PAUTA

Navier-Stokes en coordenadas cilíndricas, flujo sólo según $\hat{z}$ ($V_r = V_\theta = 0$)

Flujo axisimétrico $\Rightarrow \frac{\partial V_z}{\partial \theta} = 0$

Continuidad: $\frac{1}{r}\frac{\partial(r\cancelto{0}{V_r})}{\partial r} + \frac{1}{r}\frac{\partial \cancelto{0}{V_\theta}}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0 \iff \frac{\partial V_z}{\partial z} = 0$ (0,2)

N-S $\hat{z}$: $\overset{\text{permanente}}{\cancelto{0}{\frac{\partial V_z}{\partial t}}} + \cancelto{0}{V_r}\frac{\partial V_z}{\partial r} + \frac{\cancelto{0}{V_\theta}}{r}\frac{\partial V_z}{\partial \theta} + V_z \overset{\text{Continuidad}}{\cancelto{0}{\frac{\partial V_z}{\partial z}}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \overset{\text{axisimetría}}{\cancelto{0}{\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}}} + \overset{\text{Continuidad}}{\cancelto{0}{\frac{\partial^2 V_z}{\partial z^2}}}\right]$ (1)

N-S $\hat{r}$: $\overset{\text{Permanente}}{\cancelto{0}{\frac{\partial V_r}{\partial t}}} + \cancelto{0}{V_r\frac{\partial V_r}{\partial r}} + \cancelto{0}{\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}} - \cancelto{0}{\frac{V_\theta^2}{r}} + \cancelto{0}{V_z\frac{\partial V_r}{\partial z}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial r} + \frac{\mu}{\rho}\left[\overset{\text{Cont}}{\cancelto{0}{\frac{\partial^2 V_r}{\partial r^2}}} + \overset{\text{Cont}}{\cancelto{0}{\frac{1}{r}\frac{\partial V_r}{\partial r}}} + \cancelto{0}{\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}} + \cancelto{0}{\frac{\partial^2 V_r}{\partial z^2}} - \cancelto{0}{\frac{V_r}{r^2}} - \overset{\text{Cont}}{\cancelto{0}{\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}}}\right]$ (2)

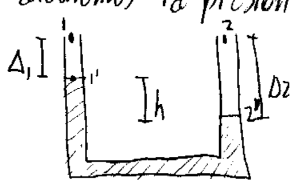
(0,2)

De (2): $\frac{\partial \hat{p}}{\partial r} = 0 \Rightarrow \hat{p}$ no depende de r (0,1)

De (1): $\frac{\partial \hat{p}}{\partial z} = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)\right] \iff \frac{\partial \hat{p}}{\partial z} = \frac{\mu}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) \iff \frac{r}{\mu}\frac{d\hat{p}}{dz} = \frac{d}{dr}\left(r\frac{dV_z}{dr}\right)$

Integrando: $\frac{r^2}{2\mu}\frac{d\hat{p}}{dz} + C_1 = r\frac{dV_z}{dr} \iff \frac{r}{2\mu}\frac{d\hat{p}}{dz} + \frac{C_1}{r} = \frac{dV_z}{dr} \iff \frac{r^2}{4\mu}\frac{d\hat{p}}{dz} + C_1\cdot\ln(r) + C_2 = V_z$ (0,5)

Calculemos la presión motriz: $\frac{\partial \hat{p}}{\partial z} = \frac{\Delta \hat{p}}{\Delta z} = \frac{\hat{P}_2 - \hat{P}_1}{z_2 - z_1} = \frac{P_2 + \rho g h_2 - P_1 - \rho g h_1}{L}$; $h_2 = h_1 \Rightarrow \frac{\Delta \hat{p}}{\Delta z} = \frac{P_2 - P_1}{L}$



Del dibujo: $P_1' = P_1 + \rho g \Delta_1$; $P_2' = P_2 + \rho g \Delta_2 \iff P_2' - P_1' = P_2 - P_1 + \rho g(\Delta_2 - \Delta_1)$ (3)

Por otro lado: $P_2' = P_1' + \rho_{Hg} g h \iff P_2' - P_1' = \rho_{Hg} g h$ (4)

(3) y (4): $P_2 - P_1 + \rho g(\Delta_2 - \Delta_1) = \rho_{Hg} g h$; pero $(\Delta_2 - \Delta_1) = h \iff P_2 - P_1 + \rho g h = \rho_{Hg} g h \iff P_2 - P_1 = (\rho_{Hg} - \rho) g h$

$\Rightarrow \frac{\partial \hat{p}}{\partial z} = \frac{(\rho_{Hg} - \rho) g h}{L}$ (1,0)

$\Rightarrow V_z(r) = \frac{r^2}{4\mu}\cdot\frac{(\rho_{Hg} - \rho)}{L} g h + C_1\cdot\ln(r) + C_2$

C.B: $V_z(r = \frac{d}{2}) = V_0$; $V_z(r = \frac{D}{2}) = -V_0$

$$\boxed{\begin{aligned} V_0 &= \frac{d^2}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh + C_1\ln\left(\frac{d}{2}\right) + C_2 \\ -V_0 &= \frac{D^2}{16\mu}\frac{(\rho_{Hg}-\rho)gh}{L} + C_1\cdot\ln\left(\frac{D}{2}\right) + C_2 \end{aligned}}$$

$$\Rightarrow 2V_0 = \frac{(d^2-D^2)}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh + C_1\cdot\ln\left(\frac{d}{D}\right)$$

$$\Rightarrow C_1 = \frac{\left[2V_0 + \frac{(D^2-d^2)}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh\right]}{\ln(d/D)}$$

$$C_2 = V_0 - \frac{d^2}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh - \left(2V_0 + \frac{(D^2-d^2)}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh\right)\cdot\frac{\ln(d/2)}{\ln(d/D)}$$

$$\Rightarrow V_z(r) = \frac{r^2}{4\mu}\cdot\frac{(\rho_{Hg}-\rho)gh}{L} + \frac{\ln(r)}{\ln(d/D)}\cdot\left(2V_0 + \frac{(D^2-d^2)}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh\right) + V_0 - \frac{d^2}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh - \left(2V_0 + \frac{(D^2-d^2)}{16\mu}\frac{(\rho_{Hg}-\rho)}{L}gh\right)\frac{\ln(d/2)}{\ln(d/D)}$$

$$\Rightarrow V_z(r) = V_0\left(1 + \frac{2\ln(r)}{\ln(d/D)} - \frac{2\ln(d/2)}{\ln(d/D)}\right) + \frac{(\rho_{Hg}-\rho)gh}{4\mu L}\left[r^2 + \frac{(D^2-d^2)}{4}\cdot\frac{\ln(r)}{\ln(d/D)} - \frac{d^2}{4} - \frac{(D^2-d^2)}{4}\cdot\frac{\ln(d/2)}{\ln(d/D)}\right]$$

$$\Rightarrow V_z(r) = V_0\left(1 + \frac{2\ln(2r/d)}{\ln(d/D)}\right) + \frac{(\rho_{Hg}-\rho)gh}{4\mu L}\left[r^2 - \frac{d^2}{4} + \frac{(D^2-d^2)}{4\ln(d/D)}\cdot\ln\left(\frac{2r}{d}\right)\right] \quad 1{,}0$$

b) $F_1 = \tau(r=\frac{D}{2})\cdot A$; $\tau = \mu\cdot\left.\frac{dV_z}{dr}\right|_{r=\frac{D}{2}}$; $\frac{dV_z}{dr} = \frac{2V_0}{\ln(d/D)}\cdot\frac{\cancel{d}}{\cancel{2}r}\cdot\frac{\cancel{2}}{\cancel{d}} + \frac{(\rho_{Hg}-\rho)gh}{4\mu L}\cdot\left(2r + \frac{(D^2-d^2)}{4\ln(d/D)}\cdot\frac{\cancel{d}}{\cancel{2}r}\cdot\frac{\cancel{2}}{\cancel{d}}\right)$

$$\Rightarrow \frac{dV_z}{dr} = \frac{2V_0}{\ln(d/D)}\cdot\frac{1}{r} + \frac{(\rho_{Hg}-\rho)}{4\mu L}\cdot gh\left(2r + \frac{(D^2-d^2)}{4\ln(d/D)}\cdot\frac{1}{r}\right) \quad 0{,}6$$

$$\Rightarrow F_1 = \mu\cdot\left[\frac{2V_0}{\ln(d/D)}\cdot\frac{2}{D} + \frac{(\rho_{Hg}-\rho)}{4\mu L}gh\left(D + \frac{(D^2-d^2)}{4\ln(d/D)}\cdot\frac{2}{D}\right)\right]\cdot\pi D = \mu\pi\left[\frac{4V_0}{\ln(d/D)} + \frac{(\rho_{Hg}-\rho)}{4\mu L}gh\left(D^2 + \frac{(D^2-d^2)}{2\ln(d/D)}\right)\right]$$

$$\Rightarrow F_2 = \mu\left[\frac{2V_0}{\ln(d/D)}\cdot\frac{2}{d} + \frac{(\rho_{Hg}-\rho)gh}{4\mu L}\left(d + \frac{(D^2-d^2)}{4\ln(d/D)}\cdot\frac{2}{d}\right)\right]\pi d = \mu\pi\left[\frac{4V_0}{\ln(d/D)} + \frac{(\rho_{Hg}-\rho)}{4\mu L}gh\left(d^2 + \frac{(D^2-d^2)}{2\ln(d/D)}\right)\right] \quad 0{,}6$$

$$\Rightarrow F_1 = 10^{-3}\cdot\pi\left[\frac{4\cdot 1}{\ln(3/5)} + \frac{12600\cdot 9{,}8\cdot 0{,}01}{4\cdot 10^{-3}\cdot 0{,}05}\left(0{,}05^2 + \frac{(0{,}05^2-0{,}03^2)}{2\ln(3/5)}\right)\right] = 18{,}09\,[N]$$

$$F_2 = 10^{-3}\pi\left[\frac{4\cdot 1}{\ln(3/5)} + \frac{12600\cdot 9{,}8\cdot 0{,}01}{4\cdot 10^{-3}\cdot 0{,}05}\left(0{,}03^2 + \frac{(0{,}05^2-0{,}03^2)}{2\cdot\ln(3/5)}\right)\right] = -12{,}94\,[N] \quad 0{,}3$$

$$Q=\int v\,dA = 2\pi\int_{d/2}^{D/2} v_z(r)\cdot r\,dr = 2\pi\int_{d/2}^{D/2}\left[v_0 r\left(1+\frac{2\ln(2r/d)}{\ln(d/D)}\right)+\frac{(\rho_{Hg}-\rho)ghr}{4\mu L}\left[r^2-\frac{d^2}{4}+\frac{(D^2-d^2)}{4\ln(d/D)}\cdot\ln\left(\frac{2r}{d}\right)\right]\right]dr \quad 0{,}6$$

$$=2\pi\left[v_0-\frac{(\rho_{Hg}-\rho)ghd^2}{16\mu L}\right]\int_{d/2}^{D/2} r\,dr+\frac{\pi(\rho_{Hg}-\rho)gh}{2\mu L}\int_{d/2}^{D/2} r^3\,dr+\frac{4\pi}{\ln(d/D)}\left[v_0+\frac{(\rho_{Hg}-\rho)gh(D^2-d^2)}{32\mu L}\right]\int_{d/2}^{D/2} r\ln\left(\frac{2r}{d}\right)dr$$

$$=\frac{\pi}{4}\left[v_0-\frac{(\rho_{Hg}-\rho)ghd^2}{16\mu L}\right](D^2-d^2)+\frac{\pi(\rho_{Hg}-\rho)gh}{128\mu L}(D^4-d^4)+\frac{4\pi}{\ln(d/D)}\left[v_0+\frac{(\rho_{Hg}-\rho)gh(D^2-d^2)}{32\mu L}\right]\left[\frac{D^2}{8}\ln\left(\frac{D}{d}\right)+\frac{(d^2-D^2)}{16}-\frac{d^2}{8}\cdot\cancel{\ln(1)}^{0}\right] \quad 0{,}6$$

$$=\frac{\pi}{4}\left[1-\frac{12600\cdot 9{,}8\cdot 0{,}0001\cdot 0{,}03^2}{16\cdot 10^{-3}\cdot 1}\right](0{,}05^2-0{,}03^2)+\frac{\pi\cdot 12600\cdot 9{,}8\cdot 0{,}0001\,(0{,}05^4-0{,}03^4)}{128\cdot 10^{-3}\cdot 1}+\frac{4\pi}{\ln(0{,}03/0{,}05)}\left[1+\frac{12600\cdot 9{,}8\cdot 0{,}0001\cdot(0{,}05^2-0{,}03^2)}{32\cdot 10^{-3}\cdot 1}\right]\left[\frac{0{,}05^2}{8}\cdot\ln\left(\frac{0{,}05}{0{,}03}\right)+\frac{(0{,}03^2-0{,}05^2)}{16}\right]$$

$$=0{,}00038+0{,}00165-0{,}00237=-0{,}00034\ [m^3/s] \quad 0{,}3$$