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SHORT NOTES AND RESEARCH COMMUNICATONS

CONSUMER'S SURPLUS AND THE VALUE OF TRAVEL TIME SAVINGS

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Abstract—The implications of evaluating travel time savings from changes in consumer's surplus derived from discrete choice models are examined, and this approach is related to and compared with the standard procedure based on time pricing.

1. INTRODUCTION

Time savings are frequently one of the most important sources of benefit in transportation projects, whether these relate to management measures or major investments in infrastructure. This is evident in the cases of traffic control problems or in larger scale traffic projects analyzed with the help of delay minimizing simulation models, but this pertains to practically all types of projects. As is well known, the standard procedure involves producing an estimate of the physical amount of time saved due to the project by all users, which is then converted into monetary units using a social *value of time* figure (VT), previously agreed upon using some procedure which normally involves the evaluation of labor productivity, the estimation of demand models, and the approval of some planning authority (see, for example, Bates and Roberts, 1986, for a good picture of U.K. practice, and Winston *et al.*, 1987, for applications to automobile operations in the US). In this approach, time is taken as a (valuable) resource, the price of which is a matter for discussion.

On the other hand, time usually enters the specification of utility in mode choice models, often decomposed in various parts (walking, waiting, travel, etc.). Thus, time savings appear as gains in utility which can be translated into money measures using the tools of welfare analysis. Then the question of which approach should be used arises, that of time pricing, consumers' surplus, or both. In this paper we will try to clarify this problem by dealing directly with the relationship between accepted welfare measures and the traditional valuation of travel time savings. The analytical development that follows concentrates on changes in utility, caused by changes in fares and/or modal quality dimensions, within the context of mode choice. We show that both measures are approximately equal under certain analytical conditions, and we discuss the economic premises behind them.

2. USERS' BENEFITS AND QUALITY CHANGES

Presently, accepted measures of welfare variation from discrete mode choice models come from various sources: Williams (1977), McFadden (1981), and Small and Rosen (1981). Although we have previously shown that the three approaches coincide analytically (Jara-Diaz and Farah, 1988), here we will use Small and Rosen's version of users' benefits (UB) directly expressed by

$$UB = \frac{N}{\lambda} \int_{U^0}^{U^1} \sum_{i=1}^{m} \pi_i(U)\, dU_i, \tag{1}$$

where

U_i^j indirect utility of mode i in state j;

U^j vector of modal utilities in state j, $\{U_i^j\}$;

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π_i probability of choosing mode i, as a function of U;
λ marginal utility of income;
N total number of users whose behaviour is described by the probabilistic choice model
m number of available modes.

Expression (1) is the probabilistic line integral counterpart of Hotelling's (1938) generalized version of the consumers' surplus measure for continuous goods. As known, this line integral will have a unique value, independent of the path of integration, if

$$\frac{\partial \pi_i}{\partial U_j} = \frac{\partial \pi_j}{\partial U_i} \quad \forall\, i, j = 1, \ldots, m. \tag{2}$$

This condition (which is valid for the popular logit model) will be assumed valid here and a linear path will be chosen. Let

$$U_i = U_i^0 + \theta(U_i^1 - U_i^0) = U_i(\theta); \tag{3}$$

then $U_i(0) = U_i^0$, $U_i(1) = U_i^1$ and $dU_i = \Delta U_i\, d\theta$, where $\Delta U_i = U_i^1 - U_i^0$. Changing variables we get

$$UB = \frac{N}{\lambda} \int_0^1 \sum_i \pi_i[U(\theta)]\, \Delta U_i\, d\theta = \frac{N}{\lambda} \sum_i \Delta U_i \int_0^1 \pi_i[U(\theta)]\, d\theta. \tag{4}$$

The line integral has been reduced to a sum of simple integrals. If we further assume linearity of π_i between the 0 and 1 states, $\forall_i$, we finally get the compact form

$$UB \doteq \frac{N}{\lambda} \sum_i \Delta U_i\, \overline{\pi}_i = \frac{1}{\lambda} \sum_i \Delta U_i\, \overline{X}_i \tag{5}$$

with $\overline{\pi}_i$ defined as $\frac{1}{2}(\pi_i^0 + \pi_i^1)$; $\overline{X}_i = N\overline{\pi}_i$ is the expected number of mode i's users.

Let us now turn our attention to the arguments of U_i, which we will define as

C_i: user money cost of a trip by mode i
q_{hi}: mode i's quality dimension h (e.g., travel time).

Then a local variation in utility, ΔU_i, can be expressed in terms of local variations of cost and quality (ΔC_i and Δq_{hi}, respectively), i.e.

$$\Delta U_i = \frac{\partial U_i}{\partial C_i} \Delta C_i + \sum_{h=1}^{n} \frac{\partial U_i}{\partial q_{hi}} \Delta q_{hi}. \tag{6}$$

Replacing eqn (6) in eqn (5) we obtain

$$UB \doteq (\partial U_i/\partial C_i/\lambda)\, \Delta C_i\, \overline{X}_i + \sum_i \sum_h (\partial U_i/\partial q_{hi}/\lambda)\, \Delta q_{hi}\, \overline{X}_i. \tag{7}$$

Here we have to recall some basic definitions and microeconomic properties of discrete choice theory. First of all, the marginal utility of income is equal to minus the partial derivative of the (conditional indirect) utility function with respect to cost; this can be intuitively understood as the discrete version of Roy's identity (see Small and Rosen, 1981, or Jara-Diaz and Farah, 1988), i.e.

$$-\frac{\partial U_i}{\partial C_i} \bigg/ \lambda = 1. \tag{8}$$

On the other hand, the implicit tradeoff between cost and quality dimension h at a constant level of utility is the *subjective value of h* (SVh) and can be calculated (using the implicit function property) as minus the ratio of two partial derivatives of utility. For simplicity, we shall assume that this is a generic (i.e., nonmode specific) property, and write,

$$SVh \doteq (\partial U_i / \partial q_{hi}) / (\partial U_i / \partial C_i) = (\partial U_i / \partial q_{hi}) / \lambda. \tag{9}$$

Replacing properties (8) and (9) in eqn (7) after elementary manipulation, we obtain the main result

$$UB \doteq - \sum_i \overline{X}_i \, \Delta C_i + \sum_h SVh \sum_i \overline{X}_i \, \Delta q_{hi}. \tag{10}$$

Equation (10) shows that the welfare measure (1) is approximately equal to the well known rule-of-a-half measure after a price change plus a series of terms induced by quality variations, each one weighted by its corresponding subjective value. Let us concentrate now in the particular case of time savings.

3. BENEFITS FROM TIME SAVING PROJECTS

Consider a project that only involves time savings; neither costs nor other quality dimensions change. As stated earlier, two possibilities are open to quantify benefits. First, if a demand model has been estimated, users' benefits can be calculated using the consumers' surplus measure given by eqn (1), which has been shown to be approximately equal to expression (10). In this case, then

$$Benefits = UB \doteq SVT \sum_i \overline{X}(t_i^0 - t_i^1), \tag{11}$$

where SVT is the subjective value of time that is implicit in the demand model (eqn 9) and t_i^j is the time spent on mode i in state j (0 or 1, without or with project, respectively).

Alternatively, benefits can be calculated as total time savings TS which are priced using an (exogenously introduced) social value of time VT, i.e.

$$Benefits = TS = VT \left(\sum_i X_i^0 t_i^0 - \sum_i X_i^1 t_i^1 \right). \tag{12}$$

By inspection, it is clear that both approaches will approximately coincide provided modal split does not vary significantly, and the subjective value of time SVT is accepted as the (social) value of time VT. Therefore, the question of whether the consumers' surplus welfare measure is appropriate or not to evaluate benefits in this type of time saving project, is equivalent to the question of whether the SVT is an adequate measure of VT. This alternative formulation actually concentrates the discussion on what is conceptually behind these values.

Behind SVT are the individual preferences, presumably well represented by a mode choice model, and models are usually calibrated depending on trip purpose. Let us concentrate on the journey to work; in this case, most of the empirical work is based upon the theoretical work developed by Train and McFadden (1978). There, they show that an individual who freely decides how many hours to work, at a prespecified wage rate w, is going to choose that available mode i which maximizes $- c_i/w - t_i$. As evident, the implicit valuation of time from such a model is w. However, the stochastic version of this model allows from potential departures from SVT $= w$, since utility is usually specified as

$$U_i = \alpha_i + \beta(c_i/w) + \gamma t_i, \tag{13}$$

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$$-\frac{\partial U_i}{\partial C_i} \Big/ \lambda = 1. \tag{8}$$

such that the implict valuation of time is given by the (estimated) figure $(\hat{\gamma}/\hat{\beta})w$; the ratio $\hat{\gamma}/\hat{\beta}$ is refered to as the subjective value of time as a proportion of the wage rate. This ratio is expected to be less than one, as the case of an inelastic demand for labour at a prespecified rate w is the only one in which the *individual* (subjective) valuation of time in a work trip (or in *any* activity) would be equal to that rate, since decisions would be made in the margin and the (marginal) utility of leisure time would be equal to the individual opportunity cost w. This is regarded as an extreme possibility. It should be noted that in the specification of modal utility, w can be directly approximated by income/working hours (Ortuzar and Espinosa, 1987) or replaced by other proxies (Swait and Ben-Akiva, 1987).

Beyond the possible misspecification arising from the fact that individual income might be fixed in most cases (which we have discussed elsewhere), subjective valuation of time exceding the wage rate in empirical work is not a pathological case. In fact, this has been the case in *all* models estimated in Santiago, Chile, using data carefully gathered in different corridors and using models with different structures, functional forms, or variable definition (see, for instance, Gaudry *et al.*, 1989; there, the value of waiting time easily reaches seven times w). Thus, accepting consumers' surplus as the measure of benefits implies, in this case, accepting a social valuation of time that exceeds the individual wage rate. This implicit acceptance might not be in agreement with explicit considerations on VT. In fact, this is the case in Santiago, where the National Planning Office (ODEPLAN) recommends a flat value which is claculated as a fraction of average income, loosely following the U.K. practice of valuing commuting time at a fraction of *individual* income.

4. DISCUSSION AND CONCLUSIONS

We have shown that behind the choice of Consumers' Surplus as the measure of benefits in time saving projects, hides the implicit pricing of time saved at its subjective value SVT (i.e., the value that each individual reveals through his/her election of mode interpreted by means of mode choice models). This might seem reasonable since, after all, SVT is what individuals are willing to pay to save one minute of traveling. However, beyond the problem of the eventual sensitivity of SVT to model specifications, remains the policy issue that has actually arisen in some cases: is this acceptable to the planning authorities when SVT significantly exceeds the wage rate? In fact, one should expect that the coupling of values or life styles that resemble those of the developed world, with relatively low figures of income per capita, yield high (subjective) valuations of time relative to income. In other words, the mixture of (subjective) perceptions framed by the values of modern life, and (objective) lack of resources will tend to reveal preferences with implicitly high SVT.

For synthesis, if we are always in a hurry, running as young executives in New York, and we have an average one tenth of their purchasing power, should we use SVT? or, what we have shown to be equivalent, should we accept neo-classical welfare measures to evaluate time savings? The hidden choice behind the use of consumers' surplus is now evident.

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