Chapter 26

MAKING PRICING WORK IN PUBLIC TRANSPORT PROVISION[a]

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1. Introduction

It is not only the operators of public transport who have to use resources to produce trips: the users themselves have a crucial input as well – their time. Users have to walk, wait, and travel inside the vehicle and, by doing so, they spend time, which is a resource consumed in the production of the trips. The microeconomic analysis of public transport must include this fact in order to find the optimal provision of services and the corresponding optimal prices. This is not new in transport analysis: car congestion pricing is the result of the inclusion of users' travel time in the pricing analysis. It is based upon the increasing relation between car users' travel time and patronage after a certain demand level due to a negative externality in the use of road space. As a consequence, the price that induces the best possible use of the road space is the difference between the value of the marginal time for all users and the value of the average travel time for each user. This chapter examines optimal public transport fares from the same viewpoint.

Section 2 discusses the cost structure of public transport. Then, optimal fares and their financial results are examined. As the existence of scale economies is shown to be crucial for the financial result of optimal fares, returns to scale in public transport are discussed in Section 4. Following that, a brief discussion about the impact of substitute modes and second best fares is presented. In Section 6 the relation between optimal fare and distance is analyzed. The chapter closes with a summary and discussion.

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2. Costs in public transport

2.1. General aspects

In the microeconomic analysis of public transport, two types of resources have to be taken into account: those provided by the operators, as vehicles, fuel, terminals, or labor, and those provided by the users, namely their time, usually divided into waiting, access, and in-vehicle time. The total cost (C_T) can be written as

$$C_T = C_{op} + C_U, \tag{1}$$

where C_{op} and C_U are the operators' and users' costs, respectively.

There are two elements that help identify the main variables for a cost analysis of the operation of public transport: route and fleet. The relevant route-related variables that affect either operators' or users' cost, or both, are the length, density, and number of stops. The corresponding fleet variables are the number of vehicles (or frequency) and vehicle size. In order to analyze the relation between costs and patronage (Y), it is necessary to establish which of these variables are fixed and which can be adapted to the demand level. In some cases, such as urban railways, the three route dimensions are fixed owing to physical reasons; for public transport, frequency and vehicle size are the operational attributes that are the easiest to adapt to demand. In the case of buses, institutional factors tend to define relatively stable routes, but length, density, and stops are easy to change in physical terms. Bus frequency is easy to adapt as well, as opposed to vehicle size, which can only be changed when buses are renewed. The flexibility of both the route and fleet decreases with the size (cost) of the type of vehicle (bus < tram < metro).

2.2. Cost structure

Given the elements discussed above, the relation between costs and patronage will now be examined. It is better to analyze the total cost structure in public transport, separating it into its two components: operators' cost and users' cost.

Considering the inputs supplied by the operators, there are operational and capital costs. The former include energy, crew, maintenance, and administration, and the latter are infrastructure and rolling stock costs. Engineering cost studies find that, in the absence of vehicle congestion, the average operators' cost decreases with demand (Meyer et al., 1965; Boyd et al., 1978; Allport, 1981).

Regarding the inputs supplied by the users, these costs are associated with travel times (waiting, access, and in-vehicle) and their money values. Each component has a clear relation to the different route and fleet elements. First note that frequency adaptation is the simplest input adjustment when the demand changes in the case of vehicles with fixed capacity. In scheduled transit systems, frequency increases with patronage (something that will be examined later), which will make in-vehicle time increase due to vehicle interactions, except in the case of rigid modes (e.g. metro) where a maximum feasible frequency exists. Nevertheless, not only vehicle congestion increases in-vehicle time, passenger boarding and alighting do as well. On the other hand, the waiting time decreases with frequency (demand), and, if the routes design is adaptable, demand expansions will induce a densification of the system, yielding reductions in access time as well.

In summary, when the demand increases, users' in-vehicle time also increases owing to vehicle congestion and passengers boarding/alighting, waiting time always diminishes, and access time decreases if routes can change. Despite an apparently mode-specific conclusion from the preceding discussion, this qualitative analysis actually yields a common scheme for the relation between users' cost and patronage. On one hand, the most flexible mode (bus) has a higher probability of congestion, but at the same time it is the easiest to densify when demand increases. On the other hand, the rigid rail-based modes have little (tram) or no (metro) congestion likelihood. So, in the first case a demand growth makes in-vehicle and access times vary with opposite signs, and in the second case both changes are low or nil. It is the waiting time, therefore, that prevails, generating a decreasing average users' cost function ($\mathrm{AC_U}$) in all cases, which means that $\partial \mathrm{AC_U}/\partial Y$ is negative and reflects the positive externality that an extra user generates on total travel time in scheduled transit systems.

The main conclusion is that the sum of the operators' and users' costs yields a total cost that increases less than proportionally with the demand, as found by Boyd et al. (1978) and Allport (1981) in their engineering cost studies. This means that the total average cost decreases with demand, which implies that there are scale economies. As a decreasing total average cost implies that the total marginal cost is lower than the total average cost, the result is exactly the opposite to the private car case. In the next section this will be shown to have a crucial impact on the optimal fares in public transport.

3. Optimal fares in public transport

3.1. The optimal fare

Given that both the operators' and users' costs have to be considered, the optimal fare for a public transport service can be found by maximizing the social benefit (SB), which is the difference between the total "willingness to pay" (in terms of both money and time) and the total cost (of both operators and users). Formally,

$$\operatorname*{Max}_{Y} \mathrm{SB} = \int_{0}^{Y} \mathrm{GC}(u)\mathrm{d}u - [C_{\mathrm{op}}(Y) + C_{\mathrm{U}}(Y)], \tag{2}$$

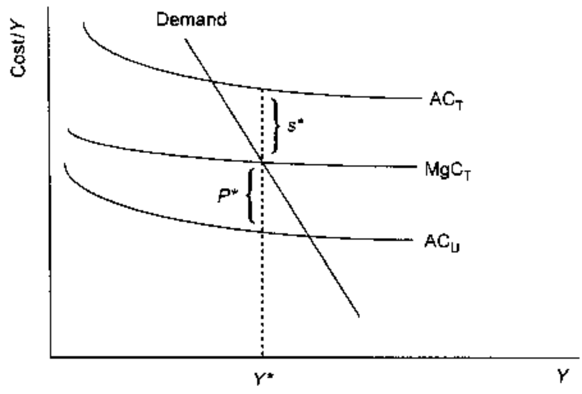

Figure 1. The optimal fare and subsidy in public transport.

where Y is the demand level, and the generalized cost (GC) is calculated as

$$\mathrm{GC} = P + \mathrm{AC_U}, \tag{3}$$

i.e. the fare (P) plus the users' average cost ($\mathrm{AC_U}$), which is the value of in-vehicle, waiting, and access times that the user experiences, given by C_U/Y. Note that GC in equation (2) is the inverse demand function.

The first-order condition for equation (2) is

$$\frac{\partial \mathrm{SB}}{\partial Y} = \mathrm{GC}^* - \mathrm{MgC_{op}}\,|_{Y^*} - \mathrm{MgC_U}\,|_{Y^*} = 0, \tag{4}$$

where $\mathrm{MgC_{op}}$ and $\mathrm{MgC_U}$ are the operators' and users' marginal costs, respectively. Using equation (3), and given that the total marginal cost ($\mathrm{MgC_T}$) is the sum of $\mathrm{MgC_{op}}$ and $\mathrm{MgC_U}$, the optimal fare is

$$P^* = \mathrm{MgC_T}|_{Y^*} - AC_U|_{Y^*}, \tag{5}$$

as shown in Figure 1. Just as in the case of the private transport optimal fare (congestion price), the user has to pay (in money) the difference between the total marginal cost and the users' average cost. By doing so, he perceives that the cost of his trip is the total marginal cost, as he already "paid" the users' average cost (the own time value).

By writing users' marginal cost as

$$\mathrm{MgC_U} = \frac{\mathrm{d}(Y\,\mathrm{AC_U})}{\mathrm{d}Y}, \tag{6}$$

the total marginal cost can be written as

$$\mathrm{MgC_T} = \mathrm{MgC_{op}} + \mathrm{AC_U} + Y\frac{\mathrm{dAC_U}}{\mathrm{d}Y}. \tag{7}$$

Replacing in equation (5) yields an alternative expression for the optimal fare (Jansson, 1979):

$$P^* = \mathrm{MgC}_{\mathrm{op}}|_{Y^*} + Y^* \left.\frac{\mathrm{dAC_U}}{\mathrm{d}Y}\right|_{Y^*}. \tag{8}$$

This expression shows that the passenger has to pay (in money) for the effect of his or her trip on the operators' cost ($\mathrm{MgC_{op}}$) and for the change that he or she produces in the users' average cost multiplied by the demand level, a negative figure that represents the total positive externality on the rest of the users, as explained above.

3.2. The financial result in the presence of scale economies

If scale economies exist, the total average cost is higher than the total marginal cost. This property yields a relevant financial result for optimal fares. Subtracting the operators' average cost from both sides of equation (5) yields

$$P^* - \mathrm{AC_{op}}|_{Y^*} = \mathrm{MgC_T}|_{Y^*} - \mathrm{AC_T}|_{Y^*}. \tag{9}$$

As deduced in the previous section, $\mathrm{AC_T}$ is larger than $\mathrm{MgC_T}$, which makes the operators' average cost larger than P^*. Therefore, the optimal fare cannot cover the operators' expenses, and an optimal subsidy per passenger (s^*) equal to the difference between $\mathrm{AC_{op}}$ and P^* is necessary. Expression (9) indicates that this optimal subsidy is equal to the difference between $\mathrm{AC_T}$ and $\mathrm{MgC_T}$ as well, as shown in Figure 1. Consequently, scale economies are a necessary and sufficient condition for optimal fares not being enough to cover operators' expenses. As said before, scale economies are equivalent to both decreasing total average cost and to $\mathrm{AC_T}$ being larger than $\mathrm{MgC_T}$.

4. Returns to scale in public transport

4.1. A microeconomic model

Due to the crucial impact of scale economies on the financial result of optimal fares, in this section returns to scale in public transport are explored in greater depth. A relevant issue for this discussion is the time period, i.e. which production factors are fixed and which are variable when fares are optimized. Jansson (1979, 1984) states that fare optimization does not make sense over a strictly defined short-term period where all inputs are fixed. In fact, the number of vehicles (fleet size) seems to be easier to change than fares. Therefore, it is reasonable to optimize fleet size first and then calculate optimal fares. Consequently, Jansson concludes that fares should be optimized in the "medium term," in which the number of vehicles is variable, implying a variable frequency as well.

The microeconomic public transport operation models available in the literature optimize different characteristics of the service, such as fleet (frequency), vehicle size, and routes spacing. If an expenditure function is considered that accounts for both operators' and users' inputs, the optimization of the variable factors yields a total cost function $C_T(Y)$, from which information regarding returns to scale can be obtained analytically. In what follows a microeconomic model is presented, in which frequency is optimized consistently with a medium-term definition that is adequate for public transport pricing. As known, deriving a cost function requires Y to be treated parametrically.

Following Jansson (1980, 1984), let us consider an isolated corridor served by one circular bus line L kilometers long, operating at a frequency f with a fleet of B vehicles. This service is used by a total of Y passengers per hour, homogeneously distributed along the corridor where each individual travels a distance l. If T denotes the time in motion of a vehicle within a cycle, and t is the boarding and alighting time per passenger, then the cycle time t_c is

$$t_c = T + t\frac{Y}{f}. \tag{10}$$

On the other hand, the frequency is given by the ratio between the fleet size and cycle time (B/t_c), which combined with equation (10) yields

$$B = fT + tY. \tag{11}$$

If c is the cost per bus-hour for the operator, and P_w and P_v are the values of the waiting and travel time, respectively, then the total value of the resources consumed (VRC) per hour is

$$\mathrm{VRC} = Bc + P_w \frac{Y}{2f} + P_v \frac{l}{L} t_c Y, \tag{12}$$

where the first term on the right-hand side of equation (12) corresponds to the operator expenses, and the second and third terms are the users' waiting and travel time values, respectively. Note that the access time is not included in VRC because the route design is not a variable and, therefore, access cost is a constant that is not relevant to the optimization of the service.

Using equations (10) and (11), we can write expression (12) as a function of B, i.e.

$$\mathrm{VRC} = Bc + P_w \frac{T}{2(B - tY)} Y + P_v \frac{l}{L}\left(T + \frac{tTY}{B - tY}\right) Y. \tag{13}$$

This expression shows that, *ceteris paribus*, increasing the number of vehicles diminishes the users' costs but increases the operators' costs. The users' cost reduction occurs because increasing frequency diminishes both the waiting and in-vehicle travel times, the latter because fewer individuals board and alight per bus.

Minimizing VRC with respect to B yields the optimal fleet size B^*, given by

$$B^* = tY + \sqrt{\frac{TY}{c}\left(\frac{1}{2}P_{\mathrm{w}} + P_{\mathrm{v}}tY\frac{l}{L}\right)}, \tag{14}$$

which from equation (11) yields the optimal frequency,

$$f^* = \sqrt{\frac{Y}{cT}\left(\frac{1}{2}P_{\mathrm{w}} + P_{\mathrm{v}}tY\frac{l}{L}\right)}, \tag{15}$$

known as the "square root formula" (Jansson, 1980, 1984). According to this result, the optimal frequency increases proportionally to the square root of total demand if the second term in parentheses is negligible relative to the first, but it can vary proportionally to the demand if the opposite occurs. A similar "square root formula" was found for the first time by Mohring (1972), and similar results are found in other single-line microeconomic models, as reviewed by Jara-Diaz and Gschwender (2003a).

Finally, on substitution of the optimal fleet size from equation (14) into equation (13) the minimum of VRC is obtained, i.e. the total cost function C_{T}:

$$C_{\mathrm{T}} = ctY + 2\sqrt{cTY\left(\frac{P_{\mathrm{w}}}{2} + P_{\mathrm{v}}tY\frac{l}{L}\right)} + P_{\mathrm{v}}TY\frac{l}{L}. \tag{16}$$

It is interesting to note that the operators' cost corresponds to the first term plus half of the second (i.e. the square root), and that the users' cost includes the square root plus the third term. The total average cost is

$$AC_{\mathrm{T}} = ct + 2\sqrt{cT\left(\frac{P_{\mathrm{w}}}{2Y} + P_{\mathrm{v}}tY\frac{l}{L}\right)} + P_{\mathrm{v}}T\frac{l}{L}, \tag{17}$$

which decreases with the demand level (Y), implying that scale economies exist. This reduction is due to the inclusion of the users' cost in the microeconomic formulation. This single-line result has been extended to various simple network cases (Jara-Diaz and Gschwender, 2003b).

4.2. *Other relevant aspects*

The model presented above does not include some important aspects that could change the cost structure. In what follows, these issues are discussed in order to ascertain their effects on scale economies.

Evans and Morrison (1997) constructed a model incorporating accident risk and non-scheduled delay. Both are included as part of the users' cost, but they also affect the operators' cost, as this has to be increased in order to reduce both risk and delays. Evans and Morrison found that the inclusion of these variables slightly increased scale economies.

On the other hand, Kocur and Hendrickson (1982) and Chang and Schonfeld (1991) included the spatial dimension optimizing the distance between parallel routes. By doing so, both optimal vehicle frequency and optimal line density are proportional to the cube root of the demand. This implies that scale economies are due not only to waiting time, which decreases at a slower rate than in the single-line case, but to access time as well.

As stated by Kerin (1992), none of these models incorporate externalities such as congestion or pollution, which would, apparently, tend to decrease the level of scale economies. If congestion is analyzed in an isolated public transport corridor in the medium term (frequency adjustment), there are two forms in which it can emerge. On one hand, as frequency grows with demand, interactions between vehicles will occur, affecting their speed. On the other hand, as frequency reaches an upper limit, growing occupancy rates will increase waiting times because of passengers not being able to board the first vehicle that arrives. The former effect is more important in the case of flexible modes (buses), and the latter is more relevant in rail modes, where a maximal frequency cannot be exceeded because of operational and safety reasons. Therefore, in the isolated corridor analysis there could be some demand level after which congestion becomes important enough to make the average total cost increase, as in the private car case.

However, the isolated corridor analysis becomes limited for high demand levels, as a possible way to adapt a public transport service to high demands is to increase the route density. The cubic root rule found by Kocur and Hendrickson and by Chang and Schonfeld for the optimal frequency shows that when route density is optimized, the optimal frequency increases slower than in the isolated corridor analysis (square root rule), slowing down congestion effects. On the other hand, the relationship between public transport and the car has to be considered when dealing with externalities, as they are usually substitute modes. This means that an increase in the total number of motorized trips has to be distributed between public and private transport. In their single public transport line optimization model, Oldfield and Bly (1988) considered that an increase in public transport demand implies a reduction in car use. Therefore, the final effect of the growth in public transport demand can be a reduction in the congestion level, due to a lower number of cars circulating. A similar analysis can be done in the case of pollution, as normally the emissions per trip are less for public transport than for cars. For synthesis, both the density analysis and the car – the transit substitution effect – make the effect of congestion on scale economies less important.

5. The impact of substitute modes and second-best fares

So far, we have implicitly assumed that all inputs and substitute or complementary modes are optimally priced (at marginal cost). Nevertheless, the car is normally underpriced. If it is a substitute for public transport, this will imply a more intensive use of the car than under the optimal modal split (congestion pricing). To correct this distortion, it is possible to underprice public transport as well, i.e. increase the amount of the optimal subsidy. Including this effect, Mohring (1979) found a negative optimal fare for a bus system. However, Kraft and Domencich (1970) state that for a given subsidy amount it is possible to attract more car users to public transport by diminishing the access, waiting, and in-vehicle times than by reducing the fare.

Nevertheless, underpricing public transport in any form (reducing fares or passenger times) to deal with the underpriced car will have an impact on the use of other modes such as the bicycle and walking (Kerin, 1992). They are substitute modes, especially for short trips, and their use will be reduced in spite of the fact that they are efficient modes in terms of congestion and pollution. Therefore, trying to optimize prices for every motorized mode would appear to be a better pricing policy than underpricing all of them.

If for any reason the optimal subsidy cannot be implemented, the second best fares are those that maximize the social benefit subject to covering the operators' cost with the fare box. In this case, those markets with larger price elasticities should have their fares closer to the optimal ones. On the other hand, other markets with lower price elasticities should have larger increases in their fares. This Ramsey pricing rule (Ramsey, 1927) allows fare revenues to be increased with the minimum modal split shift and, therefore, with the lowest loss in social benefit.

6. The optimal fare and distance

A public transport system allows trips between many origin–destination pairs. Thus, trips with different lengths will usually occur. This is not considered in models such as the one presented in Section 4, where all passengers travel the same distance and, therefore, a single marginal total cost and a single average users' cost exists, yielding a single optimal fare. Now, if passengers travel different distances, the question of the effect of trip distance on fares arises.

Cervero (1981) argued that the optimal fare should be positively related to the trip distance, but his conclusion was obtained considering only the operators' cost. When the users' cost is included in the analysis as well, the optimal fare will depend on the impact that a marginal passenger produces on the operators' cost plus the change that he or she makes to the average users' cost multiplied by the total number of passengers, as shown in equation (8). Nevertheless, it must be pointed out that this single-product analysis is not adequate when trips with different distances are considered. Thus, equation (8) only gives partial insight, as a multiproduct view is necessary for a formal analysis (e.g. Kraus, 1991).

Analyzing a feeder route and considering both the operators' and users' costs, Mohring (1972) argued that the optimal fare should be inversely related to distance, i.e. exactly the opposite to Cervero's conclusion. In the feeder route, passengers are boarding all along the line, but all of them alight at the end of the route. Thus, each passenger that boards makes those users who boarded before wait inside the vehicle for the time that he or she needs to embark. The closest to the end of the service a passenger boards (shorter trips), the fuller the vehicle will be and, therefore, more passengers will have to wait. As this makes the second term on the right-hand side of equation (8) larger, the optimal fare should be larger as well.

In a later work, Turvey and Mohring (1975) emphasized that the distance of the trip is not the relevant issue. They identify two reasons related to the occupancy of the vehicle that makes the fare higher: the first is the "Mohring effect" mentioned above, and the second (Turvey and Mohring effect) is that the higher the occupancy of the vehicle when the passenger travels in it, the higher the probability of other users not being able to board because of a full vehicle, increasing their waiting times. The latter effect is somehow positively related with trip distance.

Kraus (1991) constructed a multiproduct model where the passenger flow on each origin–destination pair is a different product. Besides including the impact of a marginal passenger on the in-vehicle time of the other users because of boarding and alighting (the Mohring effect), Kraus was the first to take into account crowding inside the vehicle. This "Kraus effect" is considered through a variable value of in-vehicle time, which increases with the occupancy factor. During the time that a passenger is inside the vehicle, he or she makes other users travel more uncomfortable. As in the case of the "Turvey and Mohring effect," this is partly positively related with trip length.

In summary, it is not very clear how trip length affects the optimal fare. Both Kraus and Turvey and Mohring found that the optimal fare should be in part positively related and in part inversely related with trip distance. Regarding the users' cost, what seems to be important for the optimal fare is the occupancy of the vehicle both when the passenger boards/alights and during the time that he or she travels in it. The first effect is that the users that are inside the vehicle have to wait when a passenger boards or alights (the Mohring effect); the higher the occupancy of the vehicle, the more time is "lost" due to the boarding/alighting of the marginal passenger. The second effect is that the space occupied inside the vehicle by a passenger increases crowding and discomfort for the other users (the Kraus effect) and increases the probability of other users not being able to board the vehicle (the Turvey and Mohring effect). Again, the fuller the vehicle, the higher the impact of the marginal passenger on the other users.

As there is no clear relation between optimal fare and trip length for a given route, flat fares seem to be a good option. Moreover, a flat fare is easier and cheaper to implement. Nevertheless, if routes with different lengths exist, it may be reasonable to have higher flat fares on the longer routes because of the operators' costs.

7. Summary and discussion

Just as in the case of private transport, public transport pricing analysis should consider both the operators' and users' costs. The former includes operational and capital costs, and the latter encompasses all types of travel time. The time frame for optimal pricing is relevant only if fixed inputs are set at a level that is different to the optimal, which will normally be the case for those services that are in operation at the time of the analysis. In general, frequency is a variable that can be adjusted easily, and, therefore, optimal pricing should always consider this as a variable. Adapting route length and route density is easier for some modes (bus) than for others (urban trains and trams). Optimal frequencies increase less than proportionally with demand, following a square root rule if everything else is fixed, and a cubic root if lines density is a variable.

Operators' cost models show that average costs diminish with patronage in general, and the same happens with those components of users' costs related with waiting and access time. In-vehicle time tends to increase because of boarding/alighting and congestion. In general, average costs diminish for both operators and users as patronage increases (economies of scale). As the optimal fare corresponds to the difference between the total marginal cost and the average users' cost, the preceding technical property translates into optimal fares that fall short of the operators' cost, making a subsidy necessary. In the absence of congestion pricing, the car is sub-priced, which makes the optimal subsidy even larger under a social optimum scheme because of the substitute nature of the car–public transport relationship. Public transport congestion would tend to diminish that subsidy.

Intuition based only on the operators' costs suggests that optimal fares should increase with distance (trip length). However, the inclusion of the users' costs in the analysis shows that the relevant variable is the occupancy rate when the (marginal) passenger boards, travels, and alights, besides the effect on the operators' costs. These two effects have opposite signs that seem to make a case for flat fares (larger for longer routes, though). Note that in this analysis of the relation between optimal fares and distance the effect on urban space has not been taken into account. Fares that increase with trip length would push toward urban concentration.

Optimal subsidies in public transport as presented here are conceived to make prices equal to the total marginal cost for the optimal operation of the system, and should not be mistaken with a capital cost subsidy. These latter subsidies may produce some distortions in the evolution of public transport systems. For example, Armour (1980) and Frankena (1987) found that capital cost subsidies caused buses to be replaced earlier. Furthermore, this type of subsidy increases the probability of constructing capital-intensive projects (e.g. metro lines) in corridors where low demands do not justify them (Small, 1992). On the other hand, when the operation is subsidized, various authors (Bly et al., 1980; Anderson, 1983; Pucher and Markstedt, 1983) found that the operators' cost increases, because of higher salaries and productivity reductions.

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