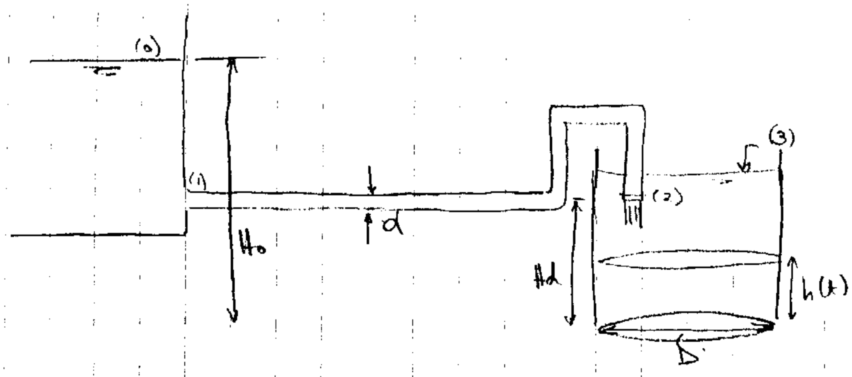

a) Caudal : $B_0 = B_2 \Rightarrow H_o = H_d + \frac{v^2}{2g}$

$\Rightarrow v^2 = 2g(H_o - H_d) \Rightarrow v = \sqrt{2g(H_o - H_d)}$

$$\therefore \boxed{Q = v \cdot A = \sqrt{2g(H_o - H_d)}\,\frac{\pi d^2}{4}}$$

b) Ec. diferencial. si $h > H_d$

$$\frac{L}{g}\frac{du}{dt} + B_2 - B_1 = 0$$

$B_1 = B_0 = H_o$

$B_2 = B_3 = h + \frac{v_e^2}{2g}$ con v_e veloc. del estanque.

$$\Rightarrow \frac{L}{g}\frac{du}{dt} + h + \frac{v_e^2}{2g} - H_o = 0$$

$$Q = v_e \frac{\pi D^2}{4} = \frac{dh}{dt}\frac{\pi D^2}{4} = u\,\frac{\pi d^2}{4} \rightarrow \frac{\cancel{\pi} D^2}{\cancel{4}}\frac{d^2h}{dt^2} = \frac{\cancel{\pi} d^2}{\cancel{4}}\frac{du}{dt}$$

$$\Rightarrow \frac{L}{g}\left(\frac{D}{d}\right)^2\frac{d^2h}{dt^2} + h + \left(\frac{dh}{dt}\right)^2\frac{1}{2g} - H_o = 0$$

$$\frac{L}{g}\frac{d^2h}{dt^2} + \left(\frac{d}{D}\right)^2 h + \cancel{\left(\frac{dh}{dt}\right)^2 \frac{1}{2g}\left(\frac{d}{D}\right)^2} - H_o\left(\frac{d}{D}\right)^2 = 0$$

$$\frac{d^2h}{dt^2} + \left(\frac{d}{D}\right)^2 \frac{g}{L} h - H_o\left(\frac{d}{D}\right)^2\frac{g}{L} = 0$$

$$\frac{d^2h}{dt^2} + \left(\frac{d}{D}\right)^2 \frac{g}{L}(h - H_o) = 0 \qquad \Delta h = h - H_o$$

$$\frac{d^2\Delta h}{dt^2} + \underbrace{\left(\frac{d}{D}\right)^2 \frac{g}{L}}_{k} \Delta h = 0$$

Resolviendo la ecuación diferencial siguiente:

$$\ddot{\Delta h} + k\,\Delta h = 0$$

$$\Rightarrow \Delta h = A \operatorname{sen}(\sqrt{k}\,t) + B\cos(\sqrt{k}\,t)$$

Aplicando las condiciones de borde.

1) $t = t_o \qquad \Delta h = \Delta h_o = H_o - H_d$

2) $\qquad \dot{\Delta h} = \dfrac{d\Delta h}{dt} = \dfrac{dh}{dt} - \cancel{\dfrac{dH_o}{dt}} = \dfrac{Q}{\frac{\pi D^2}{4}} = \sqrt{2g(H_o - H_d)}\left(\dfrac{d}{D}\right)^2$

$$\Delta h = H_d - H_d = A \cancel{\operatorname{sen}(\sqrt{k}\cdot 0)} + B\cos(\sqrt{k}\cdot 0) \Rightarrow \underline{H_o - H_d = B}$$

$$\frac{d\Delta h}{dt} = A\cos(\sqrt{k}\,t)\sqrt{k} - B\,\cancel{\operatorname{sen}}(\sqrt{k}\,t)\sqrt{k}$$

$$\sqrt{2g(H_o - H_d)}\left(\frac{d}{D}\right)^2 = A\sqrt{k}$$

$$\Rightarrow B = +\sqrt{2g(H_o - H_d)}\left(\frac{d}{D}\right)^{\cancel{2}} \cdot \frac{1}{\cancel{\left(\frac{d}{D}\right)}\sqrt{\frac{g}{L}}}$$

$$\therefore \boxed{\Delta h = (H_o - H_d)\cos(\sqrt{k}\,t) + \frac{\sqrt{2(H_o - H_d)}}{\sqrt{L}}\left(\frac{d}{D}\right)\operatorname{sen}(\sqrt{k}\,t)}$$

datos:

$H_o = 10$ m

$H_d = 5$ m

$L = 40$ m

$d = 10$ cm

$D = 2$ m

$$k = \left(\frac{d}{D}\right)^2 \cdot \frac{g}{L} \quad 6,125 \times 10^{-4} \left[\frac{1}{s^2}\right]$$

$$\rightarrow \underline{\Delta h = 5 \cdot \cos\left(2,475 \times 10^{-2}\, t\right) + 0,025 \,\sin\left(2,475 \times 10^{-2}\, t\right)}$$

c) $H_e = 7$ m

$$\Delta h = H_o - h = H_o - H_e = 3 \text{ m} \quad \rightarrow \quad t\,?$$

$$3 = 5 \cdot \cos\left(2,475 \times 10^{-2}\, t\right) + 0,025 \cdot \sin\left(2,475 \times 10^{-2}\, t\right)$$

$$\Rightarrow \underline{t_1 = 12410,3 \; [s]} \quad \rightarrow \quad \underline{3,447 \; [hr]}$$