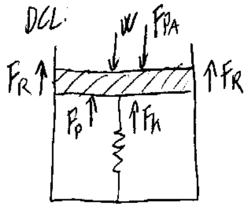
PAUTA P2

DCL:



a) $\sum \vec{F}=0 \Leftrightarrow F_P+F_K+F_R-W-F_{PA}=M\ddot{x}$ (1)

F_P: Proceso Isotérmico: $P\forall = cte \Rightarrow P(x)=\frac{P_A \forall_0}{\forall(x)} = \frac{P_A H \cancel{b^2}}{(H-x)\cancel{b^2}}$; $F_P = P\cdot A = \frac{P_A H}{(H-x)}\cdot b^2$

$F_K = Kx$; $F_{PA}=P_A\cdot A = P_A b^2$; $W=Mg$; $F_R=\tau\cdot dA = \frac{\mu\dot{x}}{e}\cdot 4Lb = \frac{4\mu\dot{x}Lb}{e}$

Reemplazando en (1): $\underbrace{\frac{P_A H b^2}{(H-x)}}_{0,5} + \underbrace{Kx}_{0,25} + \underbrace{\frac{4\mu L b}{e}\dot{x}}_{0,5} - \underbrace{Mg}_{0,25} - \underbrace{P_A b^2}_{0,25} = \underbrace{M\ddot{x}}_{0,25}$

b) Si $\mu\to 0$, y usando la indicación: $\frac{P_A H b^2}{(H-x)} + Kx - (Mg+P_A b^2) = M\dot{x}\frac{d\dot{x}}{dx}$; en $x=0, \dot{x}=0$

Integrando: $P_A H b^2\int_0^x \frac{dx}{H-x} + K\int_0^x x\,dx - (Mg+P_Ab^2)\int_0^x dx = M\int_0^{v(x)}\dot{x}\,d\dot{x}$

$\Leftrightarrow P_A H b^2 \ln\left(\frac{H}{H-x}\right) + \frac{Kx^2}{2} - (Mg+P_Ab^2)x = \frac{Mv(x)^2}{2}$ (1,0)

El pistón se detiene cuando $v=0 \Leftrightarrow P_AHb^2\ln\left(\frac{H}{H-x}\right)+\frac{Kx^2}{2} = (Mg+P_Ab^2)x$

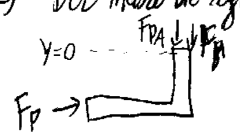
$\Leftrightarrow \overbrace{1.012\ln\left(\frac{1}{1-x}\right)+400x^2}^{A} = \overbrace{1.502x}^{B}$; Iterar para resolver:

X	A	B
0,5	801,5	751
0,4	681	600,8
0,45	686	675,9
0,43	642,8	645,9

$\Rightarrow X=0,43=X_0$ (0,5)

P Manómetro $= P(x_0) = \frac{P_A H}{(H-x_0)}\cdot b^2 = 1.775$ [kPa] (0,5)

c) DCL masa de agua:



$\sum F=0 \Leftrightarrow F_P - F_{PA} - F_\mu = M\ddot{y}$ (2)

F_P: Proceso Isotérmico $\Rightarrow P(y) = \frac{P(x_0)\forall(x_0)}{\forall(y)} = \frac{P_A H_0 b^2 \cancel{(H-x_0)}}{\cancel{(H-x_0)}\left(b^2(H-x_0)+\frac{\pi c^2}{4}y\right)}$; $F_P = P(y)\frac{\pi c^2}{4}$

$F_{PA}=P_A\cdot A = \frac{P_A\pi c^2}{4}$; $F_\mu = \rho g(h+y)\frac{\pi c^2}{4}$; $M=\rho(d+h)\frac{\pi c^2}{4}$

Reemplazando en (2): $\frac{P_AHb^2}{\left(b^2(H-x_0)+\frac{\pi c^2}{4}y\right)}\cdot\frac{\cancel{\pi c^2}}{\cancel{4}} - P_A\frac{\cancel{\pi c^2}}{\cancel{4}} - \rho g(h+y)\frac{\cancel{\pi c^2}}{\cancel{4}} = \rho(d+h)\frac{\cancel{\pi c^2}}{\cancel{4}}\ddot{y}$

$\Leftrightarrow \underbrace{\frac{P_AHb^2}{b^2(H-x_0)+\frac{\pi c^2}{4}y}}_{0,5} \underbrace{- P_A - \rho g(h+y)}_{0,5} = \underbrace{\rho(d+h)\frac{d\dot{y}}{dy}\cdot\dot{y}}_{0,5}$; En $y=0, \dot{y}=0$; Integrando.

$$P_A H b^2 \int_0^y \frac{dy}{b^2(H-X_0)+\frac{\pi c^2}{4}y} - P_A\int_0^y dy - \rho g\int_0^y (h+y)dy = \rho(d+h)\int_0^{v(y)} \dot{y}\, d\dot{y}$$

$$\frac{4P_A H b^2}{\pi c^2}\cdot \ln\left(\frac{b^2(H-X_0)+\frac{\pi c^2}{4}y}{b^2(H-X_0)}\right) - P_A y - \rho g\left(hy+\frac{y^2}{2}\right) = \rho(d+h)\frac{v^2(y)}{2}$$

Cuando la masa de agua empieza a salir del tubo, se tiene: $y=(d-h)$

$$\Rightarrow \frac{4P_A H b^2}{\pi c^2}\cdot \ln\left(\frac{b^2(H-X_0)+\frac{\pi c^2}{4}(d-h)}{b^2(H-X_0)}\right) - P_A(d-h) - \rho g\left(\cancel{hd}-h^2+\frac{d^2}{2}+\frac{h^2}{2}-\cancel{hd}\right) = \rho(d+h)\frac{v^2}{2}$$

$$\Rightarrow v = \left[\frac{2}{\rho(d+h)}\left(\frac{4P_A H b^2}{\pi c^2}\cdot \ln\left(\frac{b^2(H-X_0)+\frac{\pi c^2}{4}(d-h)}{b^2(H-X_0)}\right) - P_A(d-h) - \frac{\rho g}{2}(d^2-h^2)\right)\right]^{1/2}$$

$$\Rightarrow v = \left[\frac{2}{1{,}025(0{,}5+0{,}2)}\left(\frac{4\cdot 101{,}200\cdot 1\cdot 0{,}1^2}{\pi\cdot 0{,}02^2}\cdot \ln\left(\frac{0{,}1^2(1-0{,}43)+0{,}25\pi\cdot 0{,}02^2(0{,}5-0{,}2)}{0{,}1^2(1-0{,}43)}\right) - 101{,}200(0{,}5-0{,}2) - \frac{1{,}025\cdot 9{,}8}{2}(0{,}5^2-0{,}2^2)\right)\right]^{1/2}$$

$$\Rightarrow v = 7{,}73\ [m/s]$$ 0,5