A NOTE ON TRIP BENEFITS IN SPATIAL INTERACTION MODELS

FRANCISCO J. MARTÍNEZ AND CLAUDIO A. ARAYA
UNIVERSIDAD DE CHILE
POBox 228-3, Santiago, Chile
e-mail: fmartine@cec.uchile.cl

Submitted to the Journal of Regional Science
First Version submitted July 30, 1998
Revised Version submitted April 9, 1999
Final Version submitted March 1, 2000
A NOTE ON TRIP BENEFITS IN SPATIAL INTERACTION MODELS*

Francisco J. Martínez and Claudio A. Araya
Università de Chile, Casilla 228-3, Santiago, Chile. Email: fmartine@cec.uchile.cl

ABSTRACT: We extend the well-known transport users’ benefits measure (TUB) for the doubly-constrained spatial interaction model derived by Williams (1976). The original formula expresses the TUB as composed by two terms associated with the origin and the destination zones. First, the TUB is associated here with trips instead of zones, providing a natural interpretation as a rule-of-a-half measure of benefit under inelastic demand (or short run case). Secondly, a TUB formula for the long run case is derived, i.e. total number of trips, trip origins and destinations change. Updated measures of accessibility for location behavior are then proposed.

Journal of Economic Literature category: R41

*This research was partially funded by FONDECYT 1981206. The authors are grateful of Sergio Jara-Díaz useful comments.
1. INTRODUCTION

The importance of users’ benefits in transport projects assessments is well known by transport economists and planners in charge of defining the best project option in social terms. Therefore, measuring these benefits as well as understanding their correct interpretation and limitations is highly relevant as it may define what infrastructure should be developed given limited resources.

Measures of transport users’ benefits (TUB) are obtained by calculating the consumers’ surplus change, which depends on the travel demand function. One of the most used TUB measure was proposed by Williams (1976) for a travel demand modelled by the well-known doubly-constrained spatial interaction (or gravity) framework. These TUB measures are extensively applied as a standard method in transport project appraisals, based on the theoretical argument that travel demand is derived from activities’ demand so benefits measured in the transport system take into account all benefits transferred into the activity system (Mohring, 1976; Wheaton, 1977; Jara-Díaz, 1986).

This note contributes in extending Williams’ measures in two ways. First, the original formulae is rewritten allowing a new interpretation of users’ benefits since it allocates benefits to individual trips instead to the origin and destination zones. Thus, the relationship between zone benefits and trip benefits is clearly established providing a more complete interpretation of these benefits. As a second contribution we derive TUB measures for the long run, defined as the case where trip origins and destinations change between two situations, which allows
considering changes on land use. These long run measures provide a method to assess transport projects independently, avoiding the comparison to another alternative option (the usual with and without project comparison).

2. SHORT RUN TRANSPORT USERS’ BENEFITS

The doubly-constrained spatial interaction model for travel demand is expressed by:

\[
T_{ij} = A_i O_i B_j D_j \exp(-\beta c_{ij}) \\
\sum_j T_{ij} = O_i \\
\sum_i T_{ij} = D_j
\]

which distributes trips exogenously generated at each zone \( O_i \) to all destinations, subject to comply with the exogenously given trips destinations \( D_j \). These constraints reproduce the assumption that land use and trip frequencies are exogenous for the model, which we defined as a short run context. \( A_i \) and \( B_j \) are balancing factors that assures that estimated trips do comply with \( O_i \) and \( D_j \) constraints respectively; it is well known that balancing factors are relative values dependent upon an unknown constant. The travel demand sensitivity to the transport cost \( c_{ij} \) is contained in parameter \( \beta \).

Williams’ expression to assess the TUB comparing two different transport cost situations in a given (cross-section) time, i.e. a short run case, is
\[ \Delta TUB^{ir} = -\frac{1}{\beta} \left[ \sum_{i} O_i \ln \left( \frac{A_i^j}{A_i^0} \right) + \sum_{j} D_j \ln \left( \frac{B_j^i}{B_j^0} \right) \right] \]  

(2)

where index 0 and 1 stand for situations with and without the project that induces costs variations. Here, each term of equation (2) is relative to the unknown constant, although the total consumers’ surplus change is an absolute value. According to Williams and Senior (1978) the first term of equation (2) represents benefits remaining on travelers’ hands and while the second term represent rents at the trip destination. However, this interpretation requires a strong assumption on whether the traveler is a job seeker (residence location is fixed) or a residence seeker (job location is fixed), which weakens its applicability.

Although the original entropy formulation was not founded on a strict micro-economic framework, the demand model (1) and its associated users’ benefit measure (2) do have a microeconomic meaning. However, a rigorous relationship between the TUB and a disaggregated measure of users’ benefits based on the random utility theory, e.g. the logsum benefit measure, has not been rigorously established to the authors’ knowledge, despite the contribution showing the equivalence between parameters obtained from these frameworks made by Anas (1983). On the other hand, \( c_{ij} \) in equation (1) is said to reflect an inclusive or generalized cost, which can be interpreted as a representative dis-utility associated traveling in this particular O-D pair, measured in monetary terms. From this viewpoint, \(-\beta\) plays the role of the cost coefficient in the microeconomics of discrete choices. Therefore, Williams’ assumption that \( \beta \) is constant in deriving equation (2), is equivalent to the implicit assumption of no income effect (Jara-Díaz and Videla, 1989). Thus, in this context the Marshallian measure (2) is a Hicksian and exact measure of benefits.
In order to lessen the impact of the absence of income effects, travel demand and users’ benefits may be calculated segmenting the transport market into categories of homogeneous trip makers, clustered by socio-economic characteristics (associated to index \( n \)). The disaggregated by cluster expression of benefits was also presented in Williams (1976) paper; this is:

\[
\Delta TUB^{sr} = \sum_n \frac{1}{\beta^n} \sum_i O^n_i \ln \left( \frac{A^{in}_i}{A^{0n}_i} \right) + \frac{1}{\bar{\beta}} \sum_j D_j \ln \left( \frac{B^1_j}{B^0_j} \right)
\]  

(3)

where, \( \beta \) parameters are now cluster specific. For practical reasons, the second term aggregates all categories, but for consistency \( \bar{\beta} \) should be (Williams and Senior, 1977):

\[
\frac{1}{\bar{\beta}} = \frac{1}{T} \sum_n T^n
\]

where \( T \) and \( T^n \) represents, respectively, the total and by cluster number of trips in the study area.

A feature of Williams’ TUB measure is that it identifies benefits associated to origin and destination zones. However, it is counterintuitive that these expressions do not allow a direct association between benefits and trips. In order to extend them to allow this relationship, we have rewritten Williams’ formulae, simply replacing \( O_i \) and \( D_j \) in equation (2) by the expression of their associated constraints in equation (1) to obtain:

\[
\Delta TUB^{sr} = \frac{1}{\beta} \left[ \sum_i \sum_j T_{ij} \ln \left( \frac{A^1_i}{A^0_i} \frac{B^1_j}{B^0_j} \right) \right]
\]  

(4)
Note that $T_{ij}$ can be either $T^0_{ij}$ or $T^1_{ij}$, since both reproduce the invariant $O_i$ and $D_j$ trip totals. In fact, we could have replaced any $T_{ij}=T^*_{ij}$ such that $T^*_{ij} = \alpha T^0_{ij} + (1-\alpha)T^1_{ij}$, with $\alpha \in [0,1]$, without altering total benefits. This ambiguity is induced by the invariant condition of the spatial interaction model, which is transferred into equation (4) in the form that total benefits are invariant to $T^*_{ij}$. This new invariance raises an important interpretation issue: is it possible to establish a direct association between trips and benefits induced by a transport project? To deal with this issue, we define the elemental trip user’s benefit ($tub$) as:

$$tub_{ij} = -\frac{1}{B} \ln \left( A_i B_j \right)$$

(5)

which represent a unit of absolute benefit perceived by a user travelling between zones $i$ and $j$ for a given transport situation, subject to trips comply with total trip origins and destinations. Replace equation (5) in equation (4) and take $T^*_{ij} = (T^0_{ij} + T^1_{ij})/2$ (or $\alpha = 1/2$) to obtain:

$$\Delta TUB^SR = \frac{1}{2} \sum_i \sum_j (T^0_{ij} + T^1_{ij}) \ln (\Delta tub_{ij})$$

(6)

which represents a cross-section pseudo-rule-of-a-half (PRH) measure of benefits. By using $\Delta tub_{ij} = tub^1_{ij} - tub^0_{ij}$ instead of the usual cost variation of the rule of a half (RH), equation (6) avoids the approximation embedded in the common RH measure which approximates benefits (see Jara and Farah, 1988). This can also be written as:

$$\Delta TUB^SR = \sum_i \sum_j \left( T^0_{ij} + \frac{1}{2} (T^1_{ij} - T^0_{ij}) \right) \ln (\Delta tub_{ij})$$

which allows interpreting the total benefit variation as composed by a full benefit $\Delta tub_{ij}$ obtained by original travelers between $i$ and $j$ and half of that benefit obtained by new travelers.
or generated traffic. This describes a plausible interpretation of the relationship between trips and their associated benefits.

Analogously, a trip related expression can be written for equation (3) in terms of a benefits disaggregated by users category \( n \), \( \text{tub}^n \). These expressions of transport users’ benefits have the appealing interpretation of a direct aggregation of elemental trip benefits across origins and destinations, with the additional advantage that involves absolute values of benefits.

3. LONG RUN TRANSPORT USERS’ BENEFITS

The main limitation of the transport user benefit measure presented above is that they are valid in the short term, i.e. they assume the system dimensions \((T, O_n, D_j)\) as constant under travel cost changes from \(c_{ij}^0\) to \(c_{ij}^1\). Let us now explore the case where the change from situation 0 to 1 is caused by an exogenous change in travel costs due to a transport project, which induces a change in the trip pattern. The system evolves from \((T_i^0, O_j^0, D_j^0, T^0_{ij}, c_{ij}^0)\) to \((T_i^1, O_j^1, D_j^1, T^1_{ij}, c_{ij}^1)\), assuming total trips as well as trip origins and destinations exogenously defined for each situation. Trips matrices \(T_{ij}^0\) and \(T_{ij}^1\) comply with the gravity model, i.e. the trip matrix \(T_{ij}\) is obtained from equation (1). We define this case as the long run.

In order to derive a long run measure of transport users’ benefits, we integrate the Marshallian consumers’ surplus expression applying the dual approach with constant \( \beta \) used by Williams
(1976), but in this case trip origins and destinations are assumed elastic with respect to travel costs. Introducing the assumptions of integrability conditions (Green’s theorem) to assure that Hotellings’ linear integral is unique, we are allowed to assume a linear path between costs $c^0_{ij}$ and $c^1_{ij}$. The result of this analysis (see appendix) is the following long run transport users’ benefit:

$$\Delta TUB^{LR} = -\frac{1}{\beta} \left[ \sum_j \frac{(O^1_j + O^0_j)}{2} \ln \left( \frac{A^1_j}{A^0_j} \right) + \sum_j \frac{(D^1_j + D^0_j)}{2} \ln \left( \frac{B^1_j}{B^0_j} \right) + (T^1 - T^0) \right]$$

(7)

As an elementary test of consistency notice that if total trips origins and destinations do not change, this measure converges to Williams’ original formulae.

A plausible interpretation for each term of the right hand side of equation (7) is: the first summatory contains terms related to a change in each zone accessibility, the second one refers to changes in zone attractiveness and the third term to a macro level correction. This last term appears in the entropy framework associated to the gravity model, although in the short run analysis it is irrelevant. It is directly interpreted as the benefit associated to the expansion of the overall macro-level constraint in the entropy framework and describes the benefits of an aggregated trip generation effect.

The trip related long run expression is:

$$\Delta TUB^{LR} = -\frac{1}{\beta} \left[ \frac{1}{2} \sum_j \sum_j (T^1_{ij} + T^0_{ij}) \ln \left( \frac{A^1_{ij}B^1_{ij}}{A^0_{ij}B^0_{ij}} \right) + (T^1 - T^0) \right]$$

(8)
which can also be written as the aggregated measure of elemental trip user’s benefits:

\[ \Delta TUB^{LR} = \sum_i \sum_j \left[ T_{ij}^* \Delta tub_{ij} - \frac{1}{B} \Delta T_{ij} \right] \]  

(9)

where now \( T_{ij}^* \) unambiguously denotes the average number of trips between situations 0 and 1.

The users’ benefits defined by equations (8) and (9) are composed by a trip distribution effect, measured by the pseudo-rule-of-a-half, plus the trips generation effect, both identified in terms of the specific trips. Again, it is also possible to specify disaggregated measures of benefits by user categories.

4. INTERPRETATION OF ACCESSIBILITY

The \( tub \) measure provides the bases to define the household customized accessibility measure (denoted as \( hc-acc \)), which takes into account the actual (or expected) trip pattern of the members of a household. If the spatial interaction model provides balancing factors disaggregated by household cluster and trip purpose, then the accumulated expected trip benefit of a household that performs a given trip pattern is calculated as:

\[ hc-acc_i^n = -\sum_{k \in K} \frac{1}{B_{nk}} T_{nk} \ln \left( A_i^{nk} B_{jk} \right) \]

where \( hc-acc_i^n \) measures the total benefit obtained by household \( n \) performing a trip pattern \( K^n \) from a given location zone \( i \). \( k \) is the index for individual trips, \( p_k \) is the trip purpose and \( j_k \)
is the trip destination zone index. Note that if total number of trips change, a trip generation effect should be added according to equation (9). The aggregation of benefits across household members implicitly assumes that within the households all members’ benefits are equally valued for making residential choices, which may not be adequate in some cases. These measures may be applied, for example, in the estimation of willingness to pay for residential location (Jara-Díaz and Martínez, 1999) used in some disaggregated residential location models (see Martínez, 1995 and Martínez and Donoso, 1995).

5. RELEVANCE TO PROJECT APPRAISALS

The TUB measure opens up the scope to develop new methods for project appraisals. Indeed, the long run measure provides a tool to calculate the total expected net present value of benefits for each specific transport project, thus the usual comparison between with and without project situations is no longer required. A clear gain of an independent project assessment is that it avoids the dependency, present in current methods, to an arbitrarily defined base for comparison (usually a do nothing or do minimum option). This absolute measure of the transport project’s benefits allows a further direct comparison with other projects in the economy.
REFERENCES


Following Williams (1976)’s approach, the maximum entropy model may be generated from a non-linear problem with dual function given by:

$$\bar{F} = \sum_i \sum_j O_i D_j \exp(-\alpha_i - \gamma_j - \beta c_{ij}) + \sum_i \alpha_i O_i + \sum_j \gamma_j D_j + \beta C$$

with the notation described in the text except for the total transport cost $C$, and $\alpha_i$, $\gamma_j$ and $\beta$ the lagrangean multipliers associated with trip origins, trip destinations and total cost constraints. Let us examine the derivative of $\bar{F}$ with respect to the cost component $c_{lm}$ at the optimum:

$$\frac{\partial F}{\partial c_{lm}} = \sum_i \sum_j \frac{\partial O_i}{\partial c_{lm}} \frac{T_{ij}}{O_i} + \sum_i \sum_j \frac{\partial D_j}{\partial c_{lm}} \frac{T_{ij}}{D_j} - \sum_i \sum_j \frac{T_{ij}}{c_{lm}} \frac{\partial \alpha_i}{\partial c_{lm}} - \sum_i \sum_j \frac{T_{ij}}{c_{lm}} \frac{\partial \gamma_j}{\partial c_{lm}} - \sum_i \sum_j \frac{T_{ij} c_{ij}}{c_{lm}} \frac{\partial \beta}{\partial c_{lm}}$$

$$- \sum_i \sum_j T_{ij} \frac{\partial c_{ij}}{\partial c_{lm}} + \frac{C}{c_{lm}} \frac{\partial \beta}{\partial c_{lm}}$$

Assuming $\frac{\partial c_{ij}}{\partial c_{lm}} = 0, \forall(i, j) \neq (l, m)$, it reduces to:

$$\frac{\partial F}{\partial c_{lm}} = -\beta T_{lm} + \beta \frac{\partial C}{\partial c_{lm}} + \sum_i \frac{\partial O_i}{\partial c_{lm}} (1+\alpha_i) + \sum_j \frac{\partial D_j}{\partial c_{lm}} (1+\gamma_j)$$

Replacing $\sum_i \frac{\partial O_i}{\partial c_{lm}} = \frac{\partial}{\partial c_{lm}} \left( \sum_i O_i \right) = \frac{\partial T}{\partial c_{lm}}$ and its analogous for $D_j$, we obtain:

$$T_{lm} = \frac{1}{\beta} \left[ -\frac{\partial F}{\partial c_{lm}} + \beta \frac{\partial C}{\partial c_{lm}} + \sum_i \alpha_i \frac{\partial O_i}{\partial c_{lm}} + \frac{\partial T}{\partial c_{lm}} + \sum_j \gamma_j \frac{\partial D_j}{\partial c_{lm}} + \frac{\partial T}{\partial c_{lm}} \right]$$
As Williams, here we use the second integral mean value theorem to evaluate the Marshallian consumer surplus:

\[ \Delta TUB^R = -\sum_i \sum_m \int \frac{1}{\beta} \left[ \sum j \left( \frac{\partial F}{\partial c_{im}^j} + \beta \frac{\partial C}{\partial c_{im}^j} + 2 \frac{\partial T}{\partial c_{im}^j} + \sum i \alpha_i \frac{\partial a}{\partial c_{im}^j} + \sum j \gamma_j \frac{\partial D_j}{\partial c_{im}^j} \right) \right] dc_{im} \]

Evaluating \( \overline{F} \) and maintaining Williams’ assumption that \( \beta \) remains invariable in its survey year value (assuming no income effect), we obtain:

\[ \Delta TUB^R = \frac{1}{\beta} \left[ 2(T^1 - T^0) - \left( \beta(C^1 - C^0) + T^1 + \sum i O_i \alpha_i^1 + \sum j D_j \gamma_j^1 - T^0 - \sum i O_i \alpha_i^0 - \sum j D_j \gamma_j^0 \right) \right] \]

Then replace \( \overline{\alpha}_i = \frac{\alpha_i^1 + \alpha_i^0}{2} \) and \( \overline{\gamma}_j = \frac{\gamma_j^1 + \gamma_j^0}{2} \) to get:

\[ \Delta TUB^{JR} = -\frac{1}{\beta} \left[ T^1 - T^0 - \sum i \left( \frac{O_i^1 + O_i^0}{2} \right) \left( \alpha_i^1 - \alpha_i^0 \right) - \sum j \left( \frac{D_j^1 + D_j^0}{2} \right) \left( \gamma_j^1 - \gamma_j^0 \right) \right] \]

Finally, using the usual transformation to balancing factors: \( A_i = \exp(-\alpha_i) \) and \( B_j = \exp(-\gamma_j) \), we obtain:

\[ \Delta TUB^{JR} = -\frac{1}{\beta} \left[ \sum i \left( \frac{O_i^1 + O_i^0}{2} \ln \left( \frac{A_i^1}{A_i^0} \right) \right) + \sum j \left( \frac{D_j^1 + D_j^0}{2} \ln \left( \frac{B_j^1}{B_j^0} \right) \right) + (T^1 - T^0) \right] \]