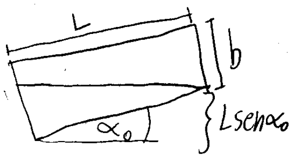
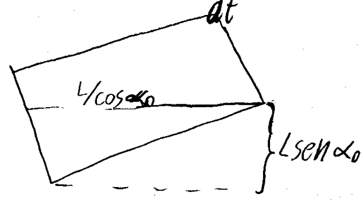
P3)



$$Q_e = \Omega\sqrt{2gh} = cte \Rightarrow h = cte = L\,sen\alpha_0 = h_0$$

Continuidad: $\frac{d\forall}{dt} = \cancel{Q_a}^{\to 0} - Q_e \Leftrightarrow \int_{\forall_0}^{\forall_f} d\forall = -Q_e\int_0^t dt$

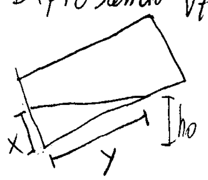
$$\Rightarrow \forall_f - \forall_0 = -Q_e \cdot t;$$



$$\forall_0 = a L\,sen\alpha_0 \cdot \frac{L}{\cos\alpha_0}\cdot\frac{1}{2} = \frac{aL^2\tan\alpha_0}{2}$$

$$\Rightarrow \forall_f(t) = \forall_0 - Q_e t = \frac{aL^2}{2}\tan\alpha_0 - \Omega\sqrt{2gh_0}\,t \quad (*)$$

Para que Q_a sea cte, h debe mantenerse cte ⇒ hay que ir aumentando α.
El tiempo máximo es en el que se tiene un volumen mínimo, manteniendo h cte.
Expresando $\forall_f$ en función de α:



$$x = \frac{h_0}{\cos\alpha}\;;\; y = \frac{h_0}{sen\alpha} \Rightarrow \forall_f = \frac{axy}{2} = \frac{a h_0^2}{2\,sen\alpha\cos\alpha} = \frac{ah_0^2}{sen(2\alpha)}$$

$$\frac{d\forall_f}{d\alpha} = 0 \Leftrightarrow \frac{-ah_0^2}{sen^2(2\alpha)}\cdot 2\cos(2\alpha) = 0 \Leftrightarrow \cos(2\alpha) = 0 \Leftrightarrow 2\alpha = 90° \Rightarrow \alpha = 45°$$

∴ Reemplazando en (*): $\frac{ah_0^2}{sen(2\cdot 45)} = \frac{aL^2\tan\alpha_0}{2} - \Omega\sqrt{2gh_0}\,t$; pero $sen(2\cdot 45) = sen(90) = 1$

$$\Rightarrow aL^2 sen^2\alpha_0 = \frac{aL^2}{2}\tan\alpha_0 - \Omega\sqrt{2gL\,sen\alpha_0}\,t \Leftrightarrow \Omega\sqrt{2gL\,sen\alpha_0}\,t = aL^2\left(sen^2\alpha_0 - \frac{\tan\alpha_0}{2}\right)$$

$$\Rightarrow t = \frac{-aL^2(2sen^2\alpha_0 - \tan\alpha_0)}{2\Omega\sqrt{2gL\,sen\alpha_0}} = \frac{-0{,}15\cdot 0{,}5^2\left(2\cdot sen^2(5°) - \tan(5°)\right)}{2\cdot 0{,}196\cdot 10^{-4}\sqrt{2\cdot 9{,}8\cdot 0{,}5\cdot sen(5°)}} = 74{,}83\,[s]$$