Pauta Auxiliar #4 CI31A 2003/01

P1) a) $\frac{d\forall}{dt} = Q_e - Q_s \qquad Q_s = v_s \cdot A_s \qquad A_s = \frac{\pi d_1^2}{4} \qquad v_s = \sqrt{2gh}$

$$Q_s = \frac{\pi d_1^2}{4}\sqrt{2gh}$$

$$\forall = \frac{\pi r(h)^2 \cdot h}{3} \qquad tg\,\alpha = \frac{r(h)}{h} \Rightarrow r(h) = h \qquad \forall = \frac{\pi h^3}{3}$$

$$\frac{d\forall}{dt} = \frac{d}{dt}\left(\frac{\pi h^3}{3}\right) = \pi h^2 \frac{dh}{dt}$$

$$\Rightarrow \pi h^2 \frac{dh}{dt} = Q_e - \frac{\pi d_1^2}{4}\sqrt{2gh}$$

$$\frac{\pi h^2}{Q_e - \frac{\pi d_1^2}{4}\sqrt{2gh}}\, dh = dt$$

b) Equilibrio final: $\frac{d\forall}{dt} = 0 \Rightarrow Q_e = Q_s$

$$Q_s(\text{en equilibrio}) = \frac{\pi d_1^2}{4}\sqrt{2g h_{eq}} = Q_e$$

$$\Rightarrow \boxed{h_{eq} = \left(\frac{4Q_e}{\pi d_1^2}\right)^2 \cdot \frac{1}{2g}}$$

c) $\frac{\pi h^2}{Q_e - \frac{\pi d_1^2}{4}\sqrt{2gh}}\, dh = dt$ Sea $A = \frac{Q_e}{\pi}$, $B = -\frac{d_1^2}{4}\sqrt{2g}$

$$\int_{h_0}^{h} \frac{h^2}{A + B\sqrt{h}}\, dh = \int_0^t dt$$

Sea $x = \sqrt{h}$, $x^5 = h^{5/2}$ | $dx = \frac{1}{2}\frac{dh}{\sqrt{h}} \,/\cdot 2h^{5/2}$, $2h^{5/2}dx = h^2 dh$, $2x^5 dx = h^2 dh$

$$\Rightarrow \int_{\sqrt{h_0}}^{\sqrt{h}} \frac{2x^5}{A+Bx}\, dx = t$$

$$\left. -\frac{2A^4x}{B^5} + \frac{A^3x^2}{B^4} - \frac{2A^2x^3}{3B^3} + \frac{Ax^4}{2B^2} - \frac{2x^5}{5B} + \frac{2A^5\ln(A+Bx)}{B^6} \right|_{\sqrt{h_0}}^{\sqrt{h}} = t$$