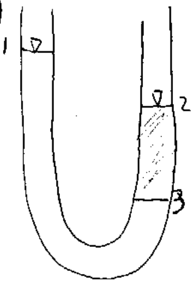
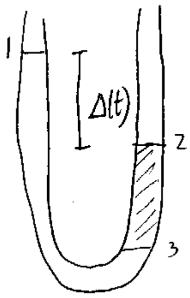
P3 a)



Hidrostática $\Rightarrow P_1 = P_2 \Leftrightarrow \gamma_1 L_1 = \gamma_2 (L_1 + \Delta h) \Leftrightarrow \Delta h = \frac{\gamma_1}{\gamma_2} L_1 - L_1 = L_1\left(\frac{\gamma_1}{\gamma_2} - 1\right) = L_1\left(\frac{\gamma_1 - \gamma_2}{\gamma_2}\right)$

b)



Euler: (1)-(2): $\frac{L_2}{g}\frac{dv}{dt} + B_2 - B_1 = 0$ (i); (2)-(3): $\frac{L_1}{g}\frac{dv}{dt} + B_3 - B_2 = 0$ (ii)

$B_i = z_i + \frac{p_i}{\gamma} + \frac{v_i^2}{2g}$; El tubo es de diámetro cte $\Rightarrow v_i = v$.

(i): $\frac{L_2}{g}\frac{dv}{dt} + h_2 + \frac{p_2}{\gamma_2} + \cancel{\frac{v^2}{2g}} - h_1 - \cancel{\frac{p_1}{\gamma_2}}^{0} - \cancel{\frac{v^2}{2g}} = 0$; (ii): $\frac{L_1}{g}\frac{dv}{dt} + h_3 + \cancel{\frac{p_3}{\gamma_1}}^{0} + \cancel{\frac{v^2}{2g}} - h_2 - \frac{p_2}{\gamma_1} - \cancel{\frac{v^2}{2g}} = 0$

Sumando (i) y (ii): $\frac{(L_1+L_2)}{g}\frac{dv}{dt} + \underbrace{h_3 - h_1}_{-\Delta} + p_2\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) = 0$ (iii)

(ii) $\Rightarrow \frac{L_1}{g}\frac{dv}{dt} + \underbrace{h_3 - h_2}_{L_1} = \frac{p_2}{\gamma_1}$; reemplazando en (iii): $\frac{(L_1+L_2)}{g}\frac{dv}{dt} = \Delta + \left(\frac{L_1}{g}\frac{dv}{dt} + L_1\right)\left(\frac{\gamma_1 - \gamma_2}{\gamma_2}\right)$

$\frac{1}{g}\Big[\overbrace{L_1 + L_2 + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)}^{L_0}\Big]\frac{dv}{dt} + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right) - \Delta = 0 \Leftrightarrow \frac{L_0}{g}\frac{dv}{dt} + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right) - \Delta = 0$

$v = -\frac{dh_1}{dt} \wedge v = \frac{dh_3}{dt} \Rightarrow 2v = \frac{d}{dt}(h_3 - h_1) = -\frac{d\Delta}{dt} \Rightarrow v = \frac{-1}{2}\frac{d\Delta}{dt} \Rightarrow \frac{dv}{dt} = -\frac{1}{2}\frac{d^2\Delta}{dt^2}$

$\Rightarrow \frac{L_0}{g}\cdot\frac{-1}{2}\frac{d^2\Delta}{dt^2} + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right) - \Delta = 0 \Leftrightarrow \frac{d^2\Delta}{dt^2} + \overbrace{\frac{2g}{L_0}}^{A}\cdot\Delta = \overbrace{\frac{2g}{L_0}L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)}^{B} \Leftrightarrow \frac{d^2\Delta}{dt^2} + A\Delta = B$

$\Rightarrow \Delta(t) = C_1 \, sen\left(\sqrt{\frac{2g}{L_0}}t\right) + C_2 \cos\left(\sqrt{\frac{2g}{L_0}}t\right) + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)$; C.B. $t=0 \Rightarrow \begin{cases} \Delta = \Delta h + 2\Delta h_0 \\ \frac{d\Delta}{dt} = 0 \end{cases}$

$\Delta(0) = \cancel{\Delta h} + 2\Delta h_0 = C_2 + \cancel{L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)} \Rightarrow C_2 = 2\Delta h_0$; $\frac{d\Delta}{dt} = \sqrt{\frac{2g}{L_0}}C_1\cos\left(\sqrt{\frac{2g}{L_0}}\cdot t\right) + \sqrt{\frac{2g}{L_0}}\cdot sen\left(\sqrt{\frac{2g}{L_0}}t\right)$

$\frac{d\Delta}{dt}(0) = 0 = \sqrt{\frac{2g}{L_0}}\cdot C_1 \Rightarrow C_1 = 0 \Rightarrow \Delta(t) = 2\Delta h_0 \cos\left(\sqrt{\frac{2g}{L_0}}\cdot t\right) = 2\cdot 0{,}05\cdot\cos\left(\sqrt{\frac{2\cdot 9{,}8}{0{,}2+0{,}9+0{,}2\left(\frac{15.000-10.000}{10.000}\right)}}\cdot t\right)$

$\Rightarrow \Delta(t) = 0{,}1\cdot\cos(4{,}04\cdot t)$