P2) Primer caso: $h_I \leq h \leq H_0$ $\quad 0 \leq t \leq \tilde{t}$

$$\frac{d\forall}{dt} = \underbrace{Q_e}_{Q_a} - \cancel{Q_s}^{\,0} \qquad \forall = h \cdot A \qquad \frac{d\forall}{dt} = A\frac{dh}{dt}$$

$$A\frac{dh}{dt} = Q_a$$

$$\int_{h_I}^{h} A\,dh = \int_0^{t} Q_a\,dt$$

$$A\,h\Big|_{h_I}^{h} = Q_a t\Big|_0^{t} \qquad A(h-h_I) = Q_a t$$

$$h(t) = \frac{Q_a t}{A} + h_I$$

$$\boxed{h(t) = 4 + 0{,}00002\,t} \quad t \text{ en seg.}$$

$\tilde{t}$: tiempo en que $h = H_0$: $\quad \underbrace{H_0}_{5} = 4 + 0{,}00002\,\tilde{t}$

$$\tilde{t} = 50000 \text{ seg.} = 0{,}579 \text{ días}$$

Segundo caso: $h > H_0$ $\quad \tilde{t} \leq t \leq T^*$

$$\frac{d\forall}{dt} = A\frac{dh}{dt} = \underbrace{Q_e}_{Q_a} - \underbrace{Q_s}_{\alpha z = \alpha(h-H_0)} \qquad A\frac{dh}{dt} = Q_a - \alpha h + \alpha H_0$$

$$\int_{H_0}^{h} \frac{A\,dh}{Q_a + \alpha H_0 - \alpha h} = \int_{\tilde{t}}^{t} dt \qquad -\frac{A}{\alpha}\ln(Q_a + \alpha H_0 - \alpha h)\Big|_{H_0}^{h} = t\Big|_{\tilde{t}}^{t}$$

$$\ln\left(\frac{Q_a + \alpha H_0 - \alpha h}{Q_a}\right) = -\frac{\alpha}{A}(t-\tilde{t})$$

$$-\alpha h = Q_a e^{\frac{\alpha}{A}(\tilde{t}-t)} - Q_a - \alpha H_0$$

$$\boxed{h = H_0 + \frac{Q_a}{\alpha}\left[1 - e^{\frac{\alpha}{A}(\tilde{t}-t)}\right]}$$

función creciente con t, Tiende a $h(t\to\infty) = H_0 + \frac{Q_a}{\alpha} = 5{,}5$

En $t = T^* = 2{,}5$ días $= 216000$ seg.

$$\boxed{h = 5{,}499 \text{ m}}$$

Tercer caso: $h > H_0 \quad t > T^*$

$$\frac{d\forall}{dt} = \cancel{Q_e}^{\,0} - \underbrace{Q_s}_{\alpha(h-H_0)} \qquad A\frac{dh}{dt} = -\alpha(h-H_0)$$

$$\int_{h(T^*)}^{h} \frac{A\,dh}{\alpha(H_0-h)} = \int_{T^*}^{t} dt$$

$$-\frac{A}{\alpha}\ln(H_0-h)\Big|_{h(T^*)}^{h} = t-T^*$$

$$\frac{A}{\alpha}\ln\left[\frac{H_0-h}{H_0-h(T^*)}\right] = T^*-t$$

$$H_0-h = \left(H_0-h(T^*)\right)\cdot e^{\frac{\alpha}{A}(T^*-t)}$$

$$\boxed{h = H_0 + \left(h(T^*)-H_0\right) e^{\frac{\alpha}{A}(T^*-t)}}$$

función decreciente con t

$$h(t\to\infty) = H_0$$