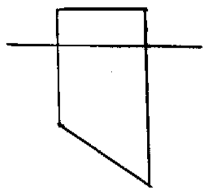
Pauta Control #1

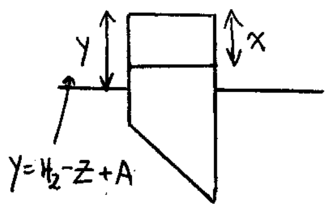
CI31A - Mecánica de Fluidos

P1) a)



Presión gas $= p_i = p_{atm}$

volumen gas $= \forall_i = \frac{\pi D^2}{4}\cdot(H_2 - Z)$



Presión gas $= p_f = \ ?$

Volumen gas $= \forall_f = \frac{\pi D^2}{4}\cdot x \quad ; x = ?$

por ley hidrostática: $p_{atm} = p_f + \gamma(y - x) \qquad (1)$

proceso isotérmico: $p_i \cdot \forall_i = p_f \cdot V_f$

$$p_{atm}\cdot \cancel{\frac{\pi D^2}{4}}(H_2 - Z) = p_f \cdot \cancel{\frac{\pi D^2}{4}}\cdot x \qquad (2)$$

De (2): $x = \frac{p_{atm}}{p_f}\cdot(H_2 - Z)$

en (1): $p_{atm} = p_f + \gamma(H_2 - Z + A - x)$

$$p_{atm} = p_f + \gamma\left[H_2 - Z + A - \frac{p_{atm}}{p_f}(H_2 - Z)\right] \quad / \cdot p_f$$

$$p_f^2 - p_{atm}\, p_f + \gamma(H_2 - Z + A)\, p_f - \gamma\, p_{atm}(H_2 - Z) = 0$$

$$p_f^2 + \left[\gamma(H_2 - Z + A) - p_{atm}\right] p_f - p_{atm}(H_2 - Z)\,\gamma = 0$$

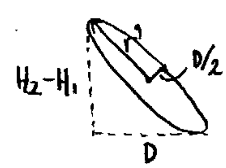
b) Por dentro:

$F_D = p_G \cdot A_r$

$$p_G = p_f + \gamma\left[\frac{H_2 + H_1}{2} - x\right]$$

$$p_G = p_f + \gamma\left[\frac{H_2 + H_1}{2} - \frac{p_{atm}}{p_f}(H_2 - Z)\right]$$

$$A_r = \pi \cdot \left(\frac{D}{2}\right)\cdot r'$$



$(2r') = \sqrt{(H_2 - H_1)^2 + D^2}$

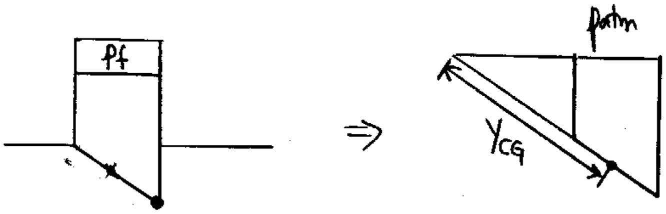
$r' = \frac{1}{2}\sqrt{(H_2 - H_1)^2 + D^2}$

$$A_r = \frac{\pi D}{4}\sqrt{(H_2 - H_1)^2 + D^2}$$

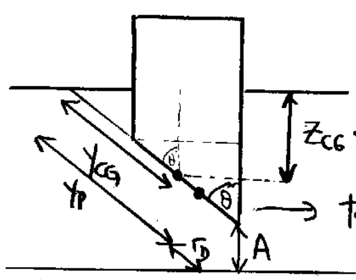
$$F_D = \left[P_f + \gamma \left\{ \frac{H_2 + H_1}{2} - \frac{p_{atm}}{P_f}(H_2 - Z) \right\} \right] \cdot \frac{\pi D}{4} \sqrt{(H_2 - H_1)^2 + D^2}$$

Punto de aplicación:

Debe establecerse la altura de una superficie equivalente, donde se encontraría la superficie libre en el exterior



Vemos que la presión dentro del tanque se mantiene desde el momento en que se cierra herméticamente la compuerta, por lo tanto es equivalente a encontrar Y_{CG} con el tanque al momento de cerrarse.



$$Z_{CG} = Z - A - \frac{H_2 - H_1}{2}$$

$$\to \operatorname{tg}\theta = \frac{D}{H_2 - H_1}$$

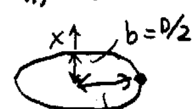
$$Y_{CG} \cdot \cos\theta = Z_{CG}$$

$$Y_{CG} = \frac{Z - A - \frac{H_2 - H_1}{2}}{\cos\left[\operatorname{atan}\left(\frac{D}{H_2 - H_1}\right)\right]}$$

$$Y_P = Y_{CG} + \frac{I_C}{Y_{CG} \cdot A_r}$$

donde $A_r = \frac{\pi D}{4}\sqrt{(H_2 - H_1)^2 + D^2}$

$$I_C = \frac{\pi a^3 b}{4}$$



$$a = \frac{1}{2}\sqrt{(H_2 - H_1)^2 + D^2}$$

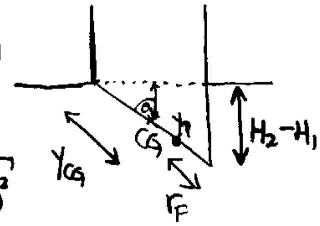
$$I_C = \frac{\pi}{4}\frac{D}{2}\frac{1}{8}\left[(H_2 - H_1)^2 + D^2\right]^{3/2}$$

Brazo: $r_D = \frac{1}{2}\sqrt{(H_2 - H_1)^2 + D^2} + (Y_P - Y_{CG}) = \frac{1}{2}\sqrt{(H_2 - H_1)^2 + D^2} + \frac{I_C}{Y_{CG} \cdot A_r}$

$$T_D = F_D \cdot r_D$$

Por fuera:

$F_F = p_G \cdot A_r \qquad p_G = p_{atm} + \gamma\left(\frac{H_2-H_1}{2}\right)$



$$A_r = \frac{\pi D}{4}\sqrt{(H_2-H_1)^2+D^2}$$

$$F_F = \left[p_{atm} + \gamma\left(\frac{H_2-H_1}{2}\right)\right]\cdot\frac{\pi D}{4}\sqrt{(H_2-H_1)^2+D^2}$$

pto. de aplicación:

$$y_{CG} = \frac{H_2-H_1}{2}\cdot\frac{1}{\cos\theta} = \frac{\frac{H_2-H_1}{2}}{\cos\left[\operatorname{atan}\left(\frac{D}{H_2-H_1}\right)\right]}$$

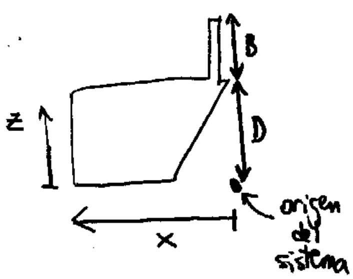
$$y_P = y_{CG} + \frac{I_c}{y_{CG}\cdot A_r}$$

Brazo: $r_F = \frac{1}{2}\sqrt{(H_2-H_1)^2+D^2} + y_P - y_{CG} = \frac{1}{2}\sqrt{(H_2-H_1)^2+D^2} + \frac{I_c}{y_{CG}\cdot A_r}$

(lo único que cambia respecto al otro brazo es y_{CG})

$T_F = F_F \cdot r_F$

c)



$$p = p_0 - \rho\left\{a_x(x-x_0) + a_y(y-y_0) + (a_z+g)(z-z_0)\right\}$$

$a_x = -a_0$
$a_y = a_z = 0$

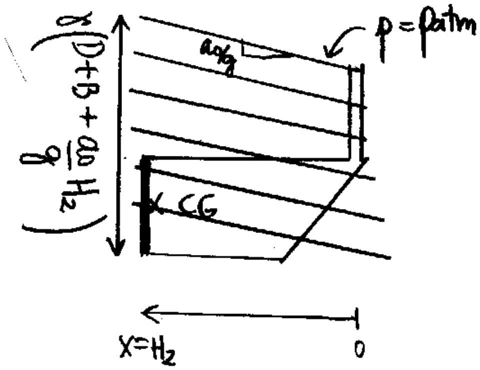
$$p = p_0 + \rho a_0(x-x_0) - \rho g(z-z_0)$$

condición de borde: $x=0;\ z = D+B\ ;\ p_0 = p_{atm}$

$$p = p_{atm} + \rho a_0 x - \rho g(z-D-B)$$

Isóbara de sup. libre ($p = p_{atm}$) $\qquad a_0 x = +g(z-D-B)$

$$z = +\frac{a_0}{g}x + D + B$$



fuerza en la cara frontal:

$$F = p_G \cdot A_r$$

$$p_G = \gamma\left[D+B+\frac{a_0}{g}H_2 - \frac{D}{2}\right]$$

$$= \gamma\left[\frac{D}{2}+B+\frac{a_0}{g}H_2\right]$$

$$A_r = \frac{\pi D^2}{4}$$

$$F = \gamma\pi\frac{D^2}{4}\left[\frac{D}{2}+B+\frac{a_0}{g}H_2\right]$$

a_0 lleva ya el signo negativo incluido, o sea, $a_0 > 0$ frenando.