THE GOODS/ACTIVITIES FRAMEWORK FOR DISCRETE TRAVEL CHOICES: INDIRECT UTILITY AND VALUE OF TIME

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ABSTRACT

We present the properties of the conditional indirect utility function corresponding to an expanded version of the goods/leisure trade-off model, which includes work and travel time as direct sources of utility. The analysis is focused on the role of the marginal utilities in the formation of a general interpretation of the subjective value of travel time. We show that this analysis depends on the exogeneity of income, and on the relation between goods consumption and consumption time. The marginal utility of work is shown to be particularly important.

INTRODUCTION

The microeconomics of travel choices is presently sustained by two powerful frameworks. One is the stream of consumer behaviour theories that include explicitly the time dimension in various forms, and the other is the theory of discrete choices, which are those decisions related with the acquisition of one unit of a general type of good within a specific set of finite alternatives. Time related consumer theories have an evident relation with travel, as travel means relocation in space-time. On the other hand, travel choices have been for long understood and modelled as a series of discrete decisions, i.e. to travel or not, where to go, and how to do it; hence the role of discrete choice theory.
When discrete choices are seen from a microeconomic viewpoint, the most important single theoretical construct is the Conditional Indirect Utility Function, which represents the maximum utility that could be obtained if a particular (discrete) choice was made. The arguments and properties of this function depend on the particular manner in which consumer behaviour is understood and modeled.

In this article I want to explore the properties of the (conditional) indirect utility function that corresponds to an expanded version of the goods/leisure framework used to model travel choices: the goods/activities framework. I will show that, in its simplest form, this framework generates neat interpretations of the value of travel time, which further complicates when some necessary relations are added, regarding a usually neglected relation between goods and leisure. The important role of the marginal utility of work is particularly highlighted.

**FROM GOODS/LEISURE TO GOODS/ACTIVITIES**

The specific approach to address the presence of time in transport models within the discrete choice framework, has been the goods/leisure trade-off model with mode choice for a specific trip. The approach rests upon a utility function that increases both with the general consumption of goods \( G \) and with time available out of work (leisure \( L \)). Two versions of this model can be built: the original one (Train and McFadden, 1978) in which the individual decides how many hours \( W \) to work at a pre-specified wage rate \( w \), and an alternative one (Jara-Díaz and Farah, 1987) in which \( W \) and \( I \) are fixed within the relevant period. The goods /leisure trade-off arises because of the inverse effect of \( W \) on \( G \) and \( L \): a high value of \( W \) makes goods consumption large and makes leisure small. A small value of \( W \) reverses the effects.

Within this framework, the individual has potentially two choices: how many hours to work and which mode to use. Each mode \( i \) has an associated cost \( c_i \) and travel time \( t_i \). If fast-expensive modes compete against slow-cheap ones, the trade-off will be present even if \( W \) (and \( I \)) are fixed, because mode choice translates into a goods-leisure choice through the income and time constraints.

On important property of the goods/leisure framework in its original version (self decision on \( W \) and \( I = wW \)) is that the **subjective value of travel time** \( SV \), calculated from the corresponding discrete travel choice model is equal to the wage rate. This result should not be a surprise, as the individual adjusts his/her hours of work such that utility is maximized. As the only reward from work time is \( w \), the level of \( W \) will be adjusted until the value of leisure time is \( w \) as well,
as if it was larger (smaller) than \( w \) the individual would work less (more). Evidently, this property vanishes in the exogenous income version.

The goods/leisure trade-off model corresponds directly to the approach by Becker (1965), who introduced time in the individual utility function in addition to market goods and thus expanded consumer’s theory. The time vector \( T \) in Becker explicitly accounted for the preparation and consumption of the so-called final commodities, therefore leaving aside working time and other activities (as travel to work) as direct sources of (dis)utility. This omission was criticized by various authors (Johnson, 1966; Oort, 1969; De Serpa, 1971; Evans, 1972) through a series of articles that ended up in the pioneering proposition by Evans (curiously ignored in the literature), who postulated that utility depends primarily on what the individual does; the goods \( X \) would play the role of a necessary input. This activity framework has appeared again in the literature (e.g., Gronau, 1986; Winston, 1987; Juster, 1990), probably due to the never-ending pressure (socially induced) on the individual’s committed time. I have recently proposed a general model of transport users’ behaviours that rescues Evans’ contribution (Jara-Díaz, 1994).

By introducing only leisure in addition to goods in the utility function, the Train and McFadden (1978) and Jara-Díaz and Farah (1987) models suffer from the same limitation as Becker’s. In what follows we extend the framework to explore its implications regarding the subjective value of time.

The main idea in the activity framework is that all activities have a potential impact on direct utility. In fact, a \( U(T) \) utility function as in Evans (1972) or Jara-Díaz (1994) has limitations because the marginal utility of an additional time unit assigned to a specific activity certainly depends on the type and amount of goods used to actually perform the activity. Thus, a \( U(X, T) \) function seems general enough (although we would like to stress the fact that it is \( T \) the basic source of satisfaction). When using the goods/leisure framework within an activity approach, we need to introduce both \( W \) and travel time using mode \( i \), \( t_i \), as potential sources of direct (dis) utility. Therefore, the most general expanded version of the model with endogenous income is

\[
\text{Max} U(G, L, W, t_i) \tag{A}
\]

subject to

\[
G + c_i = wW
\]

\[
L + W + t_i = \tau
\]

\[
i \in M
\]
where \( r \) is the reference period and \( M \) is the set of alternatives (modes).

Using the discrete choice procedure, the conditional continuous problem in \( W \) is obtained assuming \( i \) given and replacing \( G \) and \( L \) in \( U \) from the constraints. This yields

\[
\max_{W} \quad U[(wW - c_i), (r - W - t_i), W, t_i] \quad \text{(1)}
\]

The first order condition is

\[
\frac{d U}{d W} = \frac{\partial U}{\partial G} w - \frac{\partial U}{\partial L} + \frac{\partial U}{\partial W} = 0 \quad \text{(2)}
\]

from which the conditional optimal amount of work \( W^* \) (\( -c_i, w, r - t_i, t_i \)) can be obtained. Result (2) shows that the individual will choose working hours such that the marginal utility of leisure equals the marginal utility of labour, but this latter now has two components: the wage rate times the marginal utility of goods, and the marginal utility of pure labor (which was nil in the original goods/leisure model). The intuitive explanation is straightforward: those who like their jobs would be willing to work not only for the reward in terms of purchasing power, but also for the pleasure of it; everything else constant, this would make such individuals to work more, having less leisure with a higher marginal utility. The reverse occurs if work provokes disutility. From this, the conditional indirect utility function flows as

\[
V_i = U \left[(wW^* - c_i), (r - W^* - t_i), W^*, t_i \right] = V(c_i, t_i) \quad \text{(3)}
\]

which is the generic version of the so-called modal utility that commands mode choice. From this, the subjective value of travel time \( SV_i \) can be obtained in the usual manner, as the ratio between \( \partial V_i / \partial t_i \) and \( \partial V_i / \partial c_i \). First we trivially get

\[
\frac{\partial V_i}{\partial t_i} = \frac{\partial U}{\partial G} w \frac{\partial W^*}{\partial t_i} - \frac{\partial U}{\partial L} \left( \frac{\partial W^*}{\partial t_i} + 1 \right) + \frac{\partial U}{\partial W^*} \frac{\partial W^*}{\partial t_i} + \frac{\partial U}{\partial t_i} \quad \text{(4)}
\]

But \( W^* \) fulfills condition (2), which combined with eq. (4) yields
\[
\frac{\partial V_i}{\partial t_i} = -\frac{\partial U}{\partial G} w - \frac{\partial U}{\partial W} + \frac{\partial U}{\partial t_i} .
\] (5)

A similar procedure can be used to obtain

\[
\frac{\partial V_i}{\partial c_i} = \frac{\partial U}{\partial G} \left( w \frac{\partial W^*}{\partial c_i} - 1 \right) - \frac{\partial U}{\partial L} \frac{\partial W^*}{\partial c_i} + \frac{\partial U}{\partial W} \frac{\partial W^*}{\partial c_i} = -\frac{\partial U}{\partial G} .
\] (6)

From eqs. (5) and (6) we finally get

\[
SV_i = \frac{\partial V_i}{\partial t_i} = w + \frac{\partial U}{\partial W} - \frac{\partial U}{\partial G} .
\] (7)

Equation (7) is in fact a very general result for the case of endogenous income. It says that the subjective value of travel time is equal to the wage rate (which is the value in goods units of a unit time saved in travel) plus the subjective value of pure work (which is the goods equivalent of one additional unit time at work) minus the subjective value of pure travel (which is the goods equivalent of one less unit time traveling). In other words, a reduction in travel time is (individually) important because of more work (more (dis)pleasure, more money) and less travel. Note that the result is general, as it holds for positive, negative and null values for the marginal utilities of work and travel. Thus, if an individual likes the job and dislikes travelling, the \(SV_i\) is definitely higher than the wage rate as saving one minute would mean more money, more pleasure from work, and less displeasure from travel.

The case of exogenous income is simple but interesting. It presents, though, an asymmetric condition in this model with work as a source of direct (dis) satisfaction: the individual can not diminish working hours even if he or she dislikes work, but nothing prevents that person from working more if work is pleasurable, in spite of no additional money reward. The general formulation is identical to that of problem (A), but now \(wW=I\) and \(W\) has to be at least equal to the fixed amount \(W_F\). Replacing the equality constraints we get
\[
\max_{i \in M} U\left[(1 - c_i), (\tau - W - t_i), W, t_i\right] \bigg/ W - W_F \geq 0.
\]

If problem (B) is solved for \(W\) conditional on \(i\), only leisure and work contribute to the variation in \(U\) after a change in \(W\). Thus, the first order conditions are

\[
- \frac{\partial U}{\partial L} + \frac{\partial U}{\partial W} + \mu = 0, \quad \mu (W - W_F) = 0, \quad \mu \geq 0.
\]

If the individual chooses to work more than required, \((W^* > W_F)\) then the multiplier \(\mu\) will be nil and the marginal utility of leisure (positive) equals the marginal utility of pure labor, which has to be positive in this case. Then we have \(W^* \left[ (1 - c_i), (\tau - t_i), t_i \right]\) and \(\partial U / \partial W\) positive. Then the conditional indirect utility function for \(W > W_F\) is

\[
V_i = U\left[(1 - c_i), (\tau - W^* - t_i), W^*, t_i\right] \bigg/ (W - W_F).
\]

Therefore

\[
\frac{\partial V_i}{\partial t_i} = - \frac{\partial U}{\partial L} \left( \frac{\partial W^*}{\partial t_i} + 1 \right) + \frac{\partial U}{\partial W} \frac{\partial W^*}{\partial t_i} + \frac{\partial U}{\partial t_i}.
\]

Applying condition (8) for \(\mu = 0\),

\[
\frac{\partial V_i}{\partial t_i} = - \frac{\partial U}{\partial W} + \frac{\partial U}{\partial t_i}.
\]

A similar procedure for \(\partial V_i / \partial c_i\) yields the usual result - \(\partial U / \partial G\), which makes the \(SV_i\) equal to

\[
SV_i = \frac{\partial V_i / \partial t_i}{\partial V_i / \partial c_i} = \frac{\partial U / \partial W}{\partial U / \partial G} - \frac{\partial U / \partial t_i}{\partial U / \partial G}.
\]
Before making an interpretation of this result, note that the case of $\mu > 0$ (or $W^* = W_f$) makes $\partial W^* / \partial t_i$ in eq. (10) equal to zero which yields

$$SV_i = \frac{\partial V_i / \partial t_i}{\partial V_i / \partial c_i} = \frac{\partial U / \partial L}{\partial U / \partial G} - \frac{\partial U / \partial t_i}{\partial U / \partial G}.$$  

(13)

Therefore, in the case of exogenous income the subjective value of travel time is always equal to the money value of leisure minus the money value of pure travel; if the individual works more than strictly required, then the money value of leisure is equal to the subjective value of pure work (which has to be positive). A general result for the fixed income case is

$$SV_i = \frac{\partial U / \partial W}{\partial U / \partial G} + \frac{\mu}{\partial U / \partial G} - \frac{\partial U / \partial t_i}{\partial U / \partial G}.$$  

(14)

Results (7) and (14) synthesize the endogenous and exogenous income cases respectively in terms of the subjective value of time, within the "goods/activities" framework. It is interesting to note that there is no reason a priori to expect an $SV_i$ equal to either the wage rate $w$ in the first case or the ratio $I/W$ in the second case. This last property also holds for the goods/leisure model with exogenous income, as shown by Jara-Diaz and Farah (1987).

**THE MISSING LINK BETWEEN GOODS AND LEISURE**

The extended version of the goods/leisure framework is richer than its predecessor, as it allows for a direct effect of both work and travel on utility. This is not only appealing intuitively, but also generates analytical results, which contain the previous ones as particular cases. Nevertheless, the new framework still lacks an important relation among its elements, which we will now discuss.

Although Becker (1965) established an implicit relation between goods $X$ and consumption-preparation times $T$, it was DeSerpa (1971) who made it explicit that there were minimum consumption times, adding a set of constraints that account for this. Later, Evans (1972) imposed a matrix that turned goods into activities. This idea was rescued by Jara-Diaz (1994)
and Jara-Díaz et. al. (1994) in the form of a transformation function that represented a (potentially large) series of technical relations that turned goods into time and time into goods.

Within the goods/leisure and goods/activities frameworks, as presented here, the relations between physical consumption and its time counterpart become relations between $G$ and the time aggregates. In what follows, I will make the simplifying assumptions that goods are consumed during $L$ and that work and travel do not require the acquisition of goods. As goods consumption require time, let me define $\alpha$ as the rate of consumption in time units per unit $G$. According to the first simplifying assumption, $G$ and $L$ have to fulfil

$$L - \alpha G \geq O, \quad (15)$$

which means that the resulting leisure time should be large enough to permit the consumption of the resulting amount of goods.

If this framework was stated in terms of detailed consumption $X$ and activities $T$, a second type of condition should be imposed, as activities require certain minimum combination of goods to be developed (think of sports, entertainment, social life, domestic life, etc.) But, in this aggregated conceptual framework, the minimum goods requirement loose meaning as leisure activities have been fully aggregated. Thus, let us concentrate on the conditional analysis of problem $A$ with the addition of constraint (15) and its associated multiplier $\theta$.

Again, assuming $i$ given, the problem can be solved in $W$ by replacing $G$ and $L$ as functions of $W$ from the income and time constraints, in both the utility function and the new constraint (15). Thus, we get

$$\max_{W} U[(wW - c_i), (r - W - t_i), W, t_i] \quad \cdot \quad (C)$$

subject to

$$r - W - t_i - \alpha (wW - c_i) \geq 0.$$

The first order conditions for problem $C$ are
\[
\begin{align*}
\frac{\partial U}{\partial G} w - \frac{\partial U}{\partial L} + \frac{\partial U}{\partial W} + \theta \left( -1 - \alpha w \right) &= 0 \quad (16) \\
\theta \left[ r - t_i + \alpha c_i - W^* (1 + \alpha w) \right] &= 0 \quad , \quad \theta \geq 0 \quad . \quad (17)
\end{align*}
\]

Equations (16) and (17) yield generic solutions for \( W \) and \( \theta \). In general we will get \( W^*(w, c_i, t_i) \) which, once replaced in the utility of problem \( C \), yields an indirect utility function formally equal to (3) and a marginal utility of time that looks exactly as (4). In this case, by equation (16) we get

\[
\frac{\partial V_i}{\partial t_i} = \frac{\partial W^*}{\partial t_i} \left( 1 + \alpha w \right) \theta - \frac{\partial U}{\partial L} + \frac{\partial U}{\partial t_i} . \quad (18)
\]

Similarly

\[
\frac{\partial V_i}{\partial c_i} = \frac{\partial W^*}{\partial c_i} \left( 1 + \alpha w \right) \theta - \frac{\partial U}{\partial G} . \quad (19)
\]

As expected, the case of \( \theta = 0 \) yields a value of \( SV \) given by equation (7). The novel case is \( \theta > 0 \), for which

\[
W^*(c_i, t_i) = \frac{r - t_i + \alpha c_i}{1 + \alpha w} \quad . \quad (20)
\]

and

\[
\frac{\partial W^*}{\partial t_i} = -\frac{1}{1 + \alpha w} \quad , \quad \frac{\partial W^*}{\partial c_i} = \frac{\alpha}{1 + \alpha w} . \quad (21)
\]

If these are replaced in the general expressions for \( \partial V_i / \partial t_i \) and \( \partial V_i / \partial c_i \), we get
\[
\frac{\partial V_i}{\partial t_i} = -\theta - \frac{\partial U}{\partial L} + \frac{\partial U}{\partial t_i} \tag{22}
\]

\[
\frac{\partial V_i}{\partial c_i} = \alpha \theta - \frac{\partial U}{\partial G} \tag{23}
\]

and a more general result for \(SV_t\) is obtained, namely

\[
SV_t = w + \left( \frac{\partial U / \partial \bar{W}}{\frac{\partial U}{\partial G} - \alpha \theta} \right) - \left( \frac{\partial U / \partial \bar{t}_i}{\frac{\partial U}{\partial G} - \alpha \theta} \right). \tag{24}
\]

The interpretation of this general result follows. If the multiplier \(\theta\) is non-zero (positive), then constraint (15) is active, which means that goods consumption is limited by leisure time. Thus, travel time is more important and travel cost is less important than in the usual case. Accordingly, the marginal utility of travel time is (in absolute terms) larger than the sum of the gain in (more) leisure and the gain in (less) travel (equation 22). And the marginal utility of cost is, in absolute terms, smaller than the marginal utility of goods consumption (equation 23). Regarding the relation between leisure and work, equation (16) confirms that the subjective value of leisure is less than the subjective value of work plus the money value of the goods equivalent.

The subjective value of travel time somehow synthesizes the effects of leisure as an active constraint for goods consumption. The denominator of the second and third terms in equation (24) is less than the marginal utility of goods consumption and, therefore, the second term is larger than the subjective value of pure work and the third expression is larger than the direct value of travel time. Thus, if a person likes working and dislikes traveling, his/her subjective value of travel time will exceed the wage rate by a larger amount than in the non-binding leisure time case. This has a clear intuitive interpretation, as travel time not only reduces leisure and (pleasurable) working time, but also diminishes goods consumption through leisure availability.

The case we have seen in this section is one in which the individual might run out of free time. In a model with more detail than the one presented here (goods and activities), constraints
regarding goods requirement for leisure activities might cause the opposite result, in which the individual runs out of money and still have free time left. This case was in fact identified by Evans (1972) and rescued by Jara-Díaz (1997).

The fixed income case is, again, simple but interesting. As stated earlier, now the individual has to work at least \( W_F \) hours, but he/she can work more if desired. The problem in \( W \) is similar to \( B \) plus constraint (15) written as in problem \( C \) above, i.e.

\[
\begin{align*}
\max_w & \quad U[(I - c_i), (\tau - W - t_i), W, t_i] \\
\text{subject to} & \quad \tau - W - t_i - \alpha (I - c_i) \geq 0 \\
& \quad W - W_F \geq 0
\end{align*}
\]  

The first order conditions are

\[
- \frac{\partial U}{\partial L} + \frac{\partial U}{\partial W} - \theta + \mu = 0
\]  

\[
[\tau - W - t_i - \alpha (I - c_i)] \theta = 0 \quad \theta \geq 0
\]  

\[
(W - W_F) \cdot \mu = 0 \quad \mu \geq 0
\]

from which we get \( W^*(c_i, t_i) \) and the conditional indirect utility function \( V_i \) (see equation 9). The expression for \( \partial V_i / \partial i \) is similar to equation (10), and replacing from eq. (25) we get
\[ \frac{\partial V_i}{\partial t_i} = \frac{\partial W^*}{\partial t_i} (\theta - \mu) - \frac{\partial U}{\partial L} + \frac{\partial U}{\partial t_i}. \] 

(28)

On the other hand, after a few manipulations,

\[ \frac{\partial V_i}{\partial c_i} = -\frac{\partial U}{\partial G} + \frac{\partial W^*}{\partial c_i} (\theta - \mu). \] 

(29)

Out of the many possible combinations on \( \theta \) and \( \mu \), the most illustrative is the case with \( \theta > 0 \) and \( \mu = 0 \), which means that at the optimum the individual works more than the strictly necessary \( W_r \), but is limited by time availability to consume \( G \). Analytically this means

\[ W^* = t_i - c_i \alpha (1 - c_i) > W_r, \quad \frac{\partial W^*}{\partial t_i} = 1, \quad \frac{\partial W^*}{\partial c_i} = \alpha \] 

(30)

and

\[ \frac{\partial U}{\partial W} = -\frac{\partial U}{\partial L} + \theta. \] 

(31)

Note that eq. (31) proves that this case is possible only if work is pleasurable, as both \( \theta \) and the marginal utility of leisure are positive. Intuitively, the marginal utility of work should be large enough for the unpaid extra work to overweight the loss in leisure and the loss in consumption because of limited leisure.

From equations (28), (29), (30) and (31) we obtain

\[ SY_t = \frac{\partial U}{\partial U} \frac{\partial W}{\partial L} - \frac{\partial U}{\partial G} - \alpha \theta, \] 

(32)

which is equal to the endogenous income case, except for the wage rate. This was to be expected, as work is freely adjusted but the marginal money reward is nil.
Finally, note that both cases with $\theta = 0$ in fact represent conditions which we have seen previously, as this means a non-binding leisure regarding goods consumption.

**SYNTHESIS AND CONCLUSIONS**

We have expanded the goods/leisure framework to account for all activities as potential sources of utility, keeping the analysis at an aggregated level. Postulating goods, leisure, work and travel as direct sources of utility is not new, but a strict analysis through a general conditional indirect utility function (CIUF) within the framework of discrete travel choices, is a novel treatment which improves our understanding of what is behind time perception, the role of income and the subjective value of travel time.

Two families of models appear as extensions of the goods/leisure framework: those where income is endogenous (i.e. the individual decides how many hours to work at a pre-specified wage rate), and those with exogenous income. In the former case, the subjective value of travel time $SV_t$ has three components, namely the wage rate, the direct subjective value of work and the direct subjective value of travel time. In the latter case, the individual can decide to work more (unpaid), provided the marginal utility of work is positive. The corresponding $SV_t$ has two components if the individual works more than required. These components are equivalent to those in the endogenous income case with zero wage rate. If the individual works strictly according to contract, there is a third term corresponding to the difference between the direct subjective values of leisure and work.

Important results do arise after a new constraint is introduced, namely the necessary relation between goods and leisure, as goods consumption require consumption time. The first important result is that the marginal utility of travel cost, obtained from the CIUF, is not necessarily equal to the marginal utility of goods consumption. Moreover, if goods consumption happens to be limited by leisure, then its marginal utility is in fact larger than the marginal utility of income. This is not only interesting but also intuitively attractive, in line with the discussion by Evans (1972). We must consider, though, that we are assuming that all income is spent, and this needs further exploration.

If travel is seen as part of what individuals do, and utility depends primarily on activities as a result of time and money assignment, then travel decisions should be studied, modeled and understood, within the context of human activities. This has many implications like, for example, the need to understand the nature and perception of both work and leisure (e.g.
alienated or not, in the sense defined in Fromm, 1965), or the need to take into account the socially induced necessities, as highlighted by Marcuse (1968). This seems to be a very relevant point, as living in a hurry or "having no time" is becoming a new syndrome (and symbol of status) in both developed and developing societies. As stated by Lasch, "a profound shift in our sense of time has transformed work habits, values, and the definition of success" (pp.107). Thus, the social study of behaviour should become part of the effort towards meaningful microeconomic interpretations of travelers' decisions. Linking sociology, psychology and economics is not a sophisticated step towards meaningful travel choice models. It is a must.

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