P2) Navier-Stokes en coordenadas cilíndricas. El flujo es sólo según $\hat{z}$ $(V_r = V_\theta = 0)$

Flujo axisimétrico $\Rightarrow \frac{\partial V_z}{\partial \theta} = 0$

Continuidad: $\frac{1}{r}\frac{\partial (r\cancel{V_r})^{\nearrow 0}}{\partial r} + \frac{1}{r}\frac{\partial \cancel{V_\theta}^{\nearrow 0}}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0 \Rightarrow \frac{\partial V_z}{\partial z} = 0$

N-S $\hat{z}$: $\underbrace{\cancel{\frac{\partial V_z}{\partial t}}}_{\text{Perm.}} + \cancel{V_r}\frac{\partial V_z}{\partial r}^{0} + \frac{\cancel{V_\theta}}{r}\frac{\partial V_z}{\partial \theta}^{0} + V_z \underbrace{\cancel{\frac{\partial V_z}{\partial z}}}_{\text{Cont.}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\underbrace{\cancel{\frac{\partial^2 V_z}{\partial \theta^2}}}_{\text{axisim.}} + \underbrace{\cancel{\frac{\partial^2 V_z}{\partial z^2}}}_{\text{Cont}}\right)$

$\Leftrightarrow \frac{1}{\mu}\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) = cte \Leftrightarrow \frac{r}{\mu}\frac{dp}{dz} = \frac{d}{dr}\left(r\frac{dV_z}{dr}\right) \Leftrightarrow \frac{r^2}{2\mu}\frac{dp}{dz} + C_1 = r\frac{dV_z}{dr}$

$\Leftrightarrow \frac{r}{2\mu}\cdot\frac{dp}{dz} + \frac{C_1}{r} = \frac{dV_z}{dr} \Leftrightarrow \frac{r^2}{4\mu}\cdot\frac{dp}{dz} + C_1\cdot\ln(r) + C_2 = V_z(r)$

Para el flujo entre ambos tubos: $V_z\left(\frac{D}{2}\right) = 0$; $V_z\left(\frac{d}{2}\right) = 0$

$\Rightarrow 0 = \frac{D^2}{16\mu}\cdot\frac{dp}{dz} + C_1\ln\left(\frac{D}{2}\right) + C_2$

$0 = \frac{d^2}{16\mu}\cdot\frac{dp}{dz} + C_1\cdot\ln\left(\frac{d}{2}\right) + C_2$

$\Rightarrow \frac{dp}{dz}\cdot\frac{1}{16\mu}(D^2 - d^2) + C_1\ln\left(\frac{D}{d}\right) = 0$

$\Rightarrow C_1 = \frac{1}{16\mu}\cdot\frac{dp}{dz}\frac{(D^2-d^2)}{\ln(d/D)} = -0{,}0489\frac{dp}{dz}$

$\Rightarrow C_2 = -\frac{D^2}{16\mu}\cdot\frac{dp}{dz} - -0{,}0489\frac{dp}{dz}\cdot\ln\left(\frac{d}{2}\right) = -0{,}2196\frac{dp}{dz}$

$\Rightarrow V_z = 62{,}5\frac{dp}{dz}r^2 - 0{,}0489\frac{dp}{dz}\cdot\ln(r) - 0{,}2196\frac{dp}{dz} = \frac{dp}{dz}\left(62{,}5r^2 - 0{,}0489\ln(r) - 0{,}2196\right)$

Para el flujo al interior del tubo central: $V_z\left(\frac{d}{2} - e\right) = 0$; $V_z(0) < \infty$

$V_z = \frac{r^2}{4\mu}\frac{dp}{dz} + C_3\ln(r) + C_4$; para que no se indefina en $r=0 \Rightarrow C_3 = 0$

$0 = \frac{\left(\frac{d}{2}-e\right)^2}{4\mu}\frac{dp}{dz} + C_4 \Leftrightarrow C_4 = -\frac{(d/2 - e)^2}{4\mu}\frac{dp}{dz} = 0{,}00625\frac{dp}{dz} \Rightarrow V_z = \frac{dp}{dz}\left(62{,}5r^2 + 0{,}00625\right)$

b) $\tau = \mu \cdot \frac{dv_z}{dr}$; $\tau_{ext}(r=\frac{d}{2}) = \mu \cdot \frac{dv_z}{dz}\Big|_{r=\frac{d}{2}} = \frac{d\mathcal{P}}{dz}\left(125r - \frac{0,0489}{r}\right)\cdot\mu\Big|_{r=d/2} = -0,00555\,\frac{d\mathcal{P}}{dz}$

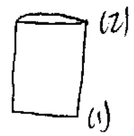
$\tau_{int}(r=\frac{d}{2}-e) = \mu\cdot\frac{dv_z}{dr}\Big|_{r=\frac{d}{2}-e} = \frac{d\mathcal{P}}{dz}\cdot 125\,r\,\mu\Big|_{r=\frac{d}{2}-e} = 0,005\cdot\frac{d\mathcal{P}}{dz}$

Equilibrio estático del tubo central: $\Sigma F_z = 0$

$\Rightarrow W = |\tau_{int}|\cdot 2\pi\left(\frac{d}{2}-e\right)\cdot L + |\tau_{ext}|\cdot 2\pi\frac{d}{2}\cdot L = 2\pi L\cdot\left(0,00555\,\frac{d\mathcal{P}}{dz}\left(\frac{d}{2}-e\right) + 0,005\,\frac{d\mathcal{P}}{dz}\cdot\frac{d}{2}\right)$

$\Rightarrow W = 8,2\cdot 10^{-4}\,\frac{d\mathcal{P}}{dz}$; $W = 0,1\,[kg_f] = 0,98\,[N]$ $\Rightarrow \frac{d\mathcal{P}}{dz} = 1.195,12\left[\frac{N}{m^3}\right]$

$\frac{d\mathcal{P}}{dz} = \frac{\Delta\mathcal{P}}{\Delta z} = \frac{(P_2+\rho g L - P_1)}{L}$



$\Rightarrow \frac{P_2-P_1}{L} + \rho g = \frac{dp}{dz} + \rho g \Leftrightarrow \frac{d\mathcal{P}}{dz} = \frac{dp}{dz} + \rho g \Leftrightarrow \frac{dp}{dz} = \frac{d\mathcal{P}}{dz} - \rho g = 1.195,12\left[N/m^3\right] - 9.800\left[N/m^3\right]$

$\Rightarrow \frac{dp}{dz} = -8.604,9\left[\frac{N}{m^3}\right]$