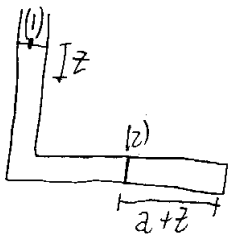
PAUTA AUX #6

P1] Ec. de Euler: $\frac{L_{12}}{g}\cdot\frac{dV}{dt}+B_2-B_1=0$



$B_1=\frac{P_0}{\gamma}+\frac{V^2}{2g}+z+H$

$B_2=\frac{P_2}{\gamma}+\frac{V^2}{2g}$

Proceso Isotérmico: $PV=cte \Leftrightarrow P_i\cdot\forall_i=P_f\forall_f$; $P_i=P_0+\gamma H$; $\forall_i=\frac{\pi d^2}{4}\cdot a$; $\forall_f=\frac{\pi d^2}{4}(a+z)$

$\Rightarrow (P_0+\gamma H)\cdot\frac{\pi d^2}{4}a=P_2\cdot\frac{\pi d^2}{4}(a+z) \Leftrightarrow P_2=\frac{(P_0+\gamma H)a}{(a+z)}$

$\Rightarrow \frac{L}{g}\cdot\frac{dV}{dt}+\frac{(P_0+\gamma H)a}{\gamma(a+z)}+\frac{V^2}{2g}-\frac{P_0}{\gamma}-\frac{V^2}{2g}-z-H$; pero, $\frac{dV}{dt}=\frac{dV}{dz}\cdot\frac{dz}{dt}=\frac{dV}{dz}\cdot V$

$\Rightarrow \frac{L}{g}\cdot V\cdot\frac{dV}{dz}+\frac{(P_0+\gamma H)a}{\gamma(a+z)}-\frac{P_0}{\gamma}-z-H \Leftrightarrow \frac{L}{g}V\frac{dV}{dz}=\frac{P_0}{\gamma}+z+H-\frac{(P_0+\gamma H)}{\gamma(a+z)}a$ (*)

$\frac{L}{g}\int V\,dV=\left(\frac{P_0}{\gamma}+H\right)\int dz+\int z\,dz-\frac{(P_0+\gamma H)a}{\gamma}\int\frac{dz}{(a+z)} \Leftrightarrow \frac{L}{g}\frac{V^2}{2}=\left(\frac{P_0}{\gamma}+H\right)z+\frac{z^2}{2}-\frac{(P_0+\gamma H)a}{\gamma}\ln(a+z)+K$

C.B: $V(z_0)=0 \Leftrightarrow 0=\left(\frac{P_0}{\gamma}+H\right)z_0+\frac{z_0^2}{2}-\frac{(P_0+\gamma H)a}{\gamma}\cdot\ln(a+z_0)+K$

$\Rightarrow K=\frac{(P_0+\gamma H)a}{\gamma}\ln(a+z_0)-\left(\frac{P_0}{\gamma}+H\right)z_0-\frac{z_0^2}{2} \Leftrightarrow V(z)=\sqrt{\frac{2g}{L}\left[\left(\frac{P_0}{\gamma}+H\right)(z-z_0)+\frac{(z^2-z_0^2)}{2}+\frac{(P_0+\gamma H)a}{\gamma}\ln\left[\frac{a+z_0}{a+z}\right]\right]}$

b) En (*) reemplazamos por $\frac{dV}{dz}=0 \Rightarrow \frac{P_0}{\gamma}+z+H=\frac{(P_0+\gamma H)a}{\gamma(a+z)} \Leftrightarrow \left(\frac{P_0}{\gamma}+z+H\right)(a+z)=\frac{(P_0+\gamma H)a}{\gamma}$

$\frac{P_0}{\gamma}a+\frac{P_0}{\gamma}z+az+z^2+Ha+Hz=\frac{(P_0+\gamma H)}{\gamma}a \Leftrightarrow z^2+\left[\frac{P_0}{\gamma}+a+H\right]z+\left(\frac{P_0}{\gamma}+a+H-\frac{P_0}{\gamma}-H\right)a=0$

$\Leftrightarrow z=\frac{-\left(\frac{P_0}{\gamma}+a+H\right)\pm\sqrt{\left(\frac{P_0}{\gamma}+a+H\right)^2-4a^2}}{2}=\frac{-\frac{101.200}{9.800}-0,1-0,2+\sqrt{\left(\frac{101.200}{9.800}+0,1+0,2\right)^2-4\cdot0,1^2}}{2}=0,0009\sim 0$

$V(0)=\sqrt{\frac{2\cdot9,8}{0,4}\left[\left(\frac{101.200}{9.800}+0,2\right)\cdot-0,05+\frac{(0,05)^2}{2}+\left(\frac{101.200+9.800\cdot0,2}{9.800}\right)\cdot0,1\cdot\ln\left(\frac{0,1+0,05}{0,05}\right)\right]}=5,55\ [m/s]$