Pauta Ejercicio #3

P2)

a) N-S s/x

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (1)$$

N-S s/y

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \quad (2)$$

N-S s/z

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \quad (3)$$

continuidad

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

unidireccional $\Rightarrow \vec{v} = u\hat{\imath} \qquad v = w = 0$

permanente $\Rightarrow \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0 \qquad (4): v = w = 0 \Rightarrow \boxed{\frac{\partial u}{\partial x} = 0}$

(1): $\frac{\partial u}{\partial t} = 0,\ v = w = 0,\ \frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial^2 u}{\partial x^2} = 0,\ \frac{\partial^2 u}{\partial z^2} = 0$ (sistema muy ancho, no hay variaciones en z)

$$\Rightarrow \boxed{\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2}}$$

(2): $v = 0 \Rightarrow \boxed{\frac{\partial \hat{p}}{\partial y} = 0} \qquad$ (3): $w = 0 \Rightarrow \boxed{\frac{\partial \hat{p}}{\partial z} = 0}$

$$\frac{\partial \hat{p}}{\partial y} = \frac{\partial \hat{p}}{\partial z} = 0 \Rightarrow \frac{\partial \hat{p}}{\partial x} \rightarrow \frac{d\hat{p}}{dx} \text{ ($\hat{p}$ solo depende de x)}$$

Además $\frac{\partial \hat{p}}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial \hat{p}}{\partial y}\right) = 0 \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial \hat{p}}{\partial x}\right) = 0$

$\Rightarrow \frac{d\hat{p}}{dx}$ no depende de y

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} = 0 \Rightarrow \frac{\partial u}{\partial y} \rightarrow \frac{du}{dy} \text{ (u solo depende de y)}$$

$$\Rightarrow \boxed{\frac{1}{\rho}\frac{d\hat{p}}{dx} = \nu\frac{d^2 u}{dy^2}}$$

Condiciones de borde:

$$\left.\begin{array}{l} u(y=0) = 0 \\ u(y=e) = -\omega R \end{array}\right\} \text{ no resbalamiento}$$

$\frac{d\hat{p}}{dx}$: en el estanque $\hat{p} = p_{atm} + \rho g H_{estanque}$

en el mar $\hat{p} = p_{atm} + \rho g H_{mar}$

$$\frac{d\hat{p}}{dx} = \frac{\hat{p}_{estanque} - \hat{p}_{mar}}{X_{estanque} - X_{mar}} = \frac{p_{atm} + \rho g H_{estanque} - p_{atm} - \rho g H_{mar}}{0 - L}$$

$$\frac{d\hat{p}}{dx} = \frac{\rho g (H_{mar} - H_{estanque})}{L} < 0$$

b) $$\frac{1}{\rho}\frac{d\hat{p}}{dx} = \nu \frac{d^2u}{dy^2} \Rightarrow \frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{\rho g (H_{mar} - H_{estanque})}{L} = K$$

$$\frac{du}{dy} = Ky + C_1$$

$$u(y) = \frac{Ky^2}{2} + C_1 y + C_2$$

cond. de borde: $u(y=0) = 0 \Rightarrow C_2 = 0$

$$u(y=e) = -\omega R \Rightarrow -\omega R = \frac{Ke^2}{2} + C_1 e$$

$$\Rightarrow C_1 = -\frac{\omega R}{e} - \frac{Ke}{2}$$

$$\Rightarrow u(y) = \frac{Ky^2}{2} - \frac{\omega R}{e}y - \frac{Key}{2}$$

$$u(y) = -\frac{\omega R}{e}y - \frac{K}{2}y(e-y) = -\frac{\omega R}{e}y - \frac{1}{2\mu}\frac{\rho g (H_{mar} - H_{estanque})}{L}y(e-y)$$

c) veloc. máxima: $$\frac{du}{dy} = 0 \Rightarrow -\frac{\omega R}{e} - \frac{K}{2}(e-2y) = 0$$

$$\frac{K}{2}(2y-e) = \frac{\omega R}{e}$$

$$\boxed{y = \frac{e}{2} + \frac{\omega R}{eK}}$$

veloc. nula : $u = 0 \Rightarrow -\frac{\omega R}{e} y_N - \frac{K}{2} y_N (e - y_N) = 0 \qquad y_N \neq 0$

$$-\frac{\omega R}{e} - \frac{K}{2}(e - y_N) = 0$$

$$\frac{K}{2}(y_N - e) = \frac{\omega R}{e}$$

$$y_N = e + \frac{2\omega R}{K}$$

Como $K < 0 \Rightarrow y < e$

si el término $\left|\frac{2\omega R}{K}\right| > e \;\Rightarrow\; y < 0 \Rightarrow$ esto no puede darse ya que $0 < y < e$

Entonces $y_N \geq 0 \Rightarrow e + \frac{2\omega R}{K} \geq 0$

$$e + \frac{2\omega R \mu L}{\rho g\,(H_{mar} - H_{estanque})} \geq 0$$

$$e \rho g\,(H_{mar} - H_{estanque}) + 2\omega R \mu L \leq 0$$

$$\Rightarrow \omega \leq \frac{e \rho g}{2 R \mu L}\,(H_{estanque} - H_{mar})$$

o bien

$$H_{mar} - H_{estanque} \leq -\frac{2\omega R \mu L}{e \rho g}$$

$$H_{estanque} - H_{mar} \geq \frac{2\omega R \mu L}{e \rho g}$$