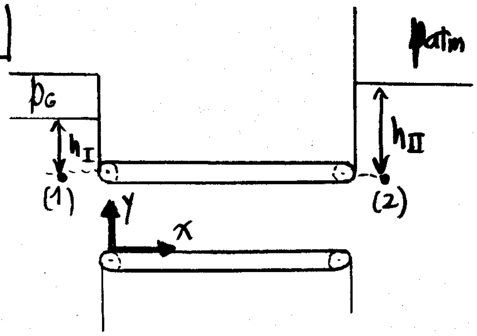
P2



$$\hat{p}_1 = p_G + \gamma h_I$$
$$\hat{p}_2 = p_{atm} + \gamma h_{II}$$
$$\frac{\partial \hat{p}}{\partial x} = \frac{p_{atm} + \gamma(h_{II} - h_I) - p_G}{L}$$

$$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$
$$v = w = 0 \quad \Rightarrow \quad \vec{v} = u\hat{i}$$

continuidad: $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} + \cancel{\frac{\partial w}{\partial z}}^{0} = 0 \quad \Rightarrow \quad \frac{\partial u}{\partial x} = 0$

Navier-Stokes: según y, según z : $\frac{\partial \hat{p}}{\partial y} = \frac{\partial \hat{p}}{\partial z} = 0$

s/x: $\underbrace{\cancel{\frac{\partial u}{\partial t}}^{0}}_{\text{permanente}} + \underbrace{u\cancel{\frac{\partial u}{\partial x}}^{0}}_{\text{continuidad}} + \cancel{v}^{0}\frac{\partial u}{\partial y} + \cancel{w}^{0}\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} + \frac{\mu}{\rho}\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}^{0}}_{\text{continuidad}} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\cancel{\frac{\partial^2 u}{\partial z^2}}^{0}}_{\text{flujo bidimensional}}\right)$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}$$

$$\Rightarrow u(y) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{y^2}{2} + cy + d$$

cond. de borde

- $u(y=0) = -\omega_2 R$

$$\Rightarrow -\omega_2 R = d$$

- $u(y=e) = -\omega_1 R$

$$\Rightarrow -\omega_1 R = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e^2}{2} + c \cdot e - \omega_2 R$$

$$\Rightarrow c = \frac{(\omega_2 - \omega_1)R}{e} - \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e}{2}$$

$$\Rightarrow u(y) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{y^2}{2} + \frac{(\omega_2 - \omega_1)R y}{e} - \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e}{2}y - \omega_2 R$$

$$\Rightarrow u(y) = \frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x} y(y-e) + (\omega_2-\omega_1)\frac{R}{e} y - \omega_2 R$$

$$q = \int_0^e u(y)\,dy = \int_0^e \left[\frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x} y(y-e) + (\omega_2-\omega_1)\frac{R}{e} y - \omega_2 R\right] dy$$

$$q = \frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x}\left[\frac{y^3}{3} - \frac{e y^2}{2}\right] + (\omega_2-\omega_1)\frac{R}{e}\frac{y^2}{2} - \omega_2 R y \Big|_0^e$$

$$q = \frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x}\left[\frac{e^3}{3} - \frac{e^3}{2}\right] + (\omega_2-\omega_1)\frac{Re}{2} - \omega_2 Re$$

$$q = -\frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x}\frac{e^3}{6} + \omega_2\frac{Re}{2} - \omega_2 Re - \frac{\omega_1 Re}{2}$$

$$q = -\frac{1}{12\mu}\frac{\partial \hat{p}}{\partial x} e^3 - (\omega_1+\omega_2)\frac{Re}{2}$$

Flujo neto nulo: $q=0 \Rightarrow \frac{1}{12\mu}\frac{\partial \hat{p}}{\partial x} e^2 = -(\omega_1+\omega_2)\frac{R}{2}$

$$\Rightarrow \frac{\partial \hat{p}}{\partial x} = \frac{-(\omega_1+\omega_2)R\cdot 12\mu}{2e^2} = \frac{-6(\omega_1+\omega_2)R\mu}{e^2}$$

$$\Rightarrow \frac{p_{atm} + \gamma(h_{II}-h_I) - p_6}{L} = -\frac{6(\omega_1+\omega_2)R\mu}{e^2}$$

$$p_6 = p_{atm} + \gamma(h_{II}-h_I) + \frac{6(\omega_1+\omega_2)R\mu L}{e^2}$$