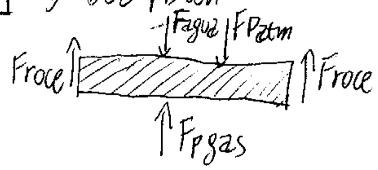
P2 a) DCL pistón



vel cte $\Rightarrow \Sigma F = 0 \Leftrightarrow F_{Pgas} + F_{roce} = F_{agua} + F_{Patm}$ (1)

$F_{Pgas} = P\cdot A$; $F_{Patm} = P_{atm}\cdot A$; $F_{agua} = \gamma (H_1 + V_0 t)\cdot A$

$F_{roce} = \tau \cdot dA = \mu \frac{dv}{dy}\cdot \pi D L$ (dist. lineal) $\Rightarrow F_{roce} = \frac{\mu V_0 \pi D L}{e}$

Laplace: $P_{int} - P_{ext} = \frac{4\sigma}{R} \Leftrightarrow P - P_{atm} = \frac{4\sigma}{R} \Leftrightarrow P = \frac{4\sigma}{R} + P_{atm}$ (2)

Reemplazando en (1): $\left(\frac{4\sigma}{R} + P_{atm}\right)\frac{\cancel{\pi D^2}}{4} + \frac{\mu V_0 \cancel{\pi D} L}{e} = \gamma (H_1 + V_0 t)\frac{\cancel{\pi D^2}}{4} + P_{atm}\cdot\frac{\cancel{\pi D^2}}{4}$

Como $t_1 = 0$; y en t_1, $R = R_1 \Rightarrow \left(\frac{4\sigma}{R_1} + \cancel{P_{atm}}\right)\frac{D}{4} + \frac{\mu V_0 L}{e} = \frac{\gamma H_1 D}{4} + \cancel{\frac{P_{atm} D}{4}}$

$\Rightarrow V_0 = \frac{De}{4\mu L}\left(\gamma H_1 - \frac{4\sigma}{R_1}\right) = 2$ [cm/seg]

b) $P\cdot\forall = nR_0 T \Rightarrow T = \frac{P\forall}{nR_0} \Rightarrow T(t_2) = \frac{P(t_2)\cdot\forall(t_2)}{nR_0}$ (3)

De (1): $P = \left[\gamma (H_1 + V_0 t) A + P_{atm} A - \frac{\mu V_0 \pi D L}{e}\right]\cdot\frac{4}{\pi D^2} \Rightarrow P(t_2) = \gamma (H_1 + V_0 t_2) + P_{atm} - \frac{4\mu V_0 L}{De}$ (4)

$\forall(t_2) = \forall_{gas}(t_2) + \forall_{burbuja}(t_2) = (h_1 - V_0 t_2)\cdot A + \frac{4}{3}\pi R^3(t_2)$ (5)

De (2): $P = \frac{4\sigma}{R} + P_{atm} \Leftrightarrow R = \frac{4\sigma}{(P - P_{atm})} \Rightarrow$ (5): $\forall(t_2) = (h_1 - V_0 t_2)\frac{\pi D^2}{4} + \frac{4}{3}\pi\left(\frac{4\sigma}{P(t_2) - P_{atm}}\right)^3$

Para conocer nR_0, usamos Ley de los gases en $t = t_1$: $nR_0 = \frac{P(t_1)\cdot\forall(t_1)}{T(t_1)}$

$\Rightarrow nR_0 = \frac{\left(\frac{4\sigma}{R_1} + P_{atm}\right)\cdot\left(\frac{\pi D^2 h_1}{4} + \frac{4}{3}\pi R_1^3\right)}{T_1}$

Reemplazando en (3): $T(t_2) = \frac{\left[\gamma (H_1 + V_0 t_2) + P_{atm} - \frac{4\mu V_0 L}{De}\right]\left[(h_1 - V_0 t_2)\frac{\cancel{\pi} D^2}{4} + \frac{4}{3}\cancel{\pi}\left(\frac{4\sigma}{\gamma (H_1 + V_0 t_2) - \frac{4\mu V_0 L}{De}}\right)^3\right]}{\left(\frac{4\sigma}{R_1} + P_{atm}\right)\left(\frac{\cancel{\pi} D^2 h_1}{4} + \frac{4}{3}\cancel{\pi} R_1^3\right)}\cdot T_1$

c) 1ª Ley Termodinámica: $\frac{dU}{dt} = \frac{d\hat{Q}}{dt} - \frac{dW}{dt}$ (6)

$dU = nC_v' dT \Rightarrow \frac{dU}{dt} = nC_v'\frac{dT}{dt}$; $\frac{d\hat{Q}}{dt} = \dot{\hat{Q}}$ (flujo de calor); $\frac{dW}{dt} = \frac{d(P\cdot\forall)}{dt} = P\cdot\frac{d\forall}{dt} + \forall\cdot\frac{dP}{dt}$

Reemplazando en (6): $nC_v'\frac{dT}{dt} = \dot{Q} - P\frac{d\forall}{dt} - \forall\frac{dP}{dt} \Leftrightarrow \dot{\hat{Q}} = nC_v'\frac{dT}{dt} + P\cdot\frac{d\forall}{dt} + \forall\cdot\frac{dP}{dt}$

Pero, $T = \frac{P\forall}{nR_0} \Rightarrow \frac{dT}{dt} = \frac{1}{nR_0}\frac{d(P\cdot\forall)}{dt}$; además, $C_v' = \frac{5}{2}R_0$ (gas diatómico)

$$\Rightarrow \dot{Q} = \frac{\cancel{n}\cdot\frac{5}{2}\cancel{R_0}}{\cancel{n}\cancel{R_0}}\left(P\frac{d\forall}{dt} + \forall\frac{dP}{dt}\right) + P\frac{d\forall}{dt} + \forall\frac{dP}{dt} = \frac{7}{2}\left(P\frac{d\forall}{dt} + \forall\frac{dP}{dt}\right)$$

$$\forall(t) = (h_1 - V_0 t)\frac{\pi D^2}{4} + \frac{4}{3}\pi\left(\frac{4\sigma}{\gamma(H_1+V_0t) - \frac{4\mu V_0 L}{De}}\right)^3 \Rightarrow \frac{d\forall}{dt} = -\frac{\pi D^2 V_0}{4} + \frac{4}{\cancel{3}}\pi\cdot\frac{(4\sigma)^3\cdot\cancel{3}\gamma V_0}{\left(\gamma(H_1+V_0t) - \frac{4\mu V_0 L}{De}\right)^2}$$

$$\Rightarrow \frac{d\forall}{dt} = -V_0\pi\left(\frac{D^2}{4} + \frac{4^4\sigma^3\gamma}{\left(\gamma(H_1+V_0t) - \frac{4\mu V_0 L}{De}\right)^2}\right)$$

$$P(t) = \gamma(H_1+V_0t) + P_{atm} - \frac{4\mu V_0 L}{De} \Rightarrow \frac{dP}{dt} = \gamma V_0$$

$$\therefore \dot{Q} = \frac{7}{2}\left\{-\left[\gamma(H_1+V_0t) + P_{atm} - \frac{4\mu V_0 L}{De}\right]\cdot V_0\pi\left[\frac{D^2}{4} + \frac{4^4\sigma^3\gamma}{\left(\gamma(H_1+V_0t) - \frac{4\mu V_0 L}{De}\right)^2}\right] + \gamma V_0\left[(h_1 - V_0t)\frac{\pi D^2}{4} + \frac{4}{3}\pi\left(\frac{4\sigma}{\gamma(H_1+V_0t - \frac{4\mu V_0 L}{De})}\right)^3\right]\right\}$$