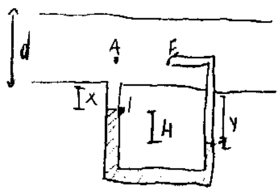
PAUTA P2

a) Hay que usar el pto. de estancamiento (E)



$$B_A = B_E \Leftrightarrow \frac{V_A^2}{2g} + \frac{P_A}{\gamma} + \cancel{z_A} = \frac{\cancel{V_E^2}^{\,0}}{2g} + \frac{P_E}{\gamma} + \cancel{z_E} \Leftrightarrow \frac{V_A^2}{2g} = \frac{P_E - P_A}{\gamma} \quad (*) \quad 0{,}6$$

Usando Hidrostática: $P_2 - P_1 = \rho_{Hg} g H$ (1)

$$\left.\begin{array}{l} P_1 = P_A + \rho g x \\ P_2 = P_E + \rho g y \end{array}\right\} P_2 - P_1 = P_E - P_A + \rho g \overbrace{(y-x)}^{H} \quad (2)$$

$$(1) = (2) \Leftrightarrow P_E - P_A + \rho g H = \rho_{Hg} g H \Leftrightarrow P_E - P_A = (\rho_{Hg} - \rho) g H \;/\cdot \frac{1}{\rho g} \Leftrightarrow \frac{P_E - P_A}{\rho g} = \left(\frac{\rho_{Hg} - \rho}{\rho}\right) H \quad 0{,}6$$

Reemplazando en (*): $\frac{V_A^2}{2g} = \left(\frac{\rho_{Hg} - \rho}{\rho}\right) H \Leftrightarrow V_A = \sqrt{2g\left[\frac{\rho_{Hg} - \rho}{\rho}\right] H}$; $Q = V \cdot A = \sqrt{2g\left[\frac{\rho_{Hg} - \rho}{\rho}\right] H} \cdot \frac{\pi d^2}{4} = 4{,}9 \cdot 10^{-4} \, [m^3/s] = 0{,}49 \, [l/s]$ 0,6 0,2

b) La pérdida de carga es la pérdida de altura piezométrica, i.e. la diferencia entre los piezómetros

$$\Rightarrow h = \frac{KV^2}{2g} = K \cdot \frac{V_A^2}{2g} = K\left(\frac{\rho_{Hg} - \rho}{\rho}\right) H = 0{,}5 \cdot 12{,}6 \cdot 0{,}01 = 0{,}063 \, [m] = 6{,}3 \, [cm] \quad 0{,}8 \quad 0{,}2$$

c) 2ª ley de Newton: $d\vec{F} = \frac{d}{dt}(\vec{v}\,dm)$; aplicando momentos: $\vec{r} \times d\vec{F} = \vec{r} \times \frac{d}{dt}(\vec{v}\,dm)$

Usando la identidad vectorial: $\vec{r} \times \frac{d}{dt}(\vec{v}\,dm) = \frac{d}{dt}(\vec{r} \times \vec{v}\,dm)$; además, $dm = \rho\, d\forall$

$\Rightarrow \vec{r} \times d\vec{F} = \frac{d}{dt}(\vec{r} \times \vec{v} \rho\, d\forall)$. Recordando el teorema del tpte. de Reynolds: $\frac{dN}{dt} = \frac{\partial}{\partial t}\int_\forall \eta \rho\, d\forall + \int_S \eta \rho \vec{v} \cdot \hat{n}\, ds$

Donde $N = \int_\forall \eta \rho\, d\forall$. En este caso, $\eta = (\vec{r} \times \vec{v})$; $N = \int_\forall (\vec{r} \times \vec{v}) \rho\, d\forall$

$\Rightarrow \frac{d}{dt}(\vec{r} \times \vec{v} \rho\, d\forall) = \frac{\partial}{\partial t}\int_\forall (\vec{r} \times \vec{v}) \rho\, d\forall + \int_S (\vec{r} \times \vec{v}) \rho \vec{v} \cdot \hat{n}\, ds$. Régimen permanente $\Rightarrow \frac{\partial}{\partial t}\int_\forall (\vec{r} \times \vec{v}) \rho\, d\forall = 0$

$\Rightarrow \vec{r} \times d\vec{F} = \int_S (\vec{r} \times \vec{v}) \rho \vec{v} \cdot \hat{n}\, ds$; integrando: $\vec{r} \times \vec{F} = (\vec{r}_S \times \vec{v}_S) \rho_S \vec{v}_S S_S - (\vec{r}_E \times \vec{v}_E) \rho_E \vec{v}_E S_E$

Pero, $\rho_E = \rho_S = \rho$; $\vec{v}_E S_E = Q_E$; $\vec{v}_S S_S = Q_S$; $Q_E = Q_S = Q \Leftrightarrow \vec{r} \times \vec{F} = \rho Q\left[(\vec{r} \times \vec{v})_S - (\vec{r} \times \vec{v})_E\right]$ 1,0

d)

$$T = \rho Q\left(R \cdot \frac{Q}{A} - R \cdot 0\right) \Leftrightarrow T = \frac{\rho Q^2 R}{\frac{\pi}{4} \cdot D^2} = \frac{4\rho Q^2 R}{\pi D^2} \quad 1{,}0$$

$$\sum \vec{T} = 0 \Leftrightarrow \frac{4\rho Q^2 R}{\pi D^2} = \beta\omega \Leftrightarrow D = \sqrt{\frac{4\rho Q^2 R}{\pi \beta \omega}} = 2Q\sqrt{\frac{\rho R}{\pi \beta \omega}} = 2 \cdot 0{,}49\sqrt{\frac{1000 \cdot 0{,}2}{\pi \cdot 1 \cdot \frac{33\frac{1}{3}}{60}}} = 0{,}01 \, [m] \doteq 1 \, [cm]$$

0,4 0,4 0,2