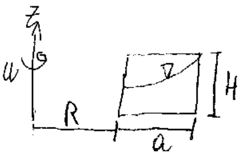
a) $p - p_0 = \frac{\rho\omega^2}{2}(r^2 - r_0^2) - \rho g (z - z_0)$



En $r = R+a$, la sup. libre debe estar a una altura tq $z \le H$

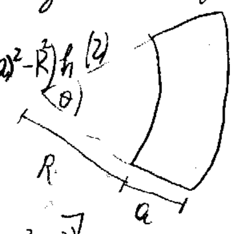
$\Rightarrow$ En la sup. libre: $p = p_0$; $r_0 = R+a$; $z_0 = H$

$$\Rightarrow \cancel{p - p_0}^{\,0} = \frac{\rho\omega^2}{2}(r^2-(R+a)^2) - \rho g(z-H) \iff \rho g(z-H) = \frac{\rho\omega^2}{2}(r^2-(R+a)^2)$$

$$\iff z = \frac{\omega^2}{2g}(r^2-(R+a)^2)+H \Rightarrow z = \frac{\omega^2}{2g}r^2 + \left[H - \frac{\omega^2}{2g}(R+a)^2\right] = A\cdot r^2 + B \ (1), \text{ con } A = \frac{\omega^2}{2g} \text{ y } B = H - \frac{\omega^2}{2g}(R+a)^2$$

1,0 1,0

Hay que aplicar conservación de volumen, $V_0 = V_f$; $V_0 = \frac{\theta}{2\pi}\pi((R+a)^2-R^2)h = \frac{\theta}{2}[(R+a)^2-R^2]h$ (2) 0,5



$$V_f = \int_R^{R+a}\int_0^z\int_0^\theta r\,d\theta\,dz\,dr = \theta\int_R^{R+a}\int_0^z r\,dz\,dr = \theta\int_R^{R+a} z\cdot r\,dr$$; pero, usando (1)

$$V_f = \theta\int_R^{R+a}(Ar^2+B)r\,dr = \theta\int_R^{R+a}(Ar^3+Br)\,dr = \theta\left[\frac{Ar^4}{4}+\frac{Br^2}{2}\right]_R^{R+a} = \left[\frac{A}{4}((R+a)^4-R^4)+\frac{B}{2}((R+a)^2-R^2)\right]\theta$$

Usando (2) y reemplazando los valores de A y B se llega a:

$$\frac{\cancel{\theta}}{2}((R+a)^2-R^2)\cdot h = \left[\frac{\omega^2}{8g}((R+a)^4-R^4) + \left(\frac{H}{2}-\frac{\omega^2}{4g}(R+a)^2\right)((R+a)^2-R^2)\right]\cancel{\theta}$$

$$\frac{h}{2}((R+a)^2-R^2) = \frac{\omega^2}{8g}\left[((R+a)^2+R^2)((R+a)^2-R^2)\right] + \frac{H}{2}((R+a)^2-R^2) - \frac{\omega^2}{4g}(R+a)^2((R+a)^2-R^2)$$

$$\frac{(h-H)}{2}\cancel{((R+a)^2-R^2)} = \frac{\omega^2}{4g}\cancel{((R+a)^2-R^2)}\left[\frac{(R+a)^2+R^2}{2}-(R+a)^2\right]$$

$$\frac{(h-H)}{\cancel{2}} = \frac{\omega^2}{4g}\left(\frac{R^2-(R+a)^2}{\cancel{2}}\right) \iff (H-h) = \frac{\omega^2}{4g}((R+a)^2-R^2) \iff (H-h) = \frac{\omega^2}{4g}(2Ra+a^2) \iff H-h = \frac{\omega^2 a}{4g}(2R+a)$$

1,5

$\iff \omega = \sqrt{\frac{4g(H-h)}{a(2R+a)}}$. Pero, se necesita calcular v: $v = R\cdot\omega = R\cdot\sqrt{\frac{4g(H-h)}{a(2+Ra)}}$ 0,5

$v = 70\cdot\sqrt{\frac{4\cdot 9{,}8(0{,}1-0{,}09)}{0{,}08(2\cdot 70+0{,}08)}} = 13{,}09$ [m/s] $= 47{,}13$ [km/h] 0,5

b) No afecta en nada, pues ω no es función de ρ 1,0