result of project appraisal in areas where users are captive, if individuals' utility was neglected. On the other hand, currently applied measures of users' benefits may present some ambiguity depending on the importance of income in the behaviour of users of the transport system; since this phenomenon is likely to be present in, at least, most developing countries, it is worth discussing carefully the analytical framework in order to understand better the type of assumptions which underlie present practice and to be able to judge the impact they may have.

Whether individual demand for goods and services is actually the result of an optimization process or is the mechanical reflection of socially determined constraints will not be discussed here. However, once a process of individual choice is accepted or assumed, the existence of a subjective perception of feasible consumption states, which are preferred to others, cannot be denied. It is true, though, that it may be regarded as unimportant or unpredictable. The choice possibility will be admitted here and, accordingly, the paper will deal with the only operational approach developed around individual preferences and observed attitudes: the neo-classical microeconomic approach, which links utility to demand. This approach not only accepts choice, but also that individuals do what is best for them: Turvey's 'customer is always right' (Turvey 1971). This is presented in the next section, where money equivalents of utility are introduced in a non-traditional way, i.e. the popular consumer's surplus is relegated to the end; strictness is favoured.

In section 3 the concept of transport demand is analysed in relation to the operational nature of welfare measures. It contains a complete analysis of the most popular tool in the area of users' benefits: the rule-of-a-half, and the more strict developments within the last decade are exposed. Finally, a critical assessment of available approaches to measure users' benefits is presented, emphasizing their strength, weakness and directions for research.

2. Turning utility into monetary units

2.1. *The neo-classical approach*

The problem of assigning money values to variations of utility have been widely treated in the economic literature. Nevertheless, there is still a lot of discussion to be carried out, especially about the strictness and/or the operational value of the different money measures.

Starting from the optimization problem that is assumed to represent consumers' behaviour in the neoclassical theory (problem A below), and using the (so-called) 'dual' of this problem (problem B below), three different forms of assigning money measures to variations of utility are derived, under a general variation of the price vector.[1]

The following notation will be used at an individual (or family) level:

$\mathbf{X}=\{X_i\}$, vector of goods and services consumed in a period.
$U=U(\mathbf{X})$, utility function.
$\mathbf{P}=\{P_i\}$, vector of prices of goods and services.
$I=$individual (or family) income.

The following problem and its solution represents consumers' behaviour.

Problem A

$$\begin{aligned}&\text{Max } U(\mathbf{X}) && \text{Solution: } \mathbf{X}=\mathbf{X}^*(\mathbf{P},I) \text{ (demand functions)}\\ &\text{s.t.: } \mathbf{P}\mathbf{X}^{\mathrm{T}}\leqslant I && \text{Optimum: } U[\mathbf{X}^*(\mathbf{P},I)]=V(\mathbf{P},I) \text{ (indirect utility function).}\\ &\qquad X_i\geqslant 0 &&\end{aligned}$$

Problem A states that, given prices and income, the individual searches for a bundle of goods and services which maximizes its utility as he or she perceives it. The amount the individual prefers is dependent on prices of all goods and income: a demand function. The maximum utility he or she can reach is that which corresponds to the preferred bundle, thus indirectly dependent on prices and income. This indirect utility function will be shown extremely useful when defining welfare measures.

A second problem, which is said to be dual to A, leads to interesting results.

[1] The mathematical properties and conditions that have to be fulfilled by the economic functions that will appear in this section, will not be listed unless strictly necessary for welfare analysis. For a full description of such properties, Varian (1978) and Malinvaud (1969) are the best references.

Problem B

Min $\mathbf{PX}^{\mathrm{T}}$ Solution: $\mathbf{X} = \mathbf{X}^c(\mathbf{P}, \bar{U})$ compensated demand

s.t.: $U(\mathbf{X}) \geqslant \bar{U}$ Optimum: $\mathbf{PX}^c(\mathbf{P}, \bar{U}) = e(\mathbf{P}, \bar{U})$ expenditure function.

$X_i \geqslant 0$

Here, utility level is given and the wanted bundle **X** is that which requires the minimum expenditure. Optimal quantities now depend on prices and on the utility level previously set as minimum acceptable. Therefore, the minimal necessary expenditure is a function of prices and utility. Using a basic property of optimization problems, it can be shown that

$$\frac{\partial e(\mathbf{P}, U)}{\partial P_i} = X_i^c(\mathbf{P}, U).^2 \tag{1}$$

The relation between problems A and B is presented graphically in figure 1. From this, it is clear that the inverse of $U = V(\mathbf{P}, I)$ in I is precisely $I = e(\mathbf{P}, U)$.

By definition, the maximum utility an individual can reach with an income equal to the minimum necessary to reach a level U at given prices, is precisely U, i.e.

$$V[\mathbf{P}, e(\mathbf{P}, U)] \equiv U \tag{2}$$

differentiating both sides of identity (2) with respect to P_i yields

$$X_i = -\frac{\partial V / \partial P_i}{\partial V / \partial I} \quad \text{(Roy's identity)} \tag{3}$$

These are all the concepts which are necessary to move into the fuzzy area of welfare.

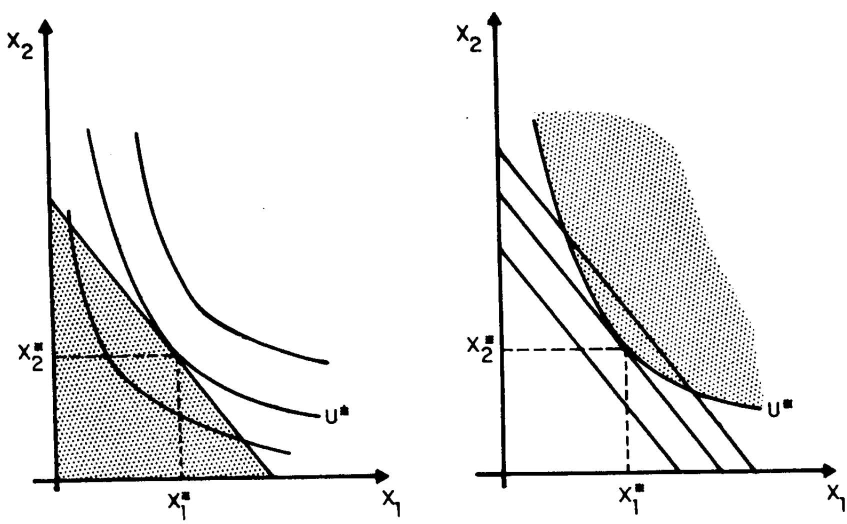

Figure 1. Utility maximization and expenditure minimization.

[2] A non-strict proof of equation (1) is given in Diamond and McFadden (1974), where other properties of the expenditure function are also explained. A strict derivation can be done applying the envelope theorem.

2.2. *The compensating and equivalent variations*

If the set of prices changes from $\mathbf{P}^0$ to $\mathbf{P}^1$, the bundle of goods consumed changes from $\mathbf{X}^0$ to $\mathbf{X}^1$, and the level of utility varies from U_0 to U_1. Money spent is the same, but utility differs. How can the difference $U_1 - U_0$ be measured in monetary terms? Hicks (1956) gave two strict answers to this question. The equivalent variation, EV, was defined as the change in income that provokes the same effect on *utility* as the price change. In the notation used here

$$U_1 = V(\mathbf{P}^1, I) = V(\mathbf{P}^0, I + EV) \tag{4}$$

It is useful to show the relation between EV and demand. This can be done making use of the expenditure function. Taking the inverse in equation (4),

$$I = e(\mathbf{P}^1, U_1) \quad \text{and} \quad I + EV = e(\mathbf{P}^0, U_1), \tag{5}$$

therefore

$$EV = e(\mathbf{P}^0, U_1) - e(\mathbf{P}^1, U_1) \tag{6}$$

Finally, using equation (1), it is easy to show that expression (6) can be turned into

$$EV = -\int_{\mathbf{P}^0}^{\mathbf{P}^1} \sum_i X_i^c(\mathbf{P}, U_1)\, dP_i{}^{3} \tag{7}$$

This shows that EV can be understood as the sum of areas to the left of compensated demands with utility held constant at the level of U_1.

The second Hicksian answer is the compensating variation, CV, which is the change in income that exactly offsets the effect of the price variation on utility, i.e.

$$U_0 = V(\mathbf{P}^0, I) = V(\mathbf{P}^1, I - CV) \tag{8}$$

such that CV is positive if prices diminish. Following the same procedure as before,

$$I = e(\mathbf{P}^0, U_0) \quad \text{and} \quad I - CV = e(\mathbf{P}^1, U_0)$$

$$\therefore CV = e(\mathbf{P}^0, U_0) - e(\mathbf{P}^1, U_0)$$

$$CV = -\int_{\mathbf{P}^0}^{\mathbf{P}^1} \sum_i X_i^c(\mathbf{P}, U_0)\, dP_i \tag{9}$$

As EV, CV is also the sum of areas to the left of compensated demands, at a different utility level (U_0).

Why not stop at this point? Both EV and CV are unambiguous income-like equivalents to utility changes and the problem seems to be solved. Unfortunately, it is not; at least, not exactly. Neither utility nor compensated demands can be observed. Thus, equations (4), (7), (8) or (9) seem only to be nice but useless constructions, unlikely

[3] Proof: for U constant, the differential of $e(\mathbf{P}, U)$ is

$$de(\mathbf{P}, U) = \sum_i \frac{\partial e}{\partial P_i} dP_i;$$

using equation (1) and integrating:

$$e(\mathbf{P}, U)|_{\mathbf{P}^1}^{\mathbf{P}^0} = -\int_{\mathbf{P}^0}^{\mathbf{P}} \sum_i X_i^c(\mathbf{P}, U) dP_i$$

to be of any help in practice. Whether this is true or not will be looked at soon. But normal market demand functions can be observed: amount of goods, prices and income are real variables, that can be measured and used for statistical demand estimation. This leads directly to the most popular (and attacked) device to assess consumer's benefits.

2.3. *The (Marshallian) consumer's surplus*

Marshall (1920) defined consumer's surplus as 'the excess of the price which [the consumer] would be willing to pay rather than go without the thing, over that which he actually does pay'. So the concept was born in terms of one good and its price. The classical textbook drawing represents the Marshallian consumer's surplus (MCS) in the (P_i, X_i) space, as the area below the demand curve, above the actual price level. This is said to reflect total willingness to pay minus actual payment. If P_i varies from P_i^0 to P_i^1, then the MCS changes in

$$\Delta MCS = -\int_{P_i^0}^{P_i^1} X_i dP_i \tag{10}$$

Note that equation (10) requires all other prices (and income) to remain constant, since demand was shown to depend on all these variables (Problem A). Hotelling (1938) provided a generalization of the consumer's surplus measure to variations in more than one price, proposing a line integral

$$\Delta MCS = -\int_{\mathbf{P}^0}^{\mathbf{P}^1} \sum_i X_i(\mathbf{P}, I) dP_i \tag{11}$$

In contrast to the equivalent and compensating variations measures, the quantities MCS and ΔMCS do not have a rigorous justification. Intuitive or not, it is not casual that the three measures can be expressed as line integrals (equations (7), (9) and (11)) which are very similar. If only one price changes, EV, CV and ΔMCS can be easily represented graphically as in figure 2 for a price reduction.

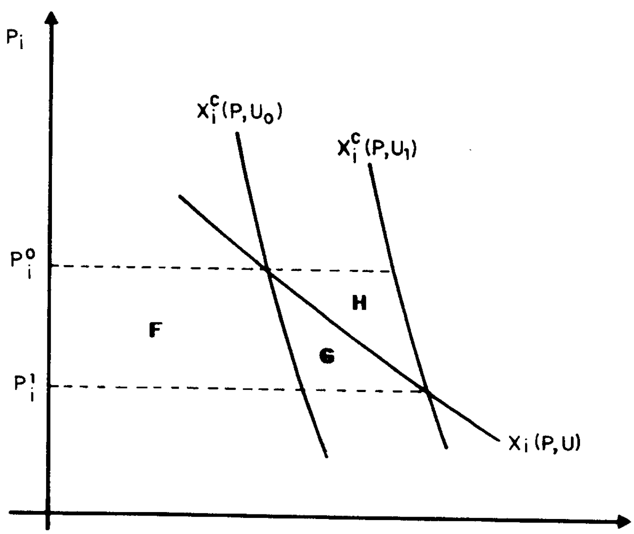

Figure 2. The relation between demand and money measures of utility variation after one price reduction. $EV = F + G + H$; $CV = F$; $\Delta MCS = F + G$.

Another form of viewing ΔMCS permits a less intuitive justification, and generates some further insights. Here is where Roy's identity (3) becomes particularly useful; replacing it in equation (11) and *assuming $\partial V/\partial I$ is constant* (constant marginal utility of income) and equal to λ, ΔMCS becomes

$$\Delta MCS = -\int_{\mathbf{P}^0}^{\mathbf{P}^1} \sum_i \left(-\frac{1}{\lambda}\frac{\partial V}{\partial P_i}\right) d P_i = \frac{1}{\lambda}[V(\mathbf{P}^1, I) - V(\mathbf{P}^0, I)] \tag{12}$$

This expression clearly shows that ΔMCS has a direct relation to utility variation.

Up to this point, the only expression that looks operational, i.e. that allows numerical calculation, is equation (11). It is stated in terms of market demands which are observable, and can be estimated and integrated; but this poses yet another problem. For line integral (11) to have a unique value, market demands have to fulfil the following conditions (Green's theorem):

$$\frac{\partial X_i}{\partial P_j} = \frac{\partial X_j}{\partial P_i} \quad i \neq j \tag{13}$$

which would not be usually present.[4] Thus, in general, the value of the only expression that appears operative is dependent on the path of integration from $\mathbf{P}^0$ to $\mathbf{P}^1$. Note that this is no problem in the line integrals for EV and CV (equations (7) and (9)), since at any level of utility

$$\frac{\partial X_i^c(\mathbf{P}, U)}{\partial P_j} = \frac{\partial(\partial e/\partial P_i)}{\partial P_j} = \frac{\partial^2 e}{\partial P_i \partial P_j} = \frac{\partial(\partial e/\partial P_j)}{\partial P_i} = \frac{\partial X_j^c(\mathbf{P}, U)}{\partial P_i} \tag{14}$$

which indicates that the result is unique in each case.

Undoubtedly, equation (12) provides ΔMCS with a more solid defence than pure intuition. Moreover, it is a relation that tolerates changes in all prices and is path-independent, as EV and CV. However, equation (12) was generated assuming constancy of the marginal utility of income between $\mathbf{P}^0$ and $\mathbf{P}^1$. In this sense, then, ΔMCS is less strict than EV and CV as money measure of ordinal preferences.[5] Willig (1976) set bounds to the difference (percentage) between ΔMCS and each of the 'sane' measures EV and CV, showing that the relative error is given by $\eta \Delta MCS/2I$, where η is the income elasticity of demand. This shows that ΔMCS may be a good approximation as a benefit measure, provided price variations are small and the consumption of the corresponding goods and services are relatively insensitive to income level.

The main advantage of ΔMCS, namely that it is related to the (observable) market demands, was weakened by the argument of Hausman (1981), who called attention to the fact that the indirect utility function can be eventually recovered directly from an estimated market demand, using equation (3) (Roy's identity). In fact, equation (3) can be seen as a differential equation in P_i and I, for a given (estimated) form of $X_i(P_i, I)$. This procedure seems operational for some forms of the demand equation and one price variation, but it can get extremely difficult (or impossible) to solve under other conditions.

Finally, it is important to note that the properties of both the expenditure and indirect utility functions can be extended to the case of discrete choice and quality changes (Small and Rosen 1981), which will be used later in this paper.

[4] Using the Slutsky equation one can show that condition (13) imply identical income elasticities of demand.

[5] For a good discussion on the relation among money measures of utility, ordinality and cardinality of preferences, see Morey (1984).

3. Consumers' benefits in transport projects

3.1. *The role of transport demand*

When viewing transport markets within the framework described in the preceding section, the role of transport demand as the basis for the valuation of users' benefits becomes obvious: it provides succintly the information on users' behaviour, captured from actual observations, which can be manipulated and converted into some monetary measure of utility. However, price variations in transport markets induce changes in supply and/or demand in many other economic activities. This is particularly clear when transport is viewed as a factor of production, i.e. as a service which is necessary both to bring inputs to and to deliver output from a particular plant. In the urban case this is also true, from a similar viewpoint, for trips with very different purposes: work, study, shopping, entertainment, etc. Then a fundamental question arises: is it necessary to add eventual benefits induced by improvements in the transport system on other economic activities? Answers to this question have been given from different viewpoints in the literature. It is worth reviewing some of them.

Mishan (1976) warns against double counting when calculating benefits due to, for instance, the construction of a new railroad. '... if this new railroad so reduces the time and increases the convenience of travel as to offer new job opportunities to a number of men, we ought *not* to include the measure of these new rents (a measure of the increase in their welfare from switching to the new jobs) as *additional* benefits. For such benefits are already subsumed in the (potential) consumers' surplus of the new railroad. Such a measure of consumers' surplus (approximated, say, by an estimate of the potential demand schedule for train journeys per annum) reveals the maximum sum each person will pay for a number of train journeys. And in determining this maximum sum, he will take into account the rents of the new jobs and, indeed, all other incidental utilities and disutilities accruing to him from the new railroad service' (p. 79).

Similarly, Mohring (1976) analyses the cost reduction achievable by substituting transport for manufacturing inputs, following a reduction in unit transport cost. He shows that a consumers' surplus type measure in the firm's transport demand schedule accounts for *all* benefits accruing to the firm. In fact, Mohring's is a particular case of the general problem regarding the relation between factor and final goods markets, treated by Carlton (1979) and, in a very strict form, by Jacobsen (1979).

The whole problem of eventual double counting rests finally on the derived nature of transport demand, whether one views that demand at an individual level, firm level or market level. In general, transport demand is determined by the spatial distribution of activities, and this is recognized by most (if not all) available models. In a recent study, Jara-Díaz (1986) explores the relation between users' benefits and the economic effects of transport improvements. An aggregated transport demand is shown to be obtainable from a description of economic activities in different zones in terms of supply and demand for goods or services. The derivation of willingness to pay for transport is accomplished both in competitive and monopolistic environments, extending the results to mixed situations. It is then shown that, at a market level, transport consumers' surplus indeed reflects the net sum of gains and losses of all producers and consumers, and it does it exactly in a competitive environment, and approximately in a monopolistic one.

Thus, transport demand is not only necessary but sufficient to account for all benefits provoked by improvements in the transport system. In the following subsections it will be shown that the role of transport demand as the basis for assessing users' benefits has been actually a very passive one, without taking full advantage of the (presumably well represented) underlying users' perceptions.

3.2. *The rule-of-a-half: from intuition to rigour*

The rule-of-a-half (RH) is the most widely used form of measuring users' benefits in transport projects. It was supported, at first, on a purely intuitive argument (Neuberger 1971, Agnello 1977). Let T^0 and T^1 denote the number of trips between a given pair of zones (by a certain mode or alternative) in some initial and final situations, respectively. Let C^0 and C^1 be the corresponding unitary costs of those trips. It will be arbitrarily assumed that $C^1 < C^0$ and, therefore, $T^1 > T^0$. The intuitive reasoning begins by dividing users in two classes: those who remain travelling between the two zones, before and after the cost reduction, and the 'new' users. Obviously, there will be T^0 'old' users and $(T^1 - T^0)$ new ones. It follows directly that the old users' benefit is $T^0(C^0 - C^1)$. Furthermore, a new user cannot perceive a benefit greater than $(C^0 - C^1)$, nor less than zero. Then, if a linearity assumption is made for the individual benefit of the *new* users, the total benefit for them will be $(T^1 - T^0) \times \frac{1}{2}(C^0 - C^1)$. So, the total consumers' surplus variation can be written as

$$\Delta MCS \simeq T^0(C^0 - C^1) + (T^1 - T^0)\tfrac{1}{2}(C^0 - C^1) \tag{15}$$

which simplifies to the well-known expression of the RH for one pair of origin–destination zones and one mode:

$$\Delta MCS \simeq \tfrac{1}{2}(T^0 + T^1)(C^0 - C^1) \tag{16}$$

A graphical interpretation of this argument is given in figure 3. The Marshallian consumers' surplus is represented here by the area C⁰–A–C–B–C¹ (joining points A and B through the demand curve). The RH quantifies the area C⁰–A–C′–B–C¹ (joining A and B by a straight line). Obviously, the less curved the demand, the better the approximation obtained with the RH. In other words, this figure tells that the RH is a good measure of user's benefit when dealing with marginal changes of costs.

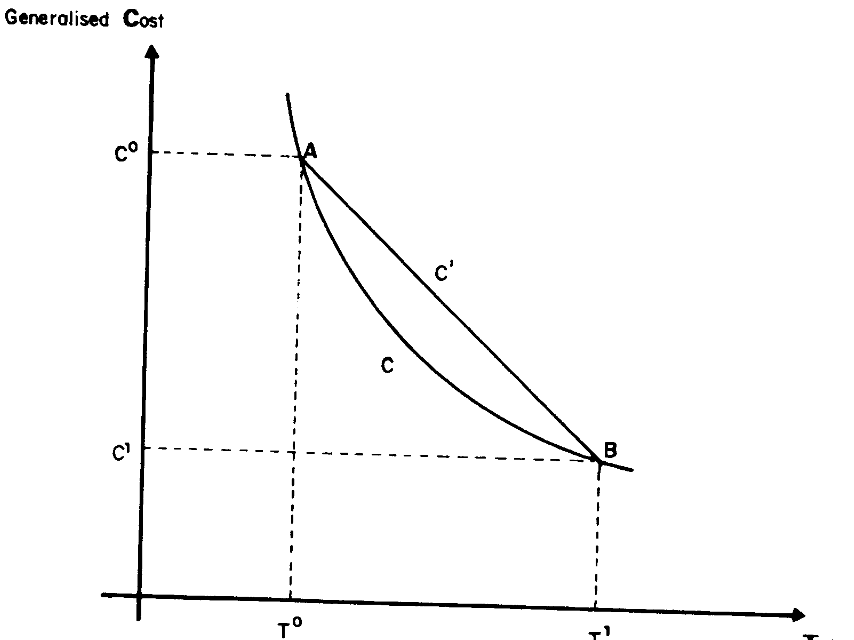

Figure 3. Graphical interpretation of the rule-of-a-half for the simple case of one mode and one pair of origin–destination zones.

In order to obtain a more general expression, let T_{ijk} be the number of trips from zone i to zone j by mode k, and let C_{ijk} be the unitary cost of a trip from zone i to zone j by mode k. If n is the total number of zones and M the total number of modes available for users of the group under analysis, it seems clear that the RH may be rewritten as:

$$\Delta MCS = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{M} (T^0_{ijk} + T^1_{ijk})(C^0_{ijk} - C^1_{ijk}) \tag{17}$$

where superscripts 0 and 1 refer to the initial and final situations.[6] However, an intuitive justification of equation (17) requires some further elaboration of the previous reasoning (Neuberger 1971). Changes may occur in several interzonal costs, and in one or more modes, while demand for trips by a given mode between a given pair of zones depends, in general, on the perceived costs of the other modes that serve not only that origin–destination pair, but other pairs as well. For each mode and zone pair, users can again be divided into two classes: those who remain travelling between the same origin and destination by the same mode, and those who modify their behaviour responding to the change. For trips between origin i and destination j, users of the first type will perceive benefits given by

$$T^0_{ijk}(C^0_{ijk} - C^1_{ijk}) \tag{18}$$

since T^0_{ijk} is the number of users that do not change their choice. The second part of the benefits accrue to those who do change. In order to simplify the explanation, consider the particular case of those users who travel from i to j by mode a before the change, and from i to h by mode b afterwards. These users will appear twice in the expression (17), as part of both T^0_{ija} and T^1_{ihb}. Assuming for simplicity that $(C^0_{ihb} - C^1_{ihb}) > (C^0_{ija} - C^1_{ija})$, benefits for this type of users cannot be larger than $(C^0_{ihb} - C^1_{ihb})$, nor less than $(C^0_{ija} - C^1_{ija})$. If benefits are assumed to lie halfway between these two extremes, it is easy to obtain expression (17) by simple addition of the two types of benefits for all origin–destination pairs and modes.

This result can also be expressed in terms of flows and costs on links of the corresponding network, i.e.

$$\Delta MCS = \frac{1}{2}\left[\sum_{k \in K} \sum_{m=1}^{M} (N^0_{km}C^0_{km} + N^1_{km}C^0_{km}) - \sum_{l \in L} \sum_{m=1}^{M} (N^0_{lm}C^1_{lm} + N^1_{lm}C^1_{lm})\right] \tag{19}$$

where

N_{im} = number of trips on link i by mode m,
C_{im} = cost of travelling along link i by mode m,
K = set of links in the base network,
L = set of links in the modified network,
M = number of modes available for the group under analysis.

[6] Strictly, the number of modes available for users of a certain socio-economic group, varies from one pair of origin–destination zones to another. The expression (17) must be:

$$\Delta MCS = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{M_{ij}} (T^0_{ijk} + T^1_{ijk})(C^0_{ijk} - C^1_{ijk})$$

where M_{ij} denotes the number of modes available for the group under analysis, for a trip from zone i to zone j. Nothing essential is lost with the (simpler) treatment given in the text.

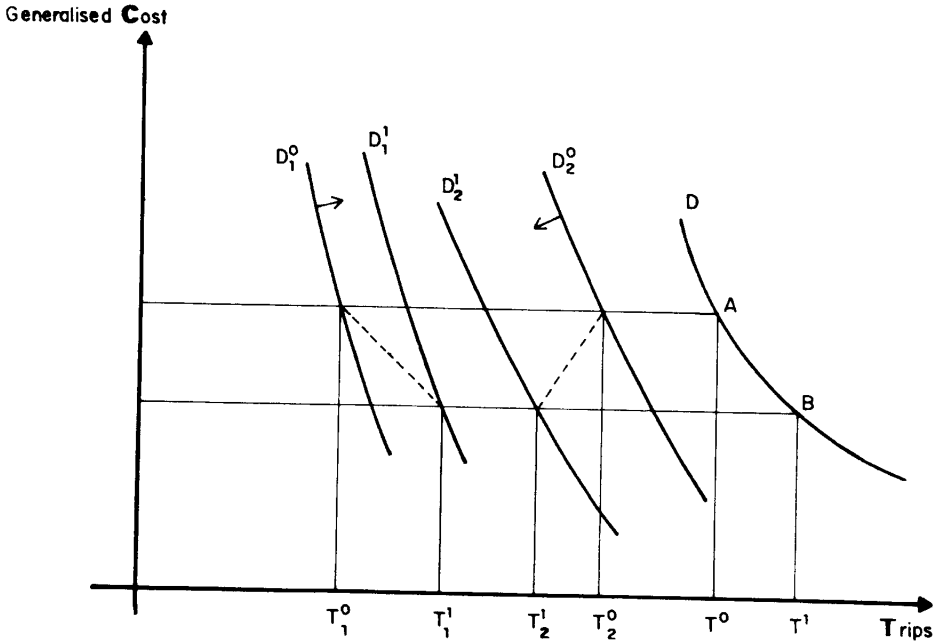

Figure 4. Graphical interpretation of the rule-of-a-half for the case of two competing modes. D = aggregate demand; D_i^0 = modal demand for mode i, before the change; and D_i^1 = modal demand for mode i, after the change.

A graphical analysis of expression (19) is somewhat complicated. Jara-Díaz and Friesz (1982) developed a method to obtain modal demands from aggregated demand for trips between a certain origin–destination pair, imposing the condition that perceived costs of all modes m, such that $T_{ijm} > 0$, are equal. They extend the analysis to several interrelated demands, and show unambiguously how modal demand curves must shift, given a set of cost changes. The simple case of two substitutable modes between a certain origin destination pair is illustrated in figure 4, where a reduction in perceived costs of travelling by mode 1 occurs (aggregate and modal supply curves are omitted in this figure).

All of these developments and reasonings contribute to give a sounder theoretical base to the RH, but to data, it still retains most of the intuitive base of its beginnings. Williams (1976) brought strictness to the derivation.

Starting from Hotelling's integral (11), Williams (1976) derived strictly the expression of the RH, clearly stating the assumptions behind it. In the one mode case, integrability conditions (13) may be expressed as:

$$\frac{\partial T_{ij}}{\partial C_{kl}} = \frac{\partial T_{kl}}{\partial C_{ij}} \tag{20}$$

Provided that integrability conditions hold, one can arbitrarily choose an integration path, because the value of Hotelling's integral is unique due to path independency. The linear path from C^0 to C^1 can be parametrically defined as

$$L(\sigma) = (l_{11}(\sigma), \ldots, l_{ij}(\sigma), \ldots, l_{nn}(\sigma)) \tag{21}$$

$$l_{ij}(\sigma)=C^{0}_{ij}+\sigma(C^{1}_{ij}-C^{0}_{ij}) \tag{22}$$

$$L(\sigma=0)=(C^{0}_{ll},\ldots,C^{0}_{ij},\ldots,C^{0}_{nn})=\mathbf{C}^{0} \tag{23}$$

$$L(\sigma=1)=(C^{1}_{ll},\ldots,C^{1}_{ij},\ldots,C^{1}_{nn})=\mathbf{C}^{1} \tag{24}$$

The demand for trips from zone i to zone j depends upon all interzonal costs, $(C_{11},\ldots,C_{nn})$, both before and after the cost changes:

$$T_{ij}=T_{ij}(C_{11},\ldots,C_{ij},\ldots,C_{nn}) \tag{25}$$

The Hotelling's integral can be written as:

$$\Delta MCS=-\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{\substack{C^0\\L(\sigma)}}^{C^1} T_{ij}(C_{11},\ldots,C_{ij},\ldots,C_{nn})dC_{ij} \tag{26}$$

or, changing variables,

$$\Delta MCS=-\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{\sigma=0}^{\sigma=1} T_{ij}(l_{11}(\sigma),\ldots,l_{ij}(\sigma),\ldots,l_{nn}(\sigma))\frac{dl_{ij}(\sigma)}{d\sigma}d\sigma \tag{27}$$

Calling $T_{ij}(L(\sigma))=T_{ij}(\sigma)$, noting that $dl_{ij}(\sigma)/d\sigma=C^{1}_{ij}-C^{0}_{ij}$, and expanding $T_{ij}(\sigma)$ in a Taylor series around $\sigma=0$, equation (27) becomes

$$\Delta MCS=\sum_{i=1}^{n}\sum_{j=1}^{n}(C^{0}_{ij}-C^{1}_{ij})\int_{\sigma=0}^{\sigma=1}\left[T_{ij}(\sigma=0)+\sigma\frac{dT_{ij}}{d\sigma}\bigg|_{\sigma=0}+\tfrac{1}{2}\sigma^{2}\frac{d^{2}T_{ij}}{d^{2}\sigma}\bigg|_{\sigma=0}+\ldots\right]d\sigma \tag{28}$$

Neglecting terms of second and higher order, which account for curvature effects in $T_{ij}(\sigma)$, and approximating $(dT_{ij}/d\sigma)|_{\sigma=0}$ by $(T^{1}_{ij}-T^{0}_{ij})$, where $T^{0}_{ij}=T_{ij}=T_{ij}(\sigma=0)$ and $T^{1}_{ij}=T_{ij}(\sigma=1)$, the final result is obtained:

$$\Delta MCS=\sum_{i=1}^{n}\sum_{j=n}^{n}(C^{0}_{ij}-C^{1}_{ij})T^{0}_{ij}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(C^{0}_{ij}+C^{1}_{ij})(T^{1}_{ij}-T^{0}_{ij}) \tag{29}$$

$$\Delta MCS=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(T^{0}_{ij}+T^{1}_{ij})(C^{0}_{ij}-C^{1}_{ij}) \tag{30}$$

Expression (30) is the RH for one mode of travel, deduced rigorously from the Hotelling's integral. It is important to identify clearly the assumptions that underlie this derivation:

(*a*) integrability conditions;
(*b*) series expansion of the function $T_{ij}(\sigma)$ around $\sigma=0$, neglecting terms of second and higher order; and
(*c*) approximation of $(dT_{ij}/d\sigma)|_{\sigma=0}$ by $(T^{1}_{ij}-T^{0}_{ij})$.

These two latter conditions indicate that the RH is favoured as a good approximation of users' benefits, by the absence of second (or higher) order effects of fares on demand, and by small variations of fares or perceived users' costs.

In summary, the RH can be seen as a simple and operational tool to assess users' benefits. It can be applied even without knowledge about the underlying demand functions, since the only information that is required to perform the calculations is contained in the set of variables that describe market equilibrium with and without the project. But this property arises only as the nice face of the coin since first derivatives of market demands had to be assumed constant. Besides, the RH is born, either intuitively or strictly, directly from the least rigorous form of money valuation of utility: the MCS. Thus, its validity further requires the assumptions behind the MCS, summarized by equations (12) and (13). Being an approximation of a non-strict measure of utility, the RH looks conceptually vulnerable once its foundations have been unveiled. But the most important shortcoming is of a more fundamental nature: the use of the RH tends to relegate to a secondary place the important relation between people's attitudes and the measure of utility. In other words, the form in which transport demand is understood and modelled should play a role in the valuation of users' benefits.

3.3. *Transport demand and more rigorous forms of users' surplus*

When some form of transport demand model is used to perform the calculation of users' benefits, it is clear that the goodness of such measure will depend not only on the strictness behind the derivation of welfare measures, but also on the quality of the demand model itself. In this subsection, emphasis will be given to users' surplus calculations, although some discussion on demand is also included.

Assume a demand model has been estimated, either at a distribution level, modal split or a combination of both. Then many possibilities for the calculation of users' benefits arise. Of course, one can always choose the RH formula, using the demand model only to predict equilibrium states. A second possibility is to use direct integration from equation (27), i.e.

$$\Delta MCS = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{\sigma=0}^{\sigma=1} T_{ij}(\sigma)(C_{ij}^{1}-C_{ij}^{0})d\sigma \tag{31}$$

If $T_{ij}(C)$ fulfils integrability conditions (20), the value of the line integral version of ΔMCS is unique and the linear path of integration in equation (31) yields the desired result. If the demand model does not meet conditions (20), then equation (31) can still be used as an operative approximation.

Yet a third possibility is to somehow look for an indirect utility function V which fulfils Roy's identity, i.e.

$$T_{ij} = -\frac{\partial V/\partial C_{ij}}{\partial V/\partial I} \tag{32}$$

Once V has been found, the expenditure function can be obtained as shown in section 2, from which exact measures of welfare as CV and EV can be derived.

The actual form to be chosen for valuing users' benefits will depend upon the form, assumptions and derivation of a demand model.

The history of demand modelling is, as in many other fields, a combination of efforts with different objectives, eventually convergent: good fit, policy sensitiveness, theoretical soundness or clean economic basis. Until today, urban transport planning models use some form of the four-steps traditional approach: generation–attraction, distribution, modal split and assignment. Although not in an urban context, demand has been also modelled in a non-sequential fashion;[7] in this case, it is not unusual to find a set of equations representing demand for each mode on every origin–destination zone pair.

In the particular case of one mode and one pair of zones, or one mode treated generically (e.g. trips), ΔMCS can be calculated directly from equation (10). Just as an

[7] For a synthetic and qualitative review of transport demand models, chapter 2 of Domencich and McFadden (1975) is still a good reference.

example, the demand for auto trips to work reported by Thomson in 1967 (in Thomson 1974) can be taken. The demand model is

$$T_a = 50118{\cdot}72\,(C_a - 50)^{-1{\cdot}66}, \tag{33}$$

where C_a represents a cost (price) index that was created to take into account differences in distances and routes, in order to explain the total number of trips using car, T_a.

The exact value of ΔMCS can be easily shown to be given by

$$\Delta MCS = 75937{\cdot}46[(C_a^0 - 50)^{-0{\cdot}66} - (C_a^1 - 50)^{-0{\cdot}66}] \tag{34}$$

The demand model indicates that cost indices of 200, 100 and 80 generate approximately 12, 76 and 177 car trips respectively. For a drop of C_a from 200 to 100, the RH yields 4402 units of benefits, a gross overestimation of ΔMCS whose value is only of 2962 units. However, when C_a drops from 100 to 80 (which increase demand in a greater number) the RH gives a figure of 2528 while the exact value of ΔMCS is 2303. The whole problem depends only upon the fact that equation (33) is extremely convex at low levels of demand, and nearly a straight line in the medium range. In short, the second case fulfils two important conditions for RH to be a good approximation of users' benefits: small curvature of demand, and little variation of the perceived cost.

But transport demand models evolved enormously since the end of the sixties, particularly in urban studies. After a whole family of more or less *ad hoc* gravity type models, the idea of entropy acquires a respectable status as the most distinguished member of that family.

Entropy appears to be a powerful method to overcome microscopic complexities when only aggregate data is available, although it is important to note that the entropy concept can be applied in a disaggregate framework as well (see, for example, Anas 1983). The whole concept is constructed upon a probabilistic basis. Assume that the total number of trips among a set of zones is known; then that number can be accommodated in an origin–destination matrix in many possible manners. However, if one trip is regarded as interchangeable with all other trips, then the same *numbers* can be generated from different (micro) configurations, by simple permutation of trips among cells (O–D pairs). If the same probability is assigned to each distinguishable micro-configuration, then the most likely set of numbers in the O–D matrix is that which maximizes the amount of possible micro-states or a monotonical transformation of it. As known, one of these possible transformations is similar to the entropy of a probabilistic system, which generates the name of the approach. If aggregate information is available, then the function representing the entropy can be maximized within the combination of matrix elements which fulfil the associated aggregate relations, i.e. a constrained maximization.

It is well known (see, for example, Wilson 1967) that the number of possible micro-states that generates the same matrix of O–D trips may be expressed as[8]

$$\omega(T_{ij}) = \frac{T!}{\prod_{i,j} T_{ij}!} \tag{35}$$

where

T_{ij} = total number of trips from zone i to zone j.
$\omega(T_{ij})$ = total number of micro-states that generates the same $\{T_{ij}\}$ matrix,
T = total number of trips.

[8] An extension to several modes and types of users can be found in Wilson (1969).

The maximization of $\omega(T_{ij})$ can be achieved by maximizing the objective function F', a monotonical transformation of it, where

$$F' = -\sum_{ij} T_{ij} \ln T_{ij} \tag{36}$$

It can be shown that, adding and subtracting convenient constants to F', the following objective function can be obtained:

$$F = -\sum_{ij} T_{ij}\left(\ln \frac{T_{ij}}{O_i D_j / T} - 1\right) \tag{37}$$

where

O_i = total number of trip origins in i,
D_j = total number of trip destinations in j.

Function F', as a measure of entropy, is more general than F, because this latter requires vectors $\{O_i\}$ and $\{D_j\}$ as additional data.

Depending on the available information, several constraints can be imposed to the maximization problem:

$$\sum_j T_{ij} = O_i \tag{38}$$

$$\sum_i T_{ij} = D_j \tag{39}$$

$$\sum_{ij} T_{ij} = T \tag{40}$$

$$\sum_{ij} C_{ij} T_{ij} = C \tag{41}$$

$$T_{ij} \geqslant 0 \tag{42}$$

where

C_{ij} = unitary cost of travelling between zones i and j,
C = total cost.

Total cost constraint (41) is always necessary, if the elements T_{ij} are assumed to depend on the interzonal trip costs, $\{C_{ij}\}$, i.e. if each T_{ij} is intended to be a demand function. Thus, the $\{C_{ij}\}$ matrix and total cost C are always required as input data. If F' is used as the objective function, there is no need of additional data. Of course, additional data, in the form of the appropriate constraints, would improve the reliability of the model. Constraints (38), (39), (40) and (41) can be imposed in any possible set including constraint (41) as an element. However, if

$$\sum_i O_i = \sum_j D_j = T,$$

equations (38), (39) and (40) will contain redundant information. The maximization of F, subject to each set of constraints, will generate a different demand model, as is pointed out by the table below.

Model	Constraints
Unconstrained	(41)
Total trip constrained	(40) and (41)
Production constrained	(41) and the family of (38) constraints
Attraction constrained	(41) and the family of (39) constraints
Doubly constrained	(41) and the families of (38) and (39) constraints

If a general problem is posed, with objective function F (equation (37)) and constraints (38), (39), (40) and (41), the solution, in terms of Lagrangian multipliers (dual variables) takes the form:

$$T_{ij} = O_i D_j \exp -(\alpha_i + \gamma_j + \rho + \phi C_{ij}) \tag{43}$$

where $\{\alpha_i\}$, $\{\gamma_j\}$, ρ and ϕ are the Lagrangian multipliers associated with constraints (38), (39), (40) and (41). As usual, the solution clearly makes the non-negativity constraints unnecessary.

It is easy to show that the dual of this problem may be written as the unconstrained minimization problem over the dual variables $\{\alpha_i\}$ $\{\gamma_j\}$, ρ and ϕ:

$$\operatorname{Min} Z = \sum_{ij} O_i D_j \exp -(\alpha_i + \gamma_j + \rho + \phi C_{ij}) + \sum_i \alpha_i O_i + \sum_j \gamma_j D_j + \rho T + \phi C \tag{44}$$

where ϕ can be viewed as the population sensitivity to transport costs. It is usually assumed that it does not depend upon the interzonal trip costs $\{C_{ij}\}$. If this assumption is made, and applying dual's first order conditions, it can be shown that:

$$\frac{\partial Z}{\partial C_{ij}} = -\phi T_{ij} + \phi \frac{\partial C}{\partial C_{ij}} \tag{45}$$

where T_{ij} is the primal solution expressed in terms of the dual variables, as in equation (43). From (45), T_{ij} can be rewritten as

$$T_{ij} = \frac{\partial C}{\partial C_{ij}} - \frac{1}{\phi} \frac{\partial Z}{\partial C_{ij}} \tag{46}$$

Deriving T_{ij} with respect to an arbitrary C_{kl}:

$$\frac{\partial T_{ij}}{\partial C_{kl}} = \frac{\partial^2 C}{\partial C_{kl} \partial C_{ij}} - \frac{1}{\phi} \frac{\partial^2 Z}{\partial C_{kl} \partial C_{ij}} \tag{47}$$

It is obvious that, with the assumption of $\partial\phi/\partial C_{ij} = 0, \forall i, j$, integrability conditions, i.e. $(\partial T_{ij}/\partial C_{kl}) = (\partial T_{kl}/\partial C_{ij})$, are satisfied. On the other hand, net Marshallian consumers' surplus can be evaluated using Hotelling's line integral between two cost situations $\mathbf{C}^0$ and $\mathbf{C}^1$, replacing T_{ij} by equation (46).

$$\Delta MCS = \sum_{ij} \int_{C^0_{ij}}^{C^1_{ij}} \left(\frac{1}{\phi} \frac{\partial Z}{\partial C_{ij}} - \frac{\partial C}{\partial C_{ij}} \right) dC_{ij} \tag{48}$$

$$\Delta MCS = \frac{1}{\phi} (Z^1 - Z^0) - (C^1 - C^0) \tag{49}$$

But optimum values of primal and dual problems must coincide, so (49) may also be expressed as (Williams 1976):

$$\Delta MCS = \frac{1}{\phi}(F^1 - F^0) + (C^0 - C^1) \tag{50}$$

Expression (50) links the entropy concept with consumers' surplus, assuming that the dual variable ϕ does not depend on costs $\{C_{ij}\}$, as is usually done. The preceding results can be extended to a distribution–modal split framework, basically keeping the same analytical properties in relation to welfare measures.

The aggregate entropy approach is indeed attractive from many viewpoints: it usually generates very good fits, it is certainly sensitive to pricing and other policies, and is the result of a clear and strict theoretical analysis. From the point of view of users' benefits, however, the entropy formulation as described here does not easily fit into the idea of choice and utility presented in sections 1 and 2. The form of T_{ij} in equation (43) undoubtedly looks like a market demand for trips, but was not constructed as such; thus, although one can actually derive ΔMCS as shown before, it is hard to link conceptually such a measure with utility, since the framework used is one of likely numbers and not one of individuals' preferences. This difficulty persists even in the disaggregate version of the entropy formulation, in spite of its compatibility with some models derived from a random utility framework (Anas 1983).[9] The family of discrete choice models presumably overcomes this problem, which should allow for a sounder welfare analysis.

In order to understand better the welfare implications of disaggregate transport demand models, it is worth presenting a version of the basic microeconomics behind them, since this approach is not generally known, which makes it difficult to understand fully welfare measures directly from a given specification. Problem A in section 2 can be restated for n (continuous) goods and one discrete good X_d (mode of transport). Then it can be solved in two steps, the first step *conditional* on mode choice, i.e.

$$\begin{aligned} &\max_{X_1,\dots X_n} U(X_1,\dots X_n, X_d) \\ &\text{subject to} \\ &\sum_{i=1}^{n} P_i X_i \leqslant I - C_d \\ &X_i \geqslant 0 \end{aligned} \qquad \textit{Problem C}$$

where X_d represents a mode that can be described by its characteristics $\{q_d\}$,[10] and C_d is the cost of using that mode. Problem C can be treated as problem A, but its solution is now a vector of *conditional* demand functions $\mathbf{X} = \mathbf{X}^*(\mathbf{P}, I - C_d, \{q_d\})$ which can be replaced in the utility function yielding a *conditional* indirect utility function

$$U[\mathbf{X}^*(\mathbf{P}, I - C_d, \{q_d\})] = V(\mathbf{P}, I - C_d, \{q_d\}) \tag{51}$$

[9] Anas (1983) showed that an entropy maximizing formulation in terms of trip probabilities, yield exactly the Logit specification if appropriate constraints are imposed. Thus, a better linkage between entropy and welfare may be expected in the near future.

[10] Description of goods or a service through its characteristics is an accepted procedure after Lancaster (1966).

As the mode should be chosen from a given finite set M, (second step) the individual maximizes utility choosing $Xb \in M$ such that

$$V(\mathbf{P}, I - C_b, \{q_b\}) > V(\mathbf{P}, I - C_i, \{q_i\}) \quad \forall i \neq b \quad \mathbf{X}i \in M \tag{52}$$

It should be noted that Roy's identity (3) holds even for the discrete good, since for the overall indirect utility function

$$V^* = V(\mathbf{P}, I - C_b, \{q_b\}) = \underset{\mathbf{X}i \in M}{\mathrm{Max}}\, V(\mathbf{P}, I - C_i, \{q_i\}) \tag{53}$$

$$\frac{\partial V^*}{\partial C_b} = \frac{\partial V}{\partial I} \frac{\partial (I - C_b)}{\partial C_b} = -\frac{\partial V}{\partial I} \tag{54}$$

$$\frac{\partial V^*}{\partial C_i} = \frac{\partial V}{\partial C_i} = 0 \quad \forall i \neq b \tag{55}$$

and

$$\frac{\partial V^*}{\partial I} = \frac{\partial V}{\partial I} \tag{56}$$

Therefore

$$-\frac{\partial V}{\partial C_i} \bigg/ \frac{\partial V}{\partial I} = \begin{cases} 1 & \text{if } i = b \\ 0 & \text{if } i \neq b \end{cases} \tag{57}$$

which is the individual's market demand for mode b (1) and for the other modes (0). Furthermore, if the conditional indirect utility function (51) is assumed to be linear in its arguments, then the comparison in (52) reduces to that portion of V which involves only cost and quality of modes.

This reduced function is usually labelled as mode i's utility (V_i) in the discrete mode choice jargon.[11] Here follows the usual treatment, assuming V_i can not be known with certainty, and can be expressed as the sum of a function U_i of the observed variables C_i and $\{q_i\}$, and a random error E_i. Therefore, the probability π_b of choosing mode b is given by the probability of $U_b + E_b$ being greater than $U_i + E_i$, $\forall i \neq b$. Then the actual form taken by that probability is dependent on the distribution assumed for the (random) error terms. What is a probability at an individual level, is a proportion of the population with similar characteristics and perceptions (i.e. with the same utility function). Let N be the size of that population. Then, as carefully shown by Small and Rosen (1981), the aggregate compensating variation after a change of transport prices or qualities which induce individual welfare changes from U_i^0 to U_i^1, is given by

$$CV = -\frac{N}{\lambda} \int_{\mathrm{U}^0}^{\mathrm{U}^1} \sum_{i=1}^{M} \pi_i(U_1, \ldots U_M) dU_i \tag{58}$$

This requires the marginal utility of income, λ, to be independent of prices and qualities of modes, and transport to be unimportant in the total consumer's expenditure (negligible income effects). This second condition is required to approximate the compensated demand by the market demand. Thus, equation (58) resembles equations (7), (9) and (11); moreover, given the assumptions behind it, equation (58) should be consistent with the procedure represented by equation (31).

[11] More general and less constraining conditions for V are stated in Small and Rosen (1981).

It is widely known that if the E_is are independently and identically distributed with the extreme value shape, the popular Logit formulation is obtained from the discrete choice approach. This means that

$$\pi_i = \exp(U_i) \Big/ \sum_{j=1}^{M} \exp(U_j) \tag{59}$$

with U_i usually specified linear in C_i and in each element of $\{q_i\}$.

Note that in this case, as in all modal split models, a given O–D pair is under analysis and welfare variations come from the change in individual modal choices following price and/or quality variations in one or more modes. Then, if one applies Williams' linear path procedure, checking integrability conditions should be made at a cross-mode level. Let T_i be the demand for trips on mode i; obviously $T_i = N\pi_i$. Then it is quite easy to prove that, for the Logit model

$$\frac{\partial T_i}{\partial C_j} = -N\theta\pi_i\pi_j = \frac{\partial T_j}{\partial C_i} \tag{60}$$

where θ is the coefficient of mode cost. Equation (60) means that integrability conditions hold and ΔMCS has a unique value. Applying equation (31), one gets (Williams 1977, Sasaki 1982)

$$\Delta MCS = \frac{N}{\Delta} \ln \sum_{i=1}^{M} \exp \Delta\bar{U}_i \Big|_{\bar{U}^0}^{\bar{U}^1} \tag{61}$$

where Δ is the parameter of the extreme value distribution and $\Delta\bar{U}_i$ is U_i from equations (58) and (59), in Williams–Sasaki notation.

On the other hand, valuing the compensating variation from equation (58) yields

$$CV = -\frac{N}{\lambda} \ln \sum_{i=1}^{M} \exp U_i \Big|_{U^0}^{U^1} \tag{62}$$

Still a third form of (directly) viewing the logarithm of the sum of the exponential of modal utilities (in short, the log-sum) as a welfare measure, is to propose the expression

$$\bar{V} = I - \frac{N}{\theta} \ln \sum_{i=1}^{M} \exp U_i \tag{63}$$

as an aggregate indirect utility function for that market segment. It is easy to check that $\bar{V}$ fulfils Roy's identity at an aggregated level, since

$$-(\partial\bar{V}/\partial C_k)/(\partial\bar{V}/\partial I) = \frac{N}{\theta} \frac{1}{\sum \exp U_i} \exp(U_k)\theta = N\pi_k = X_k \tag{64}$$

The three expressions (61), (62) and (63) are similar but not identical. It is worth asking whether they represent the same thing. In fact, they do. It is quite easy to see from equation (54) that, at an individual level, the marginal utility of income λ is given by the price coefficient θ of the utility functions, if specified linear. This makes equations (62) and (63) consistent. On the other hand, the utility level $\bar{U}_i$ in Williams–Sasaki notation is simply the so-called generalized cost with a negative sign; thus, to keep dimensions clear, $\bar{U}_i$ has to be expressed as $C_i + f(q_i)$, which makes the statistically unknown parameter Δ numerically equal to $-\theta$ (the coefficient of C_i) which results from the Logit estimation. Note that it should be no surprise that ΔMCS and CV yield the same result, since income is assumed to play no role whatsoever in modal choice.

The log-sum formula appears, then, as a fairly well-funded form of valuing users' benefits from Logit modal choice models. That it is a consistent measure of welfare can also be seen from its property as the expected maximum utility at any given level of a utility tree. Then the log-sum acts as the representative utility or composite cost when moving one level up in the (hierarchical) Logit formulation. This makes it very easy to extend the preceding results to a framework of mode–destination choice (Williams 1977, Sasaki 1982).

Finally, it is well known that a normal distribution of the error terms generates the Probit mode choice model. Although it is less popular than the Logit formulation, equation (58) can be applied illustratively to the binary case, where

$$\pi_1 = \Phi(U_1 - U_2) \tag{65}$$

with Φ being the cumulative normal distribution function. Then CV is simply given by (Small and Rosen 1981)

$$CV = -\frac{N}{\lambda}\int_{\mu 0}^{\mu 1} \Phi(\mu)d\mu \tag{66}$$

where $\mu_i = U_1^i - U_2^i$.

4. Final comments and conclusions

Improving transport systems induces conditions which are perceived as more satisfactory by users; they indeed constitute a benefit. It poses the problem of turning the subjective perception of that improvement into monetary units, for proper comparison with costs. In this paper, the operational approaches to solve this problem have been presented, emphasizing their economic foundations as welfare measures in an effort to provide an integrating view of such approaches.

Individuals' perceptions are observed through transport demand, which relates travel needs to the characteristics of transport systems. It has been shown that the information behind demand is sufficient to account for all benefits accruing to the different agents in those markets which are affected by changes in transport conditions. The relation between both market and compensated demands and the valuation of consumers' benefits has been strictly established; however, the most widely used tool to assess users' benefits, the rule-of-a-half, does not utilize the analytical form of demand, requiring only the initial and final states. The intuitively motivated RH is shown to be, even under its most general expression, an approximation to the least rigorous form of welfare measure: the Marshallian consumers' surplus. A departure from the RH leads to more rigorous forms of users' benefits calculation, which consider demand models explicitly in their derivation, thus including the information provided by the different elements involved in the economic phenomenon of transport demand. Furthermore, explicit derivation of such rigorous welfare measures permits a better interpretation of benefits in terms of demand parameters and their underlying meaning. From this viewpoint, benefit measures have been obtained for the so-called direct demand models, the family of entropy models, and the family of discrete choice models.

Most of the applied work in transport projects appraisal relies on the RH as an adequate measure of welfare variation. Even more, until now, available urban planning models bring the RH as a standard feature of their evaluation package. Should one recommend throwing the RH away and replace it by the users' benefit measure corresponding to the particular demand model used? In fact, beyond isolated experiments, the RH has not been sufficiently compared with its alternatives. So, a period of serious applications of the rigorous forms presented in section 3.3 is called for, in order to have solid empirical evidence to move definitely in the suggested direction.

On the other hand, exact or rigorous welfare measures cannot be better than the underlying demand model. Thus, demand specifications which do not reflect the actual process of choice may yield results which are as inadequate as those obtained directly from approximations. In this sense, two types of elements can be viewed as a source of problems: the analytical expression of the demand model (functional form), and the arguments within that function (explanatory variables). Though it is not the intention here to propose improvements in demand modelling, there is one aspect which cannot be regarded as further sophistication of available approaches, but as a systematically omitted element: the role of income. It is true that it can be thought of as an unimportant variable in industrialized countries, but demand models are also used in Third World countries. The usual excuse to relegate income to a secondary place has been the presumably low relevance of transport in total expenditure; the fact is that the observed structure of household expenditure in wide socio-economic groups within the southern hemisphere does not support such an assumption.[12] As seen in this paper, income elasticity does play a role in the analysis of welfare changes within the Hicksian framework, particularly in the quality of proposed approximations of market demands as compensated ones (see sections 2.3 and 3.3, especially the conditions behind equation (58) for the compensating variation). Some theoretical thinking has been devoted to income in transport demand modelling (Sasaki 1982, Hau 1983), but neither as a specific subject nor as an empirical matter.[13] A related but not identical problem, is the possibility of substitution between transport and other goods or services, which can be of some importance in low income environments (Jara-Díaz and Farah 1986).

There are a couple of related dimensions of the valuation of users' benefits which have not been analysed in this paper. These are aggregation of benefits and interpersonal comparisons. Aggregation of demand for travel forecasting is a necessary step, but aggregation of benefits for project evaluation will always have implicit value judgements, since the (money equivalent) utility of various individuals or groups of individuals have to be added. How important is the welfare of one individual relative to another is an area which lies in the boundary of project analysis and politics. The style of discussion to enter this area is different from that which has been presented in this paper. If practical advice is required, reporting users' benefits in a disaggregated fashion seems to be an adequate compromise, either if users' benefits are approximated by the RH or if they are calculated more rigorously.

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[12] For a preliminary assessment of the relation between transport and income in Chile, see Jara-Díaz and Farah (1986). There, a figure of 17 per cent of income spent in transport has been found to describe fairly well the living conditions of a significant part of the population.

[13] While this paper was being refereed, we came up with two articles on the role of income in mode choice and in users' benefits. The interested reader may wish to consult Jara-Díaz and Farah (1987) and Jara-Díaz and Videla (1987).

Foreign summaries

Tout projet de transport implique des coûts et des avantages. Les bénéfices pour les usagers prennent la forme d'une amélioration de la fréquence et de la qualité de service. Si l'on accepte le concept néo-classique de demande, l'évaluation du bénéfice des usages implique une modification de leur niveau d'utilité. Pour l'évaluation d'un projet, il faut comparer les bénéfices aux coûts, ce qui nécessite une conversion des niveaux d'utilité en termes monétaires. C'est là l'objet de cet article, qui part des relations générales entre fonction d'utilité, loi de demande et les diverses formes que peut prendre le surplus du consommateur et analyse ensuite les formes particulières que peuvent prendre ces relations dans le domaine du transport. La règle de la moitié du produit de la différence de prix par le nombre d'usagers est étudiée depuis son point de départ intuitif jusqu'à sa dérivation analytique rigoureuse. On examine ensuite des mesures plus rigoureuses de l'évaluation du surplus du consommateur, de façon très générale, de façon à les appliquer aux modèles de demande tant agrégés que désagrégés; l'accent est mis sur la façon dont on peut déduire une mesure de l'utilité dans chaque cas. Les avantages et limites des différentes méthodes possibles constituent l'essentiel du débat, avec le souci de l'amélioration de la formalisation de la demande et de la mesure des bénéfices des usagers.

Maßnahmen im Verkehr erzeugen Kosten und Nutzen. Nutzen für den Nutzer drücken sich in Form von mehr oder weniger besserer Kommunikation aus. Unterstellt man die neoklassische Theorie über die Nachfrage, so bildet die Veränderung des Nutzenniveaus die Grundlage für die Messung von Nutzen. Im Bewertungsprozeß müssen die Nutzen den Kosten gegenübergestellt werden, indem man beispielsweise Nutzen in monotären Einheiten ausdrückt. Dieses Papier beschäftigt sich mit diesem Problem. Es beginnt mit dem generellen Zusammenhang zwischen Nutzen, Nachfrage und den verschiedenen Arten von Nutzenüberschüssen und geht dann zu speziellen Formen dieser Zusammenhänge im Verkehrswesen über. 'The rule-of-a-half' von erster intuitiver Rechtfertigung zu einer strengen (und allgemeinen) analytischen Ableitung wird nachgegangen. Es werden danach strengere Formen aus dem Spektrum von Nutzer-Vorteilen für ziemlich allgemeine Fälle vorgestellt, in denen sowohl aggregierte als auch disaggregierte Verkehrsnachfragemodelle enthalten sind, wobei die Art, in der die Nutzenmessung geschieht, besonders herausgearbeitet wird. Diese Diskussion konzentriert sich auf die vergleichbaren Vorteile und Grenzen der verfügbaren Ansätze; hierbei wird nach Verbesserungen in der Nachfrageschätzung und der Nutzenmessung gesucht.

Los proyectos de transporte involucran costos y beneficios. Los beneficios a los usuarios aparecen en la forma de más o mejores viajes. Una vez que se acepta la idea neoclásica de demanda, la variación de niveles de utilidad subyace a la medida de beneficios. En el proceso de evaluación económica, éstos deben ser comparados con los costos, lo que requiere de la conversión de utilidad a dinero. En este trabajo se aborda el problema comenzando por las relaciones generales entre utilidad, demanda y las varias formas de excedentes de los consumidores, para pasar luego a las formas particulares que adquieren estas relaciones en el caso de transporte. Se expone la 'regla del medio' desde su justificación intuitiva hasta su derivación analítica estricta. Se presentan luego formas más rigurosas de medir la variación de excedentes del consumidor a partir de modelos agregados y desagregados de demanda por transporte, enfatizando la manera de deducir medidas de bienestar en cada caso. La discusión final se centra en las ventajas y limitaciones comparativas de los enfoques vigentes, y en la búsqueda de mejoramientos en la formulación de modelos de demanda y medidas de beneficios.

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