P1) a) Navier-Stokes en coordenadas cilíndricas: Continuidad: $\frac{\partial v_z}{\partial z}=0 \Rightarrow v_z$ de. cont z; $v_r=v_\theta=0$

Según $\hat{r}$: $\frac{\partial p}{\partial r}=0$; según $\hat{\theta}$: $\frac{\partial p}{\partial \theta}=0$;

Según $\hat{z}$: $-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right]=0 \quad (1)$

Como p no depende de $\hat{r}$, integramos (1) c/r a r:

$$\frac{\partial p}{\partial z}=\frac{\mu}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) \Leftrightarrow \frac{\partial p}{\partial z}\cdot\frac{r}{\mu}=\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) \Rightarrow \frac{\partial p}{\partial z}\frac{r^2}{2\mu}+C_1=r\frac{\partial v_z}{\partial r} \Leftrightarrow \frac{\partial p}{\partial z}\frac{r}{2\mu}+\frac{C_1}{r}=\frac{\partial v_z}{\partial r}$$

Integrando de nuevo: $v_z=\frac{\partial p}{\partial z}\frac{r^2}{4\mu}+C_1\cdot\ln(r)+C_2$; $\frac{\partial p}{\partial z}=\frac{\partial p}{\partial z}+\rho g\underbrace{\frac{\partial h}{\partial z}}_{0\,(\text{horizontal})}=\frac{\gamma h}{L}$

Condiciones de borde: $\begin{cases} r=D/2 \Rightarrow V_z=0 \\ r=d/2 \Rightarrow V_z=V_0 \end{cases}$; $\Rightarrow 0=\frac{\gamma h}{L}\cdot\frac{D^2}{16\mu}+C_1\cdot\ln\left(\frac{D}{2}\right)+C_2 \Rightarrow C_2=-C_1\ln\left(\frac{D}{2}\right)-\frac{\gamma h}{L}\cdot\frac{D^2}{16\mu}$

$$V_0=\frac{\gamma h}{L}\cdot\frac{d^2}{16\mu}+C_1\cdot\ln\left(\frac{d}{2}\right)-C_1\cdot\ln\left(\frac{D}{2}\right)-\frac{\gamma h}{L}\cdot\frac{D^2}{16\mu} \Leftrightarrow V_0+\frac{\gamma h}{L}\frac{(D^2-d^2)}{16\mu}=C_1\cdot\ln\left(\frac{d}{D}\right) \Rightarrow C_1=\left(V_0+\frac{\gamma h}{L}\frac{(D^2-d^2)}{16\mu}\right)\cdot\ln\left(\frac{D}{d}\right)$$

$$\Rightarrow C_2=\ln\left(\frac{d}{D}\right)\left(V_0+\frac{\gamma h}{L}\frac{(D^2-d^2)}{16\mu}\right)\cdot\ln\left(\frac{D}{2}\right)-\frac{\gamma h}{L}\cdot\frac{D^2}{16\mu}$$

$$\therefore V_z=\overbrace{\frac{\gamma h}{L}\cdot\frac{1}{4\mu}}^{A}\cdot r^2+\overbrace{\left(V_0+\frac{\gamma h}{L}\frac{(D^2-d^2)}{16\mu}\right)\cdot\ln\left(\frac{D}{d}\right)}^{B}\cdot\ln(r)+\overbrace{\ln\left(\frac{d}{D}\right)\left(V_0+\frac{\gamma h}{L}\frac{(D^2-d^2)}{16\mu}\right)\cdot\ln\left(\frac{D}{2}\right)-\frac{\gamma h}{L}\cdot\frac{D^2}{16\mu}}^{C}=Ar^2+B\cdot\ln(r)+C$$

$$A=\frac{9{,}8\cdot 1000\cdot 0{,}003}{10\cdot 4\cdot 10^{-3}}=735;\; B=\left(0{,}2+\frac{9.800\cdot 0{,}003}{10}\frac{(0{,}05^2-0{,}01^2)}{16\cdot 10^{-3}}\right)\ln\left(\frac{0{,}05}{0{,}01}\right)=1{,}032$$

$$C=\ln\left(\frac{0{,}01}{0{,}05}\right)\left(0{,}2+\frac{9.800\cdot 0{,}003}{10}\frac{(0{,}05^2-0{,}01^2)}{16\cdot 10^{-3}}\right)\cdot\ln\left(\frac{0{,}05}{2}\right)-\frac{9.800\cdot 0{,}003}{10}\cdot\frac{0{,}05^2}{16\cdot 10^{-3}}=3{,}346 \Rightarrow V_z(r)=735r^2+1{,}032\cdot\ln(r)+3{,}346$$

b) $$Q=\int_S v\cdot ds=2\pi\int_{d/2}^{D/2}\left(Ar^2+B\cdot\ln(r)+C\right)r\,dr=\int_{d/2}^{D/2}Ar^3+Br\cdot\ln(r)+Cr=\left[\frac{Ar^4}{4}+B\left(\frac{r^2}{2}\ln(r)-\frac{r^2}{4}\right)+\frac{Cr^2}{2}\right]_{d/2}^{D/2}$$

$$=\frac{A}{64}(D^4-d^4)+B\left(\frac{(D^2-d^2)}{8}\cdot\ln\left(\frac{D}{d}\right)-\frac{(D^2-d^2)}{16}\right)+\frac{C}{8}(D^2-d^2)=\frac{A}{64}(D^4-d^4)+\frac{B(D^2-d^2)}{8}\left(\ln\left(\frac{D}{d}\right)-\frac{1}{2}+C\right)$$

$$\Rightarrow Q=\frac{735}{64}(0{,}05^4-0{,}01^4)+\frac{1{,}032}{8}\cdot(0{,}05^2-0{,}01^2)\left(\ln\left(\frac{0{,}05}{0{,}01}\right)-\frac{1}{2}+3{,}346\right)=0{,}0014\ [m^3/s]$$

c) Newton-Navier: $\tau = \mu \cdot \frac{dV_z}{dr} = \mu \cdot \left(2rA + \frac{B}{r}\right)$

Alrededor de la barra $\Rightarrow \tau(d/2) = \mu\left(\frac{2dA}{2} + \frac{B2}{d}\right) = \mu\left(Ad + \frac{2B}{d}\right) = 10^{-3}\left(735 \cdot 0{,}01 + 2 \cdot 1{,}032\right) = 0{,}009 \,[N/m^2]$

Fza. F necesaria: $F = \tau \cdot dA$; en este caso, dA = Área del manto por unidad de longitud

$\Rightarrow F = \mu\left(Ad + \frac{2B}{d}\right) \cdot \pi d 1 = 0{,}009 \cdot \pi \cdot 0{,}01 = 2{,}83 \cdot 10^{-4} \,[N]$