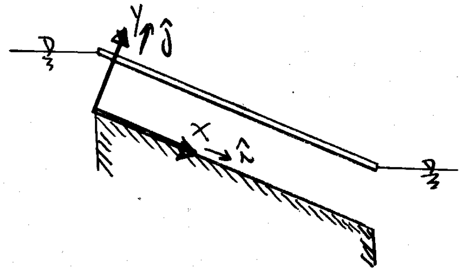
Pauta Auxiliar #4 CI31A

P1]



Equivalente a flujo entre placas planas

$$\frac{\partial \hat{p}}{\partial x} = \frac{-\gamma H}{L}$$

$$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

flujo unidireccional, $v = w = 0$

$$\vec{v} = u\hat{i}$$

continuidad

$$\Rightarrow \frac{\partial u}{\partial x} + \overbrace{\cancel{\frac{\partial v}{\partial y}}}^{0} + \overbrace{\cancel{\frac{\partial w}{\partial z}}}^{0} = 0 \quad \Rightarrow \quad \frac{\partial u}{\partial x} = 0$$

ec. de Navier Stokes

$$\text{según } y \text{ , según } z: \quad \frac{\partial \hat{p}}{\partial y} = \frac{\partial \hat{p}}{\partial z} = 0$$

$$\underbrace{\overbrace{\cancel{\frac{\partial u}{\partial t}}}^{0}}_{\text{permanente}} + \underbrace{u \overbrace{\cancel{\frac{\partial u}{\partial x}}}^{0}}_{\text{continuidad}} + \overbrace{\cancel{v}}^{0}\frac{\partial u}{\partial y} + \overbrace{\cancel{w}}^{0}\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} + \frac{\mu}{\rho}\left(\underbrace{\overbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}^{0}}_{\text{continuidad}} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\overbrace{\cancel{\frac{\partial^2 u}{\partial z^2}}}^{0}}_{\text{flujo bidimensional}}\right)$$

$$\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \quad \Rightarrow \quad \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x} y + c$$

$$u(y) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{y^2}{2} + c \cdot y + d$$

cond. de borde:

- $u(y=0) = 0 \quad \Rightarrow \quad 0 = d$
- $u(y=e) = 0 \quad \Rightarrow \quad 0 = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e^2}{2} + ce \quad \Rightarrow \quad c = -\frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e}{2}$

$$\Rightarrow u(y) = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{y^2}{2} - \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e}{2}y = \frac{1}{2\mu}\frac{\partial \hat{p}}{\partial x}y(y-e)$$

$$u(y) = \frac{1}{2\mu}\frac{-\gamma H}{L}y(y-e) = \frac{\gamma H}{2\mu L}y(e-y)$$

Esfuerzo de corte sobre la placa:

$$\tau_{xy}\Big|_{y=e} = \mu\left(\cancelto{0}{\frac{\partial v}{\partial x}} + \frac{\partial u}{\partial y}\right) = \mu \frac{\partial u}{\partial y}\Big|_{y=e}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x} y - \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\frac{e}{2} = \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\left(y - \frac{e}{2}\right)$$

$$\Rightarrow \tau_{xy}\Big|_{y=e} = \mu \frac{\partial u}{\partial y}\Big|_{y=e} = \mu \cdot \frac{1}{\mu}\frac{\partial \hat{p}}{\partial x}\left(e - \frac{e}{2}\right) = \frac{\partial \hat{p}}{\partial x}\cdot\frac{e}{2}$$

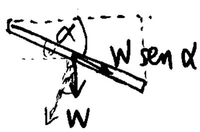
$$= -\frac{\gamma H}{L}\frac{e}{2}$$

τ sobre la placa: $\frac{\gamma H}{L}\frac{e}{2}$ $\times$ Area placa $= L \cdot 1$

ancho unitario

$$\Rightarrow F_{\text{sobre la placa}} = \frac{\gamma H e}{2}$$

falta incorporar la componente del peso: $W \operatorname{sen}\alpha$



$$\operatorname{sen}\alpha = \frac{H}{L} \quad \Rightarrow \quad W\operatorname{sen}\alpha = \frac{WH}{L}$$

Tensión en la cuerda $= \frac{\gamma H e}{2} + \frac{WH}{L}$