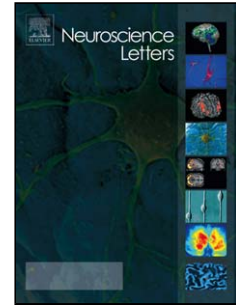


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Coincidence-Enhanced stochastic resonance: Experimental evidence challenges the psychophysical theory behind stochastic resonance

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Abstract

Stochastic Resonance (SR) is the counterintuitive phenomenon in which noise enhances detection of sub-threshold stimuli. The SR psychophysical threshold theory establishes that the required amplitude to exceed the sensory threshold barrier can be reached by adding noise to a sub-threshold stimulus. The aim of this study was to test the SR theory by comparing detection results from two different randomly-presented stimulus conditions. In the first condition, optimal noise was present during the whole attention interval; in the second, the optimal noise was restricted to the same time interval as the stimulus. SR threshold theory predicts no difference between the two conditions because noise helps the sub-threshold stimulus to reach threshold in both cases. The psychophysical experimental method used a 300 ms rectangular force pulse as a stimulus within an attention interval of 1.5s, applied to the index finger of six human subjects in the two distinct conditions. For all subjects we show that in the condition in which the noise was present only when synchronized with the stimulus, detection was better ($p < 0.05$) than in the condition in which the noise was delivered throughout the attention interval. These results provide the first direct evidence that SR threshold theory is incomplete and that a new phenomenon has been identified, which we call Coincidence-Enhanced Stochastic Resonance (CESR). We propose that CESR might occur because subject uncertainty is reduced when noise points at the same temporal window as the stimulus.

Keywords: noise-stimulus synchronization; tactile stimulus detection; uncertainty; psychophysics.

Introduction

In recent years, several experiments have shown noise-enhanced detection of weak or ‘sub-threshold’ stimuli in different sensory systems in animals and humans [1, 6, 8, 9, 18, 19, 23, 28, 32]. This phenomenon, Stochastic Resonance (SR), presumes the existence of an optimal noise level that maximizes detection [2, 5]. SR psychophysical theory establishes that the amplitude required to exceed the sensory threshold barrier is reached by adding noise to the sub-threshold stimulus [23, 30-32]. SR, therefore, requires the simultaneous presence of the stimulus and noise. In this paradigm, a sub-threshold stimulus can be detected by adding a small amount of noise to the stimulus, thus transforming the stimulus from a sub-threshold to supra-threshold level. If the noise is significant, however, it may mask the stimulus preventing stimulus detection. In the last decade, SR in the tactile-sensory domain for human subjects was demonstrated for rectangular pulse stimulation on the finger tips [6, 7, 20], for 30 Hz vibratory stimulation on the finger and the foot's first metatarsal [17], as well as for 25-400 Hz sinusoidal stimuli on the sole of the foot [32]. In all cases, the noise was superimposed on the stimulus by means of arithmetic addition in the attention interval containing the stimulus.

What remains an open question is how the temporal location of the noise relative to the stimulus affects detection, and if SR psychophysical theory can explain the results of this effect. The purpose of the present study was to test the SR model in two different conditions, each with a different temporal location of the noise relative to the stimulus. In the first condition, optimal noise was present during the complete attention interval, while in the second, the optimal noise was restricted to the same time interval as the stimulus. SR threshold theory would predict no difference between the two conditions because in both cases noise should help the sub-threshold stimulus to reach threshold. Our results demonstrate that the two conditions yield different results and that sub-threshold stimulus detection can be enhanced,

relative to the SR prediction, by synchronizing the onset and end of the noise with the stimulus. We show that in this condition the detection is better than in the case where the noise exceeds the stimulus duration, thus improving upon what the theory of SR would predict. These results show the shortcomings of the psychophysical SR theory and identify a new phenomenon we call "Coincidence-Enhanced Stochastic Resonance" (CESR), which is not explained by SR theory. CESR results may be applicable for improving tactile sensitivity in the aged or in those patients with disease-related sensory loss [18]. Also, CESR could be applied to haptic interfaces for telemedicine, telerobotics, virtual reality to improve detection and reduce power requirements [26, 27] or improve current models of tactile sensation [33].

Materials and Methods

A piezoelectric stimulator was used to deliver force pulses to the index finger. As in previous work, the tactile excitation— stimulus plus noise— was low-pass filtered at 30 Hz to limit stimulation to the NP tactile channels [6, 12].

Subjects. Six healthy young volunteers (two women and four men) participated in the study. The mean age of the group was 23 years 6 months (min = 22; max =25). All subjects were familiarized with the experimental system over several training sessions. Moreover, they all had previous experience in experiments involving vibrotactile sensing on the index finger.

The experiments were approved by the Bioethics Committee, INTA, Universidad de Chile (resolution N° 11, June 14, 2006), and the informed consent was obtained from all subjects.

Experimental system. The experimental setup consisted of a piezoelectric bimorph mounted as a cantilever from a plastic base, which was set over a balanced steel structure. This is the same setup used previously [26], except that the digital to analog converter was replaced by an AD7521 (12-bit resolution) and a fourth-order Butterworth filter, built with an LF353 operational amplifier, was added. The low-pass filter had a cut-off frequency of 30 Hz so that the generated signals stimulated only the NP channels [12]. The stimulus waveform and cut-off frequency of the filter were the same as in previous work [7].

Protocol. Each subject sat in front of a computer screen with an arm resting on a padded table with his/her index finger touching the plastic contactor. In each trial, the subject received visual and auditory warnings indicating the beginning of the attention interval that lasted 1.5 sec. The stimulus, a rectangular pulse of 300 ms [6, 7], was placed at a random position within the attention interval. The end of the attention interval was signaled on the computer screen. A research assistant recorded the subject's response.

The threshold for each subject was estimated by means of a yes/no procedure, without any non-stimulus condition, updating the stimulus level with an up-down adaptive algorithm [15]. Each time the subject said 'no,' the level of the stimulus was increased one step (0.41 mN); otherwise, it was decreased one step. After 50 trials, the stimulus level was expected to correspond to 50% correct detection, i.e., the classical threshold estimation.

In Experiment 1, the optimal noise level for each subject was determined. The stimulus strength was set at 80% of the threshold for each subject, and five different noise levels (noise levels 0-4) were used with standard deviation values equivalent to 0%, 20%, 40%, 60% and 80% of the stimulus amplitude, respectively. Stimulus amplitudes fluctuated between 9 mN and 16 mN for the six subjects. We measured the percentage of correct responses, $P(C)$, obtained in a set of 40 trials for each noise level, including equally distributed non-stimulus

and stimulus trials. Therefore, each correct response corresponded to a ‘yes’ answer when the stimulus was presented and a ‘no’ answer, otherwise.

Experiment 2 was carried out to assess the influence of the temporal location of noise on detection using two different conditions. In both conditions Gaussian noise was added to equal sub-threshold rectangular pulses. Conditions C1 and C2 shown in Fig. 1a and 1b, respectively, illustrate the two different conditions in our second experiment. In condition C1 (Fig. 1a), the noise is present during the entire attention interval that lasts 1.5 sec., while in condition C2 (Fig. 1b), the noise is present only during the sub-threshold stimulus that lasts 300 msec. In both conditions, C1 and C2, the noise helps the stimulus to exceed the threshold shown as a dashed line in Fig. 1a and 1b. Moreover, if the noise is optimal in an SR sense, then SR would predict no difference in the sensory performance of these two conditions because threshold crossings caused by the addition of noise to the stimulus are present coincident with the stimulus in both C1 and C2. In other words, according to SR theory, the optimal noise, which alone could not produce threshold crossings, is useful just in the temporal window where the stimulus is present. In Experiment 2, four hundred trials were used on the same six volunteers to compare detection for conditions C1 and C2 (Fig. 1) using the optimal noise level determined in Experiment 1. We used the same number of trials for conditions C1 and C2 and presented them in random order to avoid possible changes in the subject’s criteria that could favor one condition over another. In condition C1, two hundred trials contained noise during the entire attention interval. One hundred trials contained a stimulus, while the other hundred contained no stimulus. In condition 2, one hundred trials contained the pulse stimulus with noise synchronized in time, while the other one hundred trials were non-stimulus samples with only a 300 msec noise pulse.

Noise. The noise was generated by software using Gaussian random numbers with 12-bit resolution and was added to the stimulus signal prior to the D/A conversion. The noise's standard deviation was measured with an oscilloscope and then translated into force as specified by the piezoelectric manufacturer.

Results

Experiment 1

SR behavior is observed in the human tactile sense as shown in previous work [6, 7, 17, 32]. In Experiment 1, we determined the optimal tactile noise that allows sub-threshold stimulus detection for the six subjects in this study. We measured the percentage of correct responses, $P(C)$, obtained in a set of 40 trials for each noise level, including equally distributed non-stimulus and stimulus trials. All six subjects showed an 'SR signature', i.e., all obtained higher $P(C)$ at a particular noise level. Five subjects obtained the maximum $P(C)$ at noise level 1 (see Fig. 2a) and one subject at noise level 2 (see Fig. 2b). Figure 2c shows $P(C)$ for the six subjects showing the mean and standard deviation. All individuals obtained a maximum $P(C)$ above the dashed line in Fig. 2a and 2b. This line represents the limit above which $P(C)$ is significantly better than that expected by chance, according to a binomial test ($p < 0.05$). In this test, with an equal number of stimulus and non-stimulus trials, the chance level corresponds to $P(C) = 50\%$ or 20 of 40 correct responses, while the number of correct responses that would produce statistically significant results would be 27 or greater. The results of Experiment 1 agree with those previously found [6, 32].

Experiment 2

In Experiment 2, we assessed the influence of the temporal location of noise within the 1.5s attention interval on the stimulus detection. In condition C1 the noise was present during the

whole attention interval, while in condition C2 the noise was present only during the 300ms pulse stimulus. Detection for conditions C1 and C2 was measured on the same six volunteers using the optimal noise level determined in Experiment 1. It can be observed, in Table 1, that $P(C)$ for trials of condition C2 were greater than $P(C)$ for trials of condition C1 for all six subjects. Also, Table 1 shows that differences between conditions C1 and C2 were statistically significant for all subjects according to a binomial test. The p-values resulted in a confidence level greater than 95% for every subject. Therefore, results show that for all subjects detection for condition C2, where noise is synchronized with the stimulus, is enhanced relative to condition C1, where there is noise throughout the attention interval.

Discussion

Results show enhanced detection when noise is present only during the stimulus presentation (condition C2) rather than when noise is present during the entire attention interval (condition C1). This result violates an SR psychophysical theory prediction and thus presents a serious restriction of the theory for explaining our experimental results.

Previously, the Array-Enhanced Stochastic Resonance (AESR) was described as a phenomenon in which noise and coupling optimize the response of arrays of non-linear elements to periodic signals [14, 16]. We propose the term Coincidence-Enhanced Stochastic Resonance (CESR) to describe our measured phenomenon. In the literature, a “coincidence detection” of convergent synaptic inputs on a neuron refers to the capacity to produce an action potential when the inputs act simultaneously [21]. Also in [13] coincidence detection is described as the operation performed by a neuron when it responds maximally to synchronized inputs. In our work, the name CESR is given in the sense of the synchronized action of noise and stimulus on detection capacity. In the future it will be interesting to explore in CESR if the coincident initiation and end of the stimulus and noise within the attention interval may

produce measurable synaptic activity in the sense of coincidence detection. Also, SR has been recently described for the first time in the motor system [22]. Therefore, in the future it would be interesting to explore whether CESR may be present in the motor system.

It has been proposed that SR requires a nonlinearity early on the system under study [23]. Consistent with this, prior work involving tactile sensation has shown a type of hard nonlinearity viewed as a stimulus or energy threshold below which no sensation can be produced [10, 11]. Models of neurons have been used as surrogates for the required nonlinearity and predictions from these often fit the data quite well [1, 5, 8, 30]. However, other mechanisms could also explain the SR-type behaviour.

In both SR and CESR, the system under study performs sub-optimally, otherwise the addition of noise could only diminish performance for every increasing value of noise power. For SR, sub-optimality may be linked to the information loss in the early nonlinearity. CESR could be explained at a central rather than a peripheral level using the uncertainty theory [25, 29]. We can suppose that the human observer is uncertain about the detected signal parameters and must consider sensory information arriving from many different sensory paths. Uncertainty could be spatial (e.g. where the stimulus will be placed) or temporal (e.g. when it might occur). Prior sensory research has established that human observers may not be able to use such parametric information to the fullest even when they are informed in advance about the stimulus parameters [3, 4, 25, 29]. This phenomenon has been identified as the observer's intrinsic uncertainty [29]. The noise effect shown in CESR could reflect a reduction in the temporal uncertainty of condition C2 (Experiment 2) by using the noise to point at the same temporal window as the stimulus. Lower uncertainty may allow the observer to monitor fewer noise intrinsic sensory channels [25], although at the expense of increased noise experimentally added to the stimulus in the relevant channel(s). The information loss associated to the central sensory system is supported by the findings of Manjarrez *et al.* [19] in cats where they show SR effects in physiological measurements at a central level.

The existence of CESR may not have emerged until now because SR in the tactile sensory system is a relatively new phenomenon that was first described by Collins *et al.* in 1996 [6, 7]. In these papers the attention interval was longer than the rectangular pulse stimulus. Nevertheless, in later work by Liu *et al.* [17] and Wells *et al.* [32], noise was restricted to the same time interval of the stimulus. They compared two attention intervals, one with a sinusoidal stimulus plus noise versus one with only noise, using a Yes/No and a 2AFC (two-alternative forced choice) paradigm, respectively. Furthermore, Wells *et al.* [32] equated the energy of both intervals. They identified SR for this condition which is equivalent to condition C2 in our experiment but did not compare results to the condition where the noise exceeds the pulse duration, as in our condition C1. Therefore, CESR may have been previously measured but not identified as such, since no comparison to condition C1 was performed.

CESR can be considered a counterintuitive phenomenon since by restricting the noise to a narrower period of time (lower power) [24, 26, 27], the signal detection is improved compared to the case of SR where noise is always present. CESR and SR could have a wide range of possible applications, e.g., therapeutic applications enhancing detection for patients with sensory loss due to age or illness (diabetes, stroke, etc) by adding noise to the stimulus. Most importantly, the main contribution of this paper is in furthering our understanding of the SR phenomenon. Researchers interested in the SR phenomenon should be aware of the effects of CESR when considering applications, theoretical explanations, and possible extensions to other sensory systems.

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Figure 1. Two different samples of stimulus plus noise. (a) Condition C1: noise added to a sub-threshold square pulse of 9.6 mN during the whole attention interval of 1.5 s. (b) Condition C2: noise added only at stimulus length of 300 ms. The dashed line indicates the threshold's amplitude (12 mN in this case). In both conditions the noise is Gaussian filtered at 30 Hz (low-pass) with a standard deviation of 1.9 mN (20% of stimulus' amplitude).

Figure 2. Percentage of correct responses, $P(C)$, as a function of the noise level for the six subjects. (a) Five subjects with optimal noise at level 1, (b) one subject with optimal noise at level 2 and (c) all six subjects. The mean value and the standard deviation are shown where applicable. The noise levels from 0 to 4 correspond to Gaussian noise (low-pass filtered at 30 Hz) with standard deviation equal to 0%, 20%, 40%, 60% and 80% of stimulus' amplitude respectively. The dashed line represents the level of significance ($p < 0.05$) over which $P(C)$ was significantly better than that expected by chance.

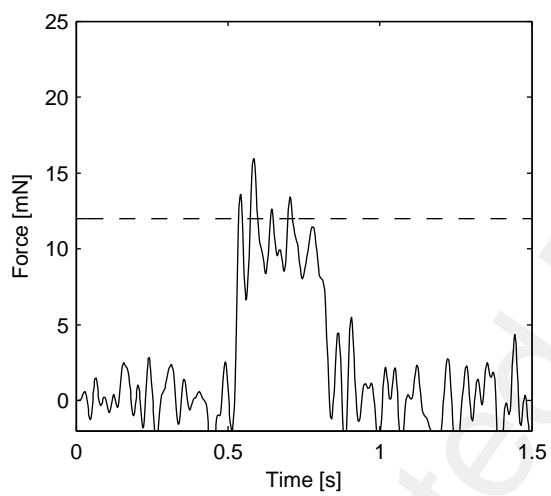
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Table 1. Percentage of correct detection $P(C)$ for the six subjects in both conditions C1 and C2 of experiment 2.

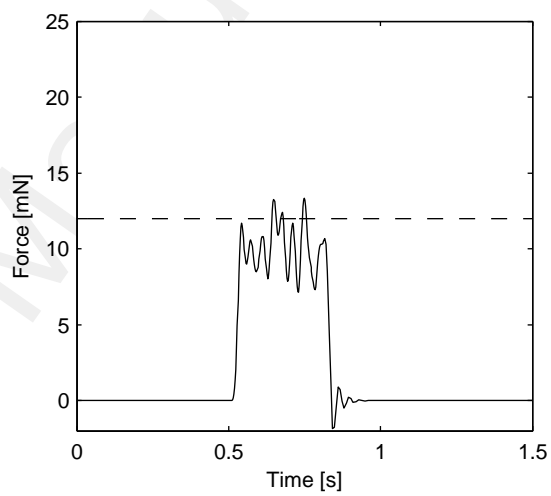
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Subject	P(C) for C1	P(C) for C2	p-value
1	67	79	0.000307
2	67.5	83	0.000003
3	71.5	78.5	0.028308
4	73	82	0.004145
5	71	80.5	0.003068
6	76.5	87.5	0.000244

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a)



b)

