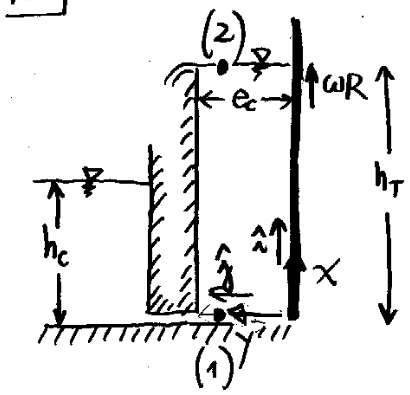
Pauta Control #2 CI31A
Semestre Primavera 2002

P2)



$$\hat{p}_1 = \underbrace{p_1}_{\gamma h_c} + \gamma_c \cancelto{0}{h_1} = \gamma_c h_c \quad \text{(igual que en el estanque)}$$

$$\hat{p}_2 = \cancelto{0}{p_2} + \underbrace{\gamma_c h_2}_{\gamma_c h_T} = \gamma_c h_T$$

$$\frac{\partial \hat{p}}{\partial x} = \frac{\hat{p}_1 - \hat{p}_2}{x_1 - x_2} = \frac{\gamma_c h_c - \gamma_c h_T}{0 - h_T} = \frac{\gamma_c (h_T - h_c)}{h_T}$$

Notar que $\frac{\partial \hat{p}}{\partial x} > 0$, no hay distribución hidrostática de presiones en la zona de flujo ascendente.

$\vec{v} = u\hat{i}$ (flujo unidireccional) $\Rightarrow$ $v = w = 0$

continuidad: $\frac{\partial u}{\partial x} + \cancelto{0}{\frac{\partial v}{\partial y}} + \cancelto{0}{\frac{\partial w}{\partial z}} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$

Ecuación de Navier-Stokes:

según y, según z: $\frac{\partial \hat{p}}{\partial y} = \frac{\partial \hat{p}}{\partial z} = 0$

según x:

$$\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{permanente}} + u \underbrace{\cancelto{0}{\frac{\partial u}{\partial x}}}_{\text{continuidad}} + \cancelto{0}{v} \frac{\partial u}{\partial y} + \cancelto{0}{w} \frac{\partial u}{\partial z} = \frac{-1}{\rho_c} \frac{\partial \hat{p}}{\partial x} + \frac{\mu_c}{\rho_c}\left(\underbrace{\cancelto{0}{\frac{\partial^2 u}{\partial x^2}}}_{\text{continuidad}} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\cancelto{0}{\frac{\partial^2 u}{\partial z^2}}}_{\text{flujo bidimensional}} \right)$$

$$\frac{1}{\rho_c} \frac{\partial \hat{p}}{\partial x} = \frac{\mu_c}{\rho_c} \frac{\partial^2 u}{\partial y^2} \Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu_c} \frac{\partial \hat{p}}{\partial x}$$

$$u(y) = \frac{1}{\mu_c} \frac{\partial \hat{p}}{\partial x} \frac{y^2}{2} + cy + d$$

cond de borde:

$$u(y=0) = \omega R$$

$$\Rightarrow \boxed{\omega R = d}$$

$$u(y=e_c) = 0$$

$$0 = \frac{1}{\mu_c}\frac{\partial \hat{p}}{\partial x}\frac{e_c^2}{2} + c\, e_c + \omega R$$

$$\Rightarrow \boxed{c = \frac{-1}{2\mu_c}\frac{\partial \hat{p}}{\partial x} e_c - \frac{\omega R}{e_c}}$$

$$\Rightarrow u(y) = \frac{1}{\mu_c}\frac{\partial \hat{p}}{\partial x}\frac{y^2}{2} - \frac{1}{2\mu_c}\frac{\partial \hat{p}}{\partial x} e_c y - \frac{\omega R}{e_c} y + \omega R$$

$$= \frac{1}{2\mu_c}\frac{\partial \hat{p}}{\partial x} y(y-e_c) + \omega R\left(1-\frac{y}{e_c}\right)$$

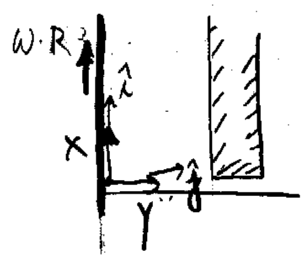
$$q_c = \int_0^{e_c} u(y)\,dy = \frac{1}{2\mu_c}\frac{\partial \hat{p}}{\partial x}\left[\frac{y^3}{3}-\frac{e_c y^2}{2}\right] + \omega R\left[y - \frac{y^2}{2e_c}\right]\Bigg|_0^{e_c}$$

$$= \frac{1}{2\mu_c}\frac{\partial \hat{p}}{\partial x}\left[\frac{e_c^3}{3}-\frac{e_c^3}{2}\right] + \omega R\left[e_c - \frac{e_c}{2}\right]$$

$$= \frac{-1}{2\mu_c}\frac{\partial \hat{p}}{\partial x}\frac{e_c^3}{6} + \omega R\frac{e_c}{2}$$

$$= \frac{-1}{12\mu_c}\frac{\partial \hat{p}}{\partial x} e_c^3 + \omega R\frac{e_c}{2} = \frac{-1}{12\mu_c}\gamma_c\frac{(h_T-h_c)}{h_T}e_c^3 + \frac{\omega R e_c}{2}$$

El problema del lado izquierda (agua) es análogo:



$$\frac{\partial \hat{p}}{\partial x} = \frac{\gamma(h_T-h_A)}{h_T}$$

$$q_A = \frac{-1}{12\mu_A}\gamma_A\frac{(h_T-h_A)}{h_T}e_A^3 + \frac{\omega R e_A}{2}$$

Se pide que el caudal de agua sea el doble del de concentrado:

$$q_A = 2 q_C$$

$$-\frac{1}{12\mu_A}\,\gamma_A \frac{(h_T - h_A)}{h_T}\, e_A^3 + \omega R \frac{e_A}{2} = 2\left[\frac{-1}{12\mu_c}\,\gamma_c \frac{(h_T - h_c)}{h_T}\, e_c^3 + \omega R \frac{e_c}{2}\right]$$

$$\omega R \frac{e_A}{2} - \omega R e_c = \frac{1}{12\mu_A}\,\gamma_A \frac{(h_T - h_A)}{h_T}\, e_A^3 - \frac{1}{6\mu_c}\,\gamma_c \frac{(h_T - h_c)}{h_T}\, e_c^3$$

$$\Rightarrow \omega = \frac{1}{R\left(\frac{e_A}{2} - e_c\right)}\left[\frac{1}{12\mu_A}\,\gamma_A \frac{(h_T - h_A)}{h_T}\, e_A^3 - \frac{1}{6\mu_c}\,\gamma_c \frac{(h_T - h_c)}{h_T}\, e_c^3\right]$$

b) Por cada lado

$$\tau = \mu \left.\frac{du}{dy}\right|_{y=0} \Rightarrow F = \tau \cdot \underbrace{A}_{h_T \cdot 1}$$

concentrado: $$\frac{du}{dy} = \frac{1}{\mu_c}\frac{\partial \hat{p}}{\partial x}\, y - \frac{1}{2\mu_c}\frac{\partial \hat{p}}{\partial x}\, e_c - \frac{\omega R}{e_c}$$

$$F = \mu_c \left.\frac{du}{dy}\right|_{y=0} \cdot h_T = -\mu_c h_T \left(\frac{1}{2\mu_c}\frac{\partial \hat{p}}{\partial x}\, e_c + \frac{\omega R}{e_c}\right)$$

(Analogo para el agua)

$$F_{TOTAL} = F_C + F_A$$

$$= \left(-\mu_c h_T \left[\frac{1}{2\mu_c}\,\gamma_c \frac{(h_T - h_c)}{h_T}\, e_c + \frac{\omega R}{e_c}\right]\right) + \left(-\mu_A h_T \left[\frac{1}{2\mu_A}\,\gamma_A \frac{(h_T - h_A)}{h_T}\, e_A + \frac{\omega R}{e_A}\right]\right)$$