Pauta Ejercicio #3 CI31A

Navier Stokes en coordenadas cilíndricas

$$\vec{v} = v_r \hat{r} + v_\theta \hat{\theta} + v_z \hat{k}$$

En este caso, flujo es unidireccional ($\vec{v} = v_z \hat{k}$, $v_r = v_\theta = 0$)

axisimetrico $\vec{v} \neq f(\theta)$

permanente $\frac{\partial \vec{v}}{\partial t} = 0$

Continuidad ($\nabla \cdot \vec{v} = 0$)

$$\frac{1}{r}\frac{\partial(r \cancelto{0}{v_r})}{\partial r} + \frac{1}{r}\frac{\partial \cancelto{0}{v_\theta}}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \Rightarrow \frac{\partial v_z}{\partial z} = 0$$

Navier Stokes (s/z)

$$\underbrace{\frac{\partial \cancelto{0}{v_z}}{\partial t}}_{\text{permanente}} + \cancelto{0}{v_r}\frac{\partial v_z}{\partial r} + \frac{\cancelto{0}{v_\theta}}{r}\frac{\partial v_z}{\partial \theta} + \underbrace{v_z \frac{\partial \cancelto{0}{v_z}}{\partial z}}_{\substack{\text{uniforme} \\ \text{(continuidad)}}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial z} + \frac{\mu}{\rho}\left\{\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \underbrace{\frac{1}{r^2}\frac{\partial^2 \cancelto{0}{v_z}}{\partial \theta^2}}_{\text{axisimet.}} + \underbrace{\frac{\partial^2 \cancelto{0}{v_z}}{\partial z^2}}_{\text{uniforme}}\right\}$$

$$\Rightarrow \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}$$

s/r , s/θ $\Rightarrow \frac{\partial \hat{p}}{\partial r} = \frac{\partial \hat{p}}{\partial \theta} = 0$

$$\Rightarrow \frac{\partial}{\partial z}\left(\frac{\partial \hat{p}}{\partial r}\right) = \frac{\partial}{\partial z}\left(\frac{\partial \hat{p}}{\partial \theta}\right) = 0 \Rightarrow \frac{\partial}{\partial r}\left(\frac{\partial \hat{p}}{\partial z}\right) = \frac{\partial}{\partial \theta}\left(\frac{\partial \hat{p}}{\partial z}\right) = 0$$

$\frac{\partial \hat{p}}{\partial z}$ es independiente de r y θ

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}$$

$$\Rightarrow \frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z} r$$

$$\Rightarrow \quad r\frac{\partial v_z}{\partial r} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\frac{r^2}{2} + C_1$$

$$\frac{\partial v_z}{\partial r} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\frac{r}{2} + \frac{C_1}{r}$$

$$v_z = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\frac{r^2}{4} + C_1 \log r + C_2$$

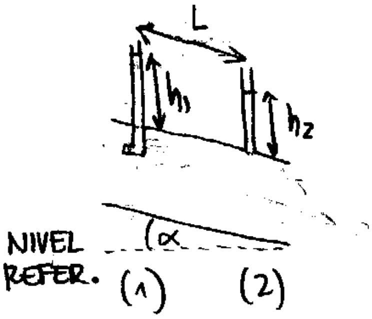
cond. borde:

- $v_z(r=0)$ es finita $\Rightarrow C_1 = 0$
- $v_z(r = d/2) = 0$

$$\Rightarrow \quad 0 = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\frac{d^2}{16} + C_2 \Rightarrow C_2 = -\frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\frac{d^2}{16}$$

$$\Rightarrow \quad v_z = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\left(\frac{r^2}{4} - \frac{d^2}{16}\right)$$

$$\frac{\partial \hat{p}}{\partial z} = \frac{\hat{p}_2 - \hat{p}_1}{L}$$



$$\hat{p}_2 = \gamma h_2$$

$$\frac{\hat{p}_1}{\gamma} + \frac{v_1^2}{2g} = h_1 + L \operatorname{sen}\alpha$$

$$\hat{p}_1 = \gamma\left[h_1 + L \operatorname{sen}\alpha - \frac{v_1^2}{2g}\right]$$

$$\frac{\partial \hat{p}}{\partial z} = \frac{1}{L}\left[\gamma(h_2 - h_1 - L \operatorname{sen}\alpha) + \rho\frac{v_1^2}{2}\right] \quad (1)$$

Por condición del tubo de Pitot

$$v_1 = v_z(r = d/4) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\left(\frac{(d/4)^2}{4} - \frac{d^2}{16}\right) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z} d^2\left(\frac{1}{64} - \frac{1}{16}\right)$$

$$v_1 = \frac{-3}{64\mu}\frac{\partial \hat{p}}{\partial z} d^2 \quad (2)$$

(1) en (2): $$v_1 = \frac{-3d^2}{64\mu}\frac{1}{L}\left[\gamma(h_2 - h_1 - L \operatorname{tg}\alpha) + \rho\frac{v_1^2}{2}\right]$$

$$\Rightarrow \frac{64\mu L}{3d^2} v_1 + \gamma(h_2 - h_1 - L\,\mathrm{sen}\,\alpha) + \frac{\rho v_1^2}{2} = 0 \qquad / \cdot \frac{2}{\rho}$$

$$\frac{128\mu L}{3d^2\rho} v_1 + 2g(h_2 - h_1 - L\,\mathrm{sen}\,\alpha) + v_1^2 = 0$$

$$a v_1^2 + b v_1 + c = 0 \qquad a = 1$$

$$b = \frac{128\mu L}{3d^2\rho}$$

$$\Rightarrow v_1 = 1{,}186 \text{ m/s} \qquad c = 2g(h_2 - h_1 - L\,\mathrm{sen}\,\alpha)$$

$$\Rightarrow \frac{\partial \hat{p}}{\partial z} = -562{,}36 \ \frac{\text{kg}}{\text{m}^2\,\text{s}}$$

$$\Rightarrow v_z = -7029{,}49\,(r^2 - 0{,}000225) = 1{,}582 - 7029{,}49\, r^2$$

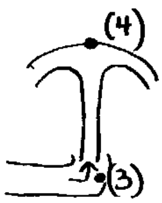
b) $$Q = \int_\Omega v_z\, dA = \int_0^{2\pi}\int_0^{d/2} v_z\, r\, dr\, d\theta = 2\pi \int_0^{d/2} v_z \cdot r\, dr$$

$$Q = 2\pi \int_0^{d/2} \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\left(\frac{r^3}{4} - \frac{d^2 r}{16}\right) dr = 2\pi\left[\frac{1}{\mu}\frac{\partial \hat{p}}{\partial z}\left(\frac{r^4}{16} - \frac{d^2 r^2}{32}\right)\right]_0^{d/2}$$

$$= \frac{2\pi}{\mu}\frac{\partial \hat{p}}{\partial z}\left[\frac{d^4}{16\cdot 16} - \frac{d^2 d^2}{4\cdot 32}\right] = \frac{2\pi d^4}{\mu}\frac{\partial \hat{p}}{\partial z}\left[\frac{1}{256} - \frac{1}{128}\right]$$

$$Q = -\frac{\pi d^4}{128\mu}\frac{\partial \hat{p}}{\partial z} = 5{,}6\times 10^{-4}\ \frac{\text{m}^3}{\text{s}} \qquad Re = \frac{4Q}{\pi d \nu} = \frac{4Q\rho}{\pi d \mu} = 1067 \approx \text{laminar} \Rightarrow \text{ok}$$

c) A la salida $p_s = p_{atm} = 0$ (en términos relativos)



$$\left.\begin{array}{l} \Rightarrow B_3 = \dfrac{\bar{v}^2}{2g} \\ \text{En } (4),\ v = 0,\ p = 0 \Rightarrow B_4 = h_3 \end{array}\right\}$$ ambos Bernouillis medidos respecto al nivel de (3) $B_3 = B_4$

$$\Rightarrow h_3 = \frac{\bar{v}^2}{2g} = \frac{1}{2g}\left(\frac{4Q}{\pi d^2}\right)^2 = 0{,}032 \text{ m} = 3{,}2 \text{ cm}$$