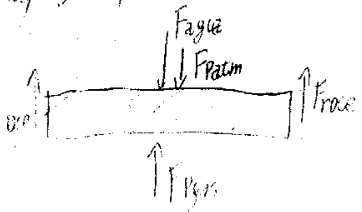
PAUTA 1:

a) DCL pistón



vel cte $\Rightarrow \sum \vec{F} = 0 \Leftrightarrow F_A + P_{atm}\cdot A = P\cdot A + F_r \quad (1)$ (0,3)

$F_A = \gamma (H_1 + v_0 t) A$; (0,3)

$F_r = \tau \cdot dA = \mu \cdot \frac{dv}{dy} \cdot \pi D L$; distribución lineal $\Rightarrow F_r = \frac{\mu v_0 \pi D L}{e}$ (0,3)

Laplace $\Rightarrow P - P_{atm} = \frac{4\sigma}{R_1} \Rightarrow P = \frac{4\sigma}{R_1} + P_{atm}$ (0,3)

Reemplazando en (1): $\gamma (H_1 + v_0 t) A + P_{atm}\cdot A = \left(\frac{4\sigma}{R_1} + P_{atm}\right) A + \frac{\mu v_0 \pi D L}{e}$; pero $A = \frac{\pi D^2}{4}$

$\Rightarrow \frac{\mu v_0 \cancel{\pi} D L}{e} = \left[\gamma (H_1 + v_0 t) + \cancel{P_{atm}} - \frac{4\sigma}{R_1} - \cancel{P_{atm}}\right] \frac{\cancel{\pi} D^2}{4}$; como $t = t_1 = 0$

$\Rightarrow v_0 = \frac{D^2 e}{4\mu D L}\left[\gamma H_1 - \frac{4\sigma}{R_1}\right] = \frac{De}{4\mu L}\left(\gamma H_1 - \frac{4\sigma}{R_1}\right)$ (0,6) $= \frac{0{,}2\cdot 0{,}005}{4\cdot 250.000\cdot 10^{-3}\cdot 0{,}05}\left(9.800\cdot 0{,}1 - \frac{4\cdot 0{,}072}{0{,}05}\right) = 0{,}0195$ [m/seg] (0,2)

b) $P\forall = n R_0 T \Rightarrow T = \frac{P\forall}{n R_0} \Rightarrow T(t_2) = \frac{P(t_2)\cdot \forall(t_2)}{n R_0}$; De (1): $P = \frac{F_A + P_{atm}\cdot A - F_r}{A} = \frac{\gamma (H_1 + v_0 t) A + P_{atm}\cdot A - \frac{\mu v_0 \pi D L}{e}}{A}$

$\Rightarrow P(t_2) = \gamma (H_1 + v_0 t_2) + P_{atm} - \frac{\mu v_0 \pi D L}{\frac{\pi D^2}{4} e} = \gamma (H_1 + v_0 t_2) + P_{atm} - \frac{4\mu v_0 L}{De}$ (0,8)

$\forall(t_2) = \forall_{gas} + \forall_{burbuja} = \gamma (h_1 - v_0 t_2) + \frac{4}{3}\pi R^3(t_2)$; Laplace: $P - P_{atm} = \frac{4\sigma}{R(t)} \Leftrightarrow R(t) = \frac{4\sigma}{P(t) - P_{atm}}$

$\Rightarrow \forall(t_2) = \gamma (h_1 - v_0 t_2) + \frac{4}{3}\pi \left(\frac{4\sigma}{P(t_2) - P_{atm}}\right)^3 = \gamma (h_1 - v_0 t_2) + \frac{4}{3}\pi \left(\frac{4\sigma}{\gamma (H_1 + v_0 t_2) + \cancel{P_{atm}} - \frac{4\mu v_0 L}{De} - \cancel{P_{atm}}}\right)^3$ (0,8)

Para conocer n, usamos la Ley de los gases en $t = t_1 = 0$:

$P(t_1)\cdot \forall(t_1) = n R_0 T(t_1) \Leftrightarrow n = \frac{P(t_1)\cdot \forall(t_1)}{R_0 T(t_1)} = \frac{\left(\frac{4\sigma}{R_1} + P_{atm}\right)\cdot\left(\gamma (h_1 - \cancel{v_0 t_1}^{\,0}) + \frac{4}{3}\pi R_1^3\right)}{R_0 \cdot T_1}$ (0,6)

Ahora, $T(t_2) = \frac{\left[\gamma (H_1 + v_0 t_2) + P_{atm} - \frac{4\mu v_0 L}{De}\right]\cdot\left[\gamma (h_1 - v_0 t_2) + \frac{4}{3}\pi \left(\frac{4\sigma}{\gamma (H_1 + v_0 t_2) - \frac{4\mu v_0 L}{De}}\right)^3\right] \cancel{R_0} T_1}{\left(\frac{4\sigma}{R_1} + P_{atm}\right)\cdot\left(\gamma h_1 + \frac{4}{3}\pi R_1^3\right) \cancel{R_0}}$

(0,5)

$\Rightarrow T(t_2) = 290{,}91\,°K = 17{,}91\,°C$ (0,3)

c) 1ª Ley Termodinámica: $\frac{dU}{dt} = \frac{d\hat{Q}}{dt} - \frac{dW}{dt}$; pero: $U = nC_v'\,dT \Rightarrow \frac{dU}{dt} = nC_v'\frac{dT}{dt}$

$\frac{d\hat{Q}}{dt} = \dot{Q}$ y $\frac{dW}{dt} = \frac{d(P\forall)}{dt} = P\cdot\frac{d\forall}{dt} + \forall\cdot\frac{dP}{dt}$

$\therefore \dot{Q} = nC_v'\frac{dT}{dt} + P\frac{d\forall}{dt} + \forall\frac{dP}{dt}$; pero $T = \frac{P\forall}{nR_0} \Rightarrow \frac{dT}{dt} = \frac{1}{nR_0}\cdot\frac{d(P\cdot\forall)}{dt} = \frac{1}{nR_0}\left(P\frac{d\forall}{dt} + \forall\frac{dP}{dt}\right)$

Además, $C_v' = \frac{5}{2}R_0$ (gases diatómicos) $\Rightarrow \dot{Q} = \frac{\cancel{n}\frac{5}{2}\cancel{R_0}}{\cancel{n}\cancel{R_0}}\left(P\frac{d\forall}{dt} + \forall\frac{dP}{dt}\right) + P\frac{d\forall}{dt} + \forall\frac{dP}{dt}$

$\Rightarrow \dot{Q} = \frac{7}{2}\left(P\frac{d\forall}{dt} + \forall\frac{dP}{dt}\right)$ (0,5)

$$\forall(t) = \gamma(H_1 - v_0t) + \frac{4}{3}\pi\left(\frac{4\sigma}{\gamma(H_1+v_0t) - \frac{4\mu v_0L}{D_e}}\right)^3 \Rightarrow \frac{d\forall}{dt} = -\gamma v_0 + \frac{\frac{4}{\cancel{3}}\pi\cdot(4\sigma)^3\cdot(-\cancel{3}\gamma v_0)}{\left(\gamma(H_1+v_0t - \frac{4\mu v_0L}{D_e}\right)^2} = -\gamma v_0\left(1 + \frac{4^4\pi\sigma^3}{\left(\gamma(H_1+v_0t) - \frac{4\mu v_0L}{D_e}\right)^2}\right)$$

$$P(t) = \gamma(H_1+v_0t) + P_{atm} - \frac{4\mu v_0L}{D_e} \Rightarrow \frac{dP}{dt} = \gamma v_0$$

$$\therefore \dot{Q} = \frac{7}{2}\left\{\left[\gamma(H_1+v_0t) + P_{atm} - \frac{4\mu v_0L}{D_e}\right]\cdot\left(-\gamma v_0\right)\left(1 + \frac{256\pi\sigma^3}{\left(\gamma(H_1+v_0t) - \frac{4\mu v_0L}{D_e}\right)^2}\right) + \left[\gamma(H_1 - v_0t) + \frac{4}{3}\pi\left(\frac{4\sigma}{\gamma(H_1+v_0t) - \frac{4\mu v_0L}{D_e}}\right)^3\right]\cdot\gamma v_0\right\}$$

$$\Rightarrow \dot{Q} = \frac{7}{2}\gamma v_0\left[\gamma(H_1 - v_0t) + \frac{4}{3}\pi\left(\frac{4\sigma}{\gamma(H_1+v_0t) - \frac{4\mu v_0L}{D_e}}\right)^3 - \left(\gamma(H_1+v_0t) + P_{atm} - \frac{4\mu v_0L}{D_e}\right)\left(1 + \frac{256\pi\sigma^3}{\left(\gamma(H_1+v_0t - \frac{4\mu v_0L}{D_e}\right)^2}\right)\right]$$ (0,5)

Nota: También es correcta la aproximación $\frac{dP}{dt} = 0 \Rightarrow \dot{Q} = -\frac{7}{2}\gamma v_0\left(\gamma(H_1+v_0t) + P_{atm} - \frac{4\mu v_0L}{D_e}\right)\left(1 + \frac{256\pi\sigma^3}{\left(\gamma(H_1+v_0t) - \frac{4\mu v_0L}{D_e}\right)^2}\right)$