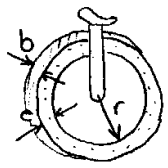
Pauta Ejercicio #1 C131A

P1]

a)



$$dT = r\,dF$$

$$dF = \tau\,dA \qquad dA = b\,r\,d\theta$$

$$\tau = \mu r \frac{d\omega}{dr}$$

$$\Rightarrow dT = \tau\, r\, dA = \mu r \frac{d\omega}{dr}\, r\, b\, r\, d\theta$$

$$dT = \mu r^3 \frac{d\omega}{dr}\, b\, d\theta$$

$$T = \int_{\Omega} dT = \mu r^3 \frac{d\omega}{dr}\, b \int_0^{2\pi} d\theta = 2\pi \mu r^3 b \frac{d\omega}{dr}$$

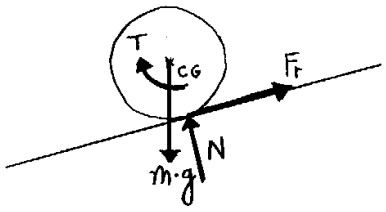
Separando variables:

$$T \int_r^{r+e} \frac{dr}{r^3} = 2\pi\mu b \int_0^{\omega_0} d\omega \qquad \begin{matrix} r = r & \omega = 0 \\ r = r+e & \omega = \omega_0 \end{matrix}$$

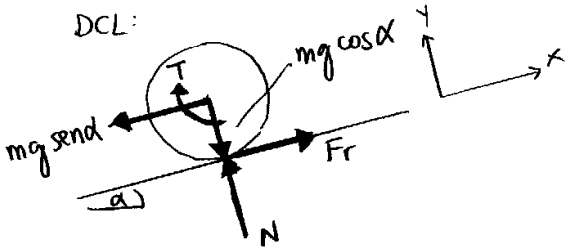
$$\frac{T}{2}\left[\frac{1}{r^2} - \frac{1}{(r+e)^2}\right] = 2\pi\mu b\,\omega_0$$

$$\omega_0 = \frac{T}{4\pi\mu b}\left[\frac{1}{r^2} - \frac{1}{(r+e)^2}\right]$$

Cálculo de T:



DCL:



Como el sistema se mueve con velocidad constante.

$$\Sigma F_x = 0 \qquad \bar{F}r = mg\,\text{sen}\alpha$$

$$\Sigma F_y = 0 \qquad N = mg\cos\alpha$$

$$\Sigma M_z = 0 \qquad T = \bar{F}r \cdot (r+e)$$

$$\Rightarrow T = mg\,\text{Sen}\alpha\,(r+e) = 11{,}27\ [N\cdot m]$$

$$\Rightarrow \omega_0 = \frac{mg\,\text{Sen}\alpha\,(r+e)}{4\pi\mu b}\left[\frac{1}{r^2} - \frac{1}{(r+e)^2}\right] = 162{,}2\ [\text{rad/s}]$$

$V_o = \omega_o (r+e)$ (rueda sin resbalar

$\Rightarrow V_o = 35{,}7 \ [m/s]$

$T_o = \frac{L}{V_o} = 84 \ [s] = 1{,}40 \ [min]$

b) Volumen de aceite en el artefacto:

$\forall_{ac} = \pi b \left[(r+e)^2 - r^2 \right] = 5{,}28 \ [lt]$

$\forall_{ac}'$ (después del sabotaje) $= \frac{\forall_{ac}}{2} = 2{,}64 \ [lt]$

$\forall_{ac}' = \pi b \left[(r+e')^2 - r^2 \right]$ e' : nuevo espesor.

$\Rightarrow e' = \sqrt{\frac{\forall_{ac}'}{\pi b} + r^2} - r = 10{,}2 \ [mm]$

En esta nueva condición:

$\omega_o' = \frac{mg \operatorname{sen}\alpha \, (r+e')}{4\pi\mu b} \left[\frac{1}{r^2} - \frac{1}{(r+e')^2} \right] = 84{,}9 \ [rad/s]$

$V_o' = \omega_o' (r+e') = 17{,}8 \ [m/s]$

$T_o' = \frac{L}{V_o'} = 168{,}1 \ [s] = 2{,}80 \ [min]$

$\frac{T_o' - T_o}{T_o} \cdot 100 = 100 \ \%$ El tiempo de viaje aumenta al doble (en un 100%).

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