# ON THE GOODS-ACTIVITIES TECHNICAL RELATIONS IN THE TIME ALLOCATION THEORY. 

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#### Abstract

In areas like household production and travel choice, time assigned to the different activities plays a key role in addition to consumption as the main variables in utility within the consumer behaviour framework. However, a comprehensive conceptual structure to understand the technological relations between goods consumption and the assignment of time to activities is still lacking. In this paper the problem is reviewed and all possible relations between goods and time are re-formulated. Two general functions are defined and proposed to account for all these relations, forming a new taxonomy for the technical constraints. The resulting consumer behaviour model is used to obtain general expressions for both the value of saving time in constrained activities like travel, and the value of leisure.


Key words: value of time, time allocation, consumer theory, travel behaviour.

## 1. Introduction

Time allocation theory has received contributions from many perspectives. Home production (household work), labour supply and travel choice are probably the most fruitful from the viewpoint of understanding both consumer behaviour and the individual valuation of time. On the other hand, at this stage of human evolution the "lack of time" complaint is part of daily life in most developed and developing countries, with all its legacy of anguish and stress that makes Woody Allen's films so attractive and psychiatrists so rich. Although expectancy of life at birth increases constantly and we have access to gadgets that permit a more efficient use of time like never before, social, professional and personal commitments seem to swallow this potential freedom in a never-ending and paradoxical process of mutual reinforcement. Nevertheless, time is still only a guest in consumer behaviour theory.

A first glimpse at the relevance of time in consumer theory can be obtained using the money budget constraint for one individual: ceteris paribus, more work usually means more money. As a consequence, however, a time constraint becomes mandatory because one cannot work continuously (although some would claim they could). But not only resting is necessary and appreciated, which is why leisure was eventually introduced in the utility function for the labour supply theory; the fact is that consumption requires time as well and activities require goods. It is this crucial though neglected point that we want to address here, namely the relations between goods consumption and time assignment to activities and its impact on the value of saving time in activities like travel. Identifying clearly these relations seems like a necessary step to keep on building modelling capabilities to improve our understanding of consumer behaviour.

The evolution of time allocation theory within the consumer behaviour framework can be looked at from many angles. One is, of course, the variables considered in utility. The traditional approach included goods only, but it evolved to include goods and consumption time as inputs for "final goods" (Becker, 1965), and then goods and activities as direct sources of satisfaction (DeSerpa, 1971). A revolutionary step (not sufficiently recognised) was taken by Evans (1972), who postulated time assigned to activities as the only argument in utility. In later years we have witnessed variations on these, depending on what is the authors' emphasis (for instance, the dynamic formulation of Winston, 1986).

A second (complementary) perspective is that of the type of constraints considered. In addition to the traditional income budget constraint, a time availability constraint was first included (Becker, 1965; Johnson, 1966; Oort, 1969). Soon after, it was necessary to account for what are usually called technological constraints, representing feasibility relations between goods $X$ and activity times $T$. It was DeSerpa (1971) who first included a very simple technical constraint, stating that time assigned to the consumption of a given set of goods has a minimum that depends on the amount of those goods. It was this observation that led to a first clear definition of leisure as those activities that are assigned more time than the minimum necessary. And this led to the (now well-known and understood) distinction between the value of saving time, the value of time as a (personal) resource and the value of time as a commodity (for the evolution in the value of time approaches see Bruzelius, 1979, or Jara-Diaz, 2000).

In this paper we want to concentrate on the technological relations between goods consumption and time assigned to activities, with two objectives. First, to build a
conceptual framework and a taxonomy in order to discuss the type of constraints that are needed for a complete description of consumer behaviour. Second, to set a more solid basis to discuss and interpret the value of time (assigned or saved) and its many components. This is particularly relevant as the value of saving time in activities like urban travel can be actually calculated from discrete choice models, as shown by Bates (1987) in his interchange of ideas with Truong and Hensher (1985). In the next section, we describe briefly the different forms in which technological constraints have been included in the literature. Then we identify all possible relations between goods and activities in the third section, where we show that some of these functions are in fact interrelated. A reduced set of two families of technical functions are defined and identified as the minimum necessary to be included in a time allocation-goods consumption consumer behaviour framework. These relations and functions form a system of definitions that is used to make an interpretation of some constraints that are present in the literature. In section four we include these relations as explicit constraints in a general model, obtaining an expanded interpretation of the value of saving time in activities like travel. A synthesis and conclusions are presented in the final section. Throughout the paper we will refer to goods consumed during a single period only. Capital stock (Juster, 1990) or durable goods will be left out of the analysis. Also, we will not be dealing with the sequence of actions (Winston, 1987; Small, 1982), but with the set of activities during a reference period, assumed to be undertaken one at a time.

## 2. The technological relations between goods and time: an overview.

The technological relations between goods consumption (described by a vector $X$ ) and time assigned to activities (described by a vector $T$ ) during the same period, have been
established in many different ways in the time economics literature. In some articles (mostly within the home production literature dealing with household work), an intermediate artefact called "final goods" (vector $Z$ ) was created, as is the case in Becker (1965), Lancaster (1966), Michael and Becker (1973), De Donnea (1972), Pollak and Wachter (1975), Dalvi (1978) and Gronau (1986). In essence, the final goods (e.g. a prepared meal) are assumed to be a function of goods (vegetables, salt, oil) and time assigned (shopping, cooking), which act as inputs in a production function, i.e.

$$
\begin{equation*}
Z=f(X, T) \tag{1}
\end{equation*}
$$

The emphasis here is on how a final good is prepared ${ }^{1}$. Becker (1965) assumed explicit fixed-coefficient-like relations between $Z_{i}, X$ and $T$, which can be written as

$$
\begin{align*}
& T_{i}=\sum_{j} a_{i j} Z_{j}  \tag{2}\\
& X_{k}=\sum_{j} b_{k j} Z_{j} \tag{3}
\end{align*}
$$

Here $a_{i j}$ and $b_{k j}$ are coefficients that convert one unit of final good $j$ into necessary time and necessary goods respectively. Thus, if $A$ and $B$ are matrices of elements $a_{i j}$ and $b_{k j}$ respectively, then the technical relations between $T$ and $X$ are given by

$$
\begin{align*}
& T=A Z \quad \text { and }  \tag{4}\\
& X=B Z . \tag{5}
\end{align*}
$$

As matrices $A$ and $B$ are not necessarily square, we can not solve for $Z$ in either equation and find a relation between $X$ and $T$.

[^0]Another stream of articles has established what we can call direct relations that involve $X$ and $T$. This is the case of the classic approach by DeSerpa (1971), who was the first to include an explicit set of technical constraints, originally stated as

$$
\begin{equation*}
T_{i} \geq a_{i} X_{i} \tag{6}
\end{equation*}
$$

It is important to recall that DeSerpa gave a number of explanations regarding what $T$ and $X$ meant in his theory. First, the $X$ 's were assumed to be consumed one at a time, which implied that $X_{i}$ acted like a single composite good (e.g. sport garments), associated with an activity $i$ (a soccer game) whose duration was $T_{i}$ ( 90 minutes). Note that the original denomination for $T_{i}$ in the article was "consumption time", which quickly turned into "activity" as DeSerpa’s text progressed. Thus, equation (6) simply says that an activity has a minimum duration depending on the amount of goods consumed.

A relevant contribution to the identification of technological relations can be found in the paper by Evans (1972). This is the only published article that presents a consumer behaviour framework entirely formulated in terms of activity times, which are the only arguments in utility. The budget constraint was explicitly written as

$$
P^{\prime} Q T \leq O
$$

where $P$ is a goods price vector and $Q$ is a matrix that turns $T$ into the amount of goods that are necessary to undertake activities in $T$. As is evident, the vector $T$ includes work, and the corresponding price is the negative value of the wage rate. Therefore, the $Q$ matrix is an implicit fixed-coefficients transformation function of activities into the necessary goods such that an explicit relation can be established, namely

$$
\begin{equation*}
X=Q T \tag{7}
\end{equation*}
$$

In other words, given a vector of activities, the amount of good $i$ consumed, $X_{i}$, is

$$
\begin{equation*}
X_{i}=\sum_{j} q_{i j} T_{j} \tag{8}
\end{equation*}
$$

where $q_{i j}$ are the elements of the $j$-th column of $Q$, denoting the amount of good $i$ needed to undertake one time unit of activity $j$. Therefore, $X$ in equation (7) and $X_{i}$ in equation (8) can be interpreted as the minimum necessary amount of goods to undertake activities in $T$, as technical feasibility is preserved if both equations are taken as inequalities $(\geq)$. Note that Evans also included a second type of relation, which establishes the possible interdependence between activity times. This takes the form

$$
\begin{equation*}
J T \leq O, \tag{9}
\end{equation*}
$$

which represents a set of linear relations between activity duration.

Finally, in a note Collings (1973) added a series of maximum time restrictions like

$$
\begin{equation*}
T_{i} \leq b_{i} X_{i} \tag{10}
\end{equation*}
$$

to DeSerpa's minimum time requirements. No further discussion was attempted, though. ${ }^{2}$

## 3. A complete system of technological relations between $X$ and $T$.

In this section we postulate and define a complete set of relations between the type and amount of goods consumed in a certain period and the type and duration of activities performed within that same period. Let us begin with a relation that associates a given amount of goods with the feasible duration of a set of activities. In other words, given an

[^1]amount and combination of goods $X^{o}$, there are some activity combinations that can actually take place, and some that are not feasible because of lack of available goods. This is represented in figure 1 in a two-activities space. If $X^{o}$ permits a combination $\left(T_{1}, T_{2}\right)$ of activities, it defines a feasible point (shaded area). The frontier is efficient, as opposed to an interior point where either $T_{1}$ or $T_{2}$ or both can be increased, keeping $X^{o}$ constant. This defines what we will call the Activity Possibility Frontier. On the other hand, given a vector of activities $T^{\mathbf{0}}$, goods are required to perform them. There are some combinations of goods that permit $T^{0}$ to take place, and there are others that do not (e.g. minimum amount of food to run an evening dinner). This is represented in figure 2, where we define the boundary as an Isoactivity Locus or curve, such that the combinations below the curve do not permit the bundle of activities in $T^{\circ}$ to occur. Note that the boundaries in figures 1 and 2 resemble the production possibility frontier for given inputs and the isoquant for a given output level respectively, where $T$ plays the role of outputs and $X$ that of inputs.


Figure 1: Activity Possibility Frontier for a given amount of goods.


Figure 2: Isoactivity Locus

A third type of relation deals with the consumption of goods that is allowed by assigning time to a $T^{0}$ activity vector. In other words, given $T^{0}$ there are some combinations and
amounts of goods $X$ which can be consumed and others that can not. Such combinations are shown in figure 3, where the boundary can be labelled as a Consumption Possibility Frontier. Lastly, a given set of goods $X^{\mathrm{o}}$ can be consumed during certain combinations of activity durations. This means that there are some activity structures $T$ which are not compatible with the consumption of $X^{\circ}$. This is represented in figure 4 , where the frontier between the feasible and not feasible combinations we have named the Isoconsumption Locus or curve. In this case, the resemblance with production theory is such that $X$ plays the role of outputs and $T$ that of inputs.


Figure 3: The Consumption Possibility
Frontier for a given activity structure


Figure 4: Isoconsumption Locus

Let us verify that the general relations and definitions that we have presented here encompass those in the literature as particular cases. First, the inequalities set by DeSerpa (1971) in equation (6) represent minimum activity times for a given level of goods consumption $X^{\mathrm{o}}$, but they can also be interpreted as maximum consumption levels for a given activity structure $T^{\circ}$. The first interpretation (which reflects DeSerpa's intention) is in fact a special case (fixed coefficients) of the Isoconsumption Locus of figure 4, shown in
figure 5 a , while the second interpretation is a particular representation of the Consumption Possibility Frontier of figure 3, shown in figure 5b. This confirms that figures 3 and 4 represent the same technical function shown in different spaces, holding either $T$ or $X$ constant.


Figure 5. Interpretation of DeSerpa's technological constraints.

On the other hand note that, by analogy with the DeSerpa constraints, Collings' maximum time restrictions represented by eq. (10) correspond not only to a fixed-coefficients like Activity Possibility Frontier, but also to an Isoactivity Locus. Curves with the same meaning can be obtained using the implicit transformation function from activities into goods represented by the $Q$ matrix in Evans' model (eq. 7), but this requires some elaboration. This matrix has only one interpretation in terms of our definitions, which is the Isoactivity locus or curve in figure 2. But, as argued above, the amounts $X_{i}$ in eq. (8) can be interpreted as the minimum necessary to undertake a set of activities with duration $T_{j}$, represented in figure 6 . Further, the interpretation of equation (8) as an inequality permits
the illustration of a somewhat hidden (although intuitive) property, which can be explicitly derived in the two goods - two activities case where

$$
\begin{align*}
& X_{1} \geq q_{11} T_{1}+q_{12} T_{2}  \tag{11}\\
& X_{2} \geq q_{21} T_{1}+q_{22} T_{2} \tag{12}
\end{align*}
$$

For a given $X^{\mathrm{o}}, T_{1}$ and $T_{2}$ have to fulfil inequalities (11) and (12), graphically represented in figure $6 b$, which clearly shows that the relations that define the Isoactivity Locus also define the Activity Possibility Frontier. Note that relations (11) and (12) are more general than Colling's maximum time restrictions.


Figure 6. Interpretation of Evans' transformation matrix

The analysis synthesised by Figures 5 and 6 confirm the idea that the general relations represented in figures 1 and 2 , or 3 and 4, are two ways of looking at the same technical relations. As suggested earlier, the Isoconsumption Locus and the Consumption Possibility Frontier are simply the same production or transformation function represented in two spaces. Similarly, behind the Isoactivity Locus and the Activity Possibility Frontier there is
another single technical function. This shows that the idea of isoquants and production frontiers from production theory, has not one but two counterparts when activities and goods are considered in consumer theory.

This discussion leads to the recognition of two types of technological relations between goods consumption and time assigned to activities. One represents a generalisation of (our interpretation of) Evans' transformation matrix which, as shown above, gives not only the combinations of goods that are necessary to undertake a given set of activities $T^{0}$, but also describes the combination of activities that can be performed with a given amount of goods $X^{0}$. The fact that activities require goods makes us define a single activity possibility function $A$ that encompasses both relations, namely

$$
\begin{equation*}
A(X, T) \geq 0 \tag{13}
\end{equation*}
$$

such that $A\left(X, T^{0}\right) \geq 0$ represents the combinations in $X$ that allow $T^{\text {o }}$, and $A\left(X^{0}, T\right) \geq 0$ represents the combination in $T$ that are allowed by $X^{\circ}$. Obviously $A\left(X, T^{0}\right)=0$ defines the Isoactivity Locus in the $X$ space and $A\left(X^{0}, T\right)=0$ defines the Activity Possibility Frontier in the $T$ space.

The second type of technological relations between $X$ and $T$ represents a generalisation of DeSerpa's technical constraints, and gives the combination of activities that allow a certain consumption structure $X^{0}$, and also the combination in $X$ that are permitted by a given activity structure $T^{0}$. In these cases goods consumption requires the assignment of time to activities, such that we can define a single consumption possibility function $G$

$$
\begin{equation*}
G(X, T) \geq 0 \tag{14}
\end{equation*}
$$

such that $G\left(X^{0}, T\right) \geq 0$ gives the combination of activities that are compatible with consumption $X^{\mathrm{o}}$, and $G\left(X, T^{0}\right) \geq 0$ describes the consumption sets that are permitted by an activity structure $T^{0}$. In this case, $G\left(X^{0}, T\right)=0$ represents the Isoconsumption Locus in the $T$ space, and $G\left(X, T^{0}\right)=0$ is the Consumption Possibility Frontier in the $X$ space. The $G$ and $F$ functions exhaust the technical relations between goods and activity times.

## 4. A revised interpretation of the value of saving time

The classic (although simple) technical constraints imposed by DeSerpa (1971) can be combined with our expanded interpretation of Evans' implicit transformation function in order to represent the general functions $G$ and $F$ in a fairly simplified manner ${ }^{3}$, i.e.

$$
\begin{array}{ll}
T_{i} \geq f_{i}(X) & \forall i=1, \ldots a \\
X_{i} \geq g_{i}(T) & \forall i=1, \ldots g \tag{16}
\end{array}
$$

Inequalities (15) state that goods consumption impose minimum levels on activity duration, and represent technical relation (14), graphically shown in figures 3 and 4. Inequalities (16) state that activities impose minimum levels on goods consumption, and take care of relation (13) representing figures 1 and 2 . Both equations can be taken as necessary restrictions to be included in a general framework for consumer behaviour that includes time. This is important not only to represent in a better way consumer behaviour within a time assignment framework, but also to re-examine what is behind the value of saving time in constrained activities.

[^2]Let us consider a general static model in which individuals obtain satisfaction from activities $T$, at a level that is dependent on consumption $X$. This is present in all time allocation microeconomic models since DeSerpa (1971), in some form or another. Intuitively, this means that the marginal utility of a time unit assigned to a given activity depends on the goods that are used (a property that is missing in Evans' model). We will examine only the case of endogenous income in which the individual decides how many hours $T_{W}$ to work at a wage rate $w$. Under this framework, the model $(A)$ including the new constraints is

$$
\begin{equation*}
\operatorname{Max} U(X, T) \tag{A}
\end{equation*}
$$

subject to

$$
\begin{array}{ll}
w T_{w}-\sum P_{i} X_{i} & \geq 0 \\
\tau-\sum T_{j} & =0 \quad(\lambda) \\
T_{j}-f_{j}(X) & \geq 0 \quad\left(\kappa_{j}\right) \quad \forall_{j}=1, \ldots a \\
X_{i}-g_{i}(T) & \geq 0 \quad\left(\psi_{i}\right) \quad \forall_{i}=1, \ldots g
\end{array}
$$

Parameters $\lambda, \mu, \kappa$ and $\psi$ are the Lagrange multipliers representing by how much utility increases when the corresponding constraint is relaxed by one unit. Thus, $\lambda$ is the marginal utility of income and $\mu$ is the marginal utility of time available. As $\tau$ can be looked at as a (limited) resource, DeSerpa (1971) defined the ratio $\mu / \lambda$ as the value of time as a resource. By the same token, $\kappa_{j}$ is the marginal utility of diminishing the constraint on activity $j$ by one unit, which makes $\kappa_{j} / \lambda$ what DeSerpa called the value of saving time in an activity.

The first order conditions for a non-working activity $k$ are ${ }^{4}$

$$
\begin{gather*}
\frac{\partial U}{\partial T_{k}}-\mu+\kappa_{k}-\sum_{i=1}^{g} \Psi_{i} \frac{\partial g_{i}}{\partial T_{k}}=0  \tag{17}\\
{\left[T_{k}-f_{k}(X)\right] \kappa_{k}=0} \tag{18}
\end{gather*}
$$

From equation (17) one can obtain a new expression for $\kappa_{k} / \lambda$, which is positive only if the activity is restricted to its minimum (i.e. $\kappa_{k} \neq 0$, as is the case of mandatory travel time) and nil otherwise by virtue of equation (18). This is

$$
\begin{equation*}
\frac{\kappa_{k}}{\lambda}=\frac{\mu}{\lambda}-\frac{\partial U / \partial T_{k}}{\lambda}+\frac{1}{\lambda} \sum_{i=1}^{g} \psi_{i} \frac{\partial g_{i}}{\partial T_{k}} . \tag{19}
\end{equation*}
$$

For synthesis, eq. (19) states that there is a value in reducing the minimum necessary time assigned to an activity because of three effects: the re-assignment of time to other activities, the direct variation in utility, and a variation in consumption, which is a new term explained below. Please note that this has nothing to do with De Donnea's (1972) effect of goods on "the circumstances under which the time is spent", that is related with the way $X$ and $T$ are specified in utility and not with some form of technical constraint. His "comfort effect" is unrelated with the impact on goods consumption that we have derived.

Note that DeSerpa (1971) defined leisure activities as those whose duration exceeds the minimum necessary. From equation (18) in this case $\kappa_{k}=0$ and, according to equation (19), the money value of its marginal utility $\left(\left(\partial U / \partial T_{k}\right) / \lambda\right)$ would be equal to the value of time as a personal resource $\mu / \lambda$ if not for the consumption related term that involves the multipliers $\psi$. In other words, if the last constraint in problem $A$ was not included, the

[^3]money value of the marginal utility for all leisure activities would be equal to $\mu / \lambda$, which is exactly the reason why DeSerpa called it the value of leisure. This equality is no longer valid, as the value of the marginal utility of a leisure activity includes the variation in consumption of those goods whose minimum are affected by a variation in activity duration $\left(\frac{\partial g_{i}}{\partial T_{k}} \neq 0\right)$, and whose consumption is binding $\left(\boldsymbol{\psi}_{i} \neq 0\right)$. In other words, it includes the value of saving consumption. This implies, among other things, that the value of time assigned to leisure activities $\left(\left(\partial U / \partial T_{k}\right) / \lambda\right)$ is no longer equal across activities because of the effect on goods consumption.

On the other hand, the first order condition for work is

$$
\begin{equation*}
\frac{\partial U}{\partial T_{w}}+\lambda w-\mu+\kappa_{w}-\sum_{i=1}^{g} \psi_{i} \frac{\partial g_{i}}{\partial T_{w}}=0 \tag{20}
\end{equation*}
$$

It is reasonable to assume that the consumption of some goods do vary with the amount of work (e.g. clothing), and also that the work period is not technically restricted by consumption, i.e $T_{w}>f_{w}(X)$, which means that $\kappa_{w}$ could be set to zero from equation (18). This implies that, from eq. (20)

$$
\begin{equation*}
\frac{\mu}{\lambda}=w+\frac{\partial U / \partial T_{w}}{\lambda}-\frac{1}{\lambda} \sum_{i=1}^{g} \Psi_{i} \frac{\partial g_{i}}{\partial T_{w}} \tag{21}
\end{equation*}
$$

which shows that the value of time as a personal resource would be equal to the wage rate either if work time is not valued per se and restricted consumption is not affected by work, or if both effects cancel out. Note that the right hand side is a new (expanded) interpretation for the value of work, including the wage rate, the value of its marginal utility and the technical impact of the work duration on goods consumption. Replacing equation (21) in
expression (19) yields a new form for the value of saving time in an activity that is restricted to its minimum (i.e. $\kappa_{k} \neq 0$, the travel time case) that is equal to

$$
\begin{equation*}
\frac{\kappa_{k}}{\lambda}=w+\frac{\partial U / \partial T_{w}}{\lambda}-\frac{\partial U / \partial T_{k}}{\lambda}+\sum_{i=1}^{g} \frac{\Psi_{i}}{\lambda}\left(\frac{\partial g_{i}}{\partial T_{k}}-\frac{\partial g_{i}}{\partial T_{w}}\right) . \tag{22}
\end{equation*}
$$

The three first terms in the right hand side of eq. (22) can be recognised as the usual three terms originally obtained by Oort (1969), later exposed by DeSerpa (1971), also derived by Bates (1987) in the context of discrete travel choice models. The novelty here is the value of the change in the consumption pattern. For synthesis, eq. (22) states that the willingness to pay for a reduction in activity $k$ is given by the wage rate, plus the value of the marginal utility of work, plus the value of a reduction of activity $k$ in direct utility, plus the value of the change in the consumption pattern. Note that the new term involving $\partial g_{i} / \partial T_{k}$ has a positive sign; this is perfectly intuitive, as $\kappa_{k} / \lambda$ is the willingness-to-pay to save time in activity $k$, which increases if that activity requires a minimum consumption (as is the case of gas in car trips).

In order to fully understand the "goods effect" in the value of saving time in activity $k$, it is worth examining the first order condition of problem (A) with respect to a good $i$. This is

$$
\begin{equation*}
\frac{\partial U}{\partial X_{i}}-\lambda P_{i}-\sum_{j} \kappa_{j} \frac{\partial f_{j}}{\partial X_{i}}+\psi_{i}=0 \tag{23}
\end{equation*}
$$

from which one can obtain

$$
\begin{equation*}
\frac{\Psi_{i}}{\lambda}=P_{i}-\frac{\partial U / \partial X_{i}}{\lambda}+\sum_{j} \frac{\kappa_{j}}{\lambda} \frac{\partial f_{j}}{\partial X_{i}} . \tag{24}
\end{equation*}
$$

Eq. (23) shows that $\psi_{i} / \lambda$ has three components: price of the good, the value of its marginal utility, and the value of those activities that are constrained by consumption including good $i$. Thus, $\psi_{i} / \lambda$ is the value of saving consumption in good $i$, and the term

$$
\sum_{i=1}^{g} \frac{\psi_{i}}{\lambda} \frac{\partial g_{i}}{\partial T_{k}}
$$

is the value of total consumption saved when activity $k$ is reduced. This clearly explains the positive sign in both equations (19) and (22). The larger its value, the larger the willingness to pay to reduce the corresponding constrained activity. How important this is should be explored empirically.

## 5. Synthesis and conclusions

We have presented the many forms in which goods consumption and time assigned to activities have been linked in the microeconomic literature on time allocation, through (implicit or explicit) technological feasibility constraints. Next we identified four type of relations between goods and activities that refer to minimum and maximum time-dependent consumption levels, and minimum and maximum goods-dependent time allocation levels. The frontiers of these relations were labelled the Activity Possibility Frontier (APF), the Isoactivity Locus (AL), the Consumption Possibility Frontier (CPF) and the Isoconsumption Locus (CL).

We have shown that the minimum time requirements of DeSerpa (1971) and our interpretation of Evans' (1972) matrix that convert activities into goods were among the clearest (but not the only) expressions of such relations. The former correspond to the
definition of both the CPF and the CL, depending on what is left constant. Similarly, an expanded interpretation of Evans' transformation matrix allowed us to show that it corresponds to the notions of both the AL and the APF. This made us define two type of functions, which we defended as necessary to account for all technological relations between goods and activity times. We called them the activity possibility function $A(X, T)$ and the consumption possibility function $G(X, T)$. These generic functions constitute a complete system of technological constraints that should be added to the budget and time constraints in a consumer behaviour framework. Together with the frontiers previously defined, these functions form a taxonomy for the relations between goods and activities within such framework. Which frontiers should be preferred to represent the $A$ and $G$ functions is subject to debate and research.

Expanded expressions of DeSerpa's and Evans' constraints were chosen to represent in a simplified form the functions defined above as particular cases, which we then introduced in the consumer behaviour framework that includes time allocation. The impact of this innovation on the value of time savings for constrained activities was examined, showing that a new effect had to be considered, namely the value of re-assigning consumption (the value of total consumption saved when a constrained activity is reduced). Besides, the value of leisure activities was shown to differ across activities precisely because of the variation in goods consumption. We believe that this analysis of the technological relations between goods and activities completes the discussion of the generic (static) microeconomic model of consumer behaviour including time.

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[^0]:    ${ }^{1}$ To be specific, in this literature the elements of $T$ are said to be related with the production of the final goods. This is made explicit in Becker (1965).

[^1]:    ${ }^{2}$ Bruzelius (1979) sees equation (1) as a scalar function of a single $X_{i}$ and a single $T_{i}$. Although inversion of such function in $X_{i}$ yields a relation between $X_{i}$ and $T_{i}$, it is parametrical in $Z_{i}$ (unlike equation 8).

[^2]:    ${ }^{3}$ Note that these are not the only form to represent the $A$ and $G$ functions.

[^3]:    ${ }^{4}$ Note that equation (18) holds for all activities.

