PAUTA AUX#5

P1) El espesor del flujo se mantiene cte $\Rightarrow$ líneas de corriente paralelas (Vel. sólo según x)

Continuidad: $\frac{\partial u}{\partial x} + \cancelto{0}{\frac{\partial v}{\partial y}} = 0 \Leftrightarrow \frac{\partial u}{\partial x} = 0$

N-S $\hat{x}$: $\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{permanente}} + \underbrace{\cancel{u\frac{\partial u}{\partial x}}}_{\text{continuidad}} + v\cancel{\frac{\partial u}{\partial y}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{\text{continuidad}} + \frac{\partial^2 u}{\partial y^2}\right) \Leftrightarrow \frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} \quad (1)$

N-S $\hat{y}$: $\underbrace{\cancel{\frac{\partial v}{\partial t}}}_{\text{permanente}} + u\cancelto{0}{\frac{\partial v}{\partial x}} + v\cancelto{0}{\frac{\partial v}{\partial y}} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\cdot\cancelto{0}{\nabla v} \Leftrightarrow \frac{dp}{dy} = 0 \Rightarrow p$ no depende de y

Integrando (1): $\mu\frac{du}{dy} = \frac{dp}{dx}y + C_1 \Leftrightarrow \mu u = \frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2 \Leftrightarrow u(y) = \frac{dp}{dx}\cdot\frac{y^2}{2\mu} + C_1 y + C_2$

Para $y \in [0,h]$; C.B: $u(0) = 0$; $u(h) = -2V_0$

$\Rightarrow 0 = C_2$

$-2V_0 = \frac{1}{2\mu}\cdot\frac{dp}{dx}\cdot h^2 + C_1 h \Leftrightarrow C_1 = -\frac{2V_0}{h} - \frac{1}{2\mu}\frac{dp}{dx}h \Rightarrow u(y) = \frac{1}{2\mu}\cdot\frac{dp}{dx}y(y-h) - \frac{2V_0}{h}y$

Para $y \in (h, 2h)$; C.B. $u(h) = -2V_0$; $u(2h) = V_0$

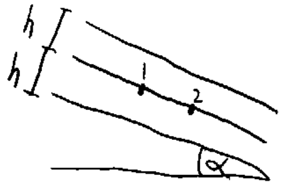
$\Rightarrow -2V_0 = \frac{1}{2\mu}\cdot\frac{dp}{dx}h^2 + C_1 h + C_2$

$V_0 = \frac{1}{2\mu}\cdot\frac{dp}{dx}4h^2 + 2C_1 h + C_2$

$\Rightarrow 3V_0 = \frac{3}{2\mu}\frac{dp}{dx}h^2 + C_1 h \Leftrightarrow C_1 = \frac{3V_0}{h} - \frac{3}{2\mu}\cdot\frac{dp}{dx}h$

$\Rightarrow C_2 = V_0 - \frac{1}{2\mu}\frac{dp}{dx}\cdot 4h^2 - 2h\left(\frac{3V_0}{h} - \frac{3}{2\mu}\frac{dp}{dx}\cdot h\right) = \frac{h^2}{\mu}\cdot\frac{dp}{dx} - 5V_0$

$\Rightarrow u(y) = \frac{dp}{dx}\cdot\frac{1}{2\mu}\cdot y^2 + \left(\frac{3V_0}{h} - \frac{3}{2\mu}\cdot\frac{dp}{dx}h\right)y + \frac{h^2}{\mu}\frac{dp}{dx} - 5V_0 = \frac{1}{2\mu}\cdot\frac{dp}{dx}\left(y^2 - 3hy + 2h^2\right) + V_0\left(\frac{3y}{h} - 5\right)$



$\frac{dp}{dx} = \frac{\Delta p}{\Delta x} = \frac{p_1 - p_2}{x_1 - x_2} = \frac{\cancel{p_1} + \rho g h_1 - \cancel{p_2} - \rho g h_2}{x_1 - x_2} = \frac{\rho g (h_1 - h_2)}{x_1 - x_2} = -\rho g \,\text{sen}\,\alpha$

$\therefore\ u(y) = -\frac{\rho g \,sen\alpha}{2\mu} y(y-h) - \frac{2V_0}{h} y$; para y entre 0 y h

$u(y) = -\frac{\rho g \,sen\alpha}{2\mu}(y^2-3hy+2h^2) + V_0\left(\frac{3y}{h}-5\right)$

$Q = \int u(y)\,dy$; hay que hacerlo por separado para cada zona:

$Q(0,h): -\frac{\rho g\, sen\alpha}{2\mu}\int_0^h y^2 dy + \frac{\rho g\, sen\alpha\, h}{2\mu}\int_0^h y dy - \frac{2V_0}{h}\int_0^h y dy = -\frac{\rho g h^3 sen\alpha}{6\mu} + \frac{\rho g h^3 sen\alpha}{4\mu} - V_0 h$

$\Rightarrow Q(0,h) = \frac{\rho g h^3 sen\alpha}{12\mu} - V_0 h$

$Q(h,2h) = -\frac{\rho g\, sen\alpha}{2\mu}\int_h^{2h}(y^2-3hy+2h^2)dy + V_0\int_h^{2h}\left(\frac{3y}{h}-5\right)dy = -\frac{\rho g\, sen\alpha}{2\mu}\left[\frac{y^3}{3} - \frac{3hy^2}{2} + 2h^2 y\right]_h^{2h} + V_0\left[\frac{3y^2}{2h} - 5y\right]_h^{2h}$

$\Rightarrow Q(h,2h) = -\frac{\rho g\, sen\alpha}{2\mu}\left[\frac{(8h^3-h^3)}{3} - \frac{(12h^3-3h^3)}{2} + (4h^3-2h^3)\right] + V_0\left[\frac{(12h-3h)}{2} - (10h-5h)\right]$

$\Rightarrow Q(h,2h) = \frac{\rho g h^3 sen\alpha}{12\mu} - \frac{V_0 h}{2}$

$Q = \frac{\rho g h^3 sen\alpha}{12\mu} - V_0 h + \frac{\rho g h^3 sen\alpha}{12\mu} - \frac{V_0 h}{2} = \frac{\rho g h^3 sen\alpha}{6\mu} - \frac{3V_0 h}{2}$

c) $Q = 0 \Leftrightarrow \frac{\rho g h^3 sen\alpha}{6\mu} - \frac{3V_0 h}{2} = 0 \Leftrightarrow \frac{\rho g h^{\cancel{3}} sen\alpha}{6\mu} = \frac{3V_0 \cancel{h}}{2} \Leftrightarrow h = \sqrt{\frac{9V_0\mu}{\rho g\, sen\alpha}}$