P2) a) La distribución de velocidades en la cáscara está dado por la ecuación.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = \frac{1}{\mu}\frac{d\hat{P}}{dz} \Rightarrow d\left(r\frac{dv_z}{dr}\right) = \frac{1}{\mu}\frac{d\hat{P}}{dz}\, r\,dr$$

$$r\frac{dv_z}{dr} = \frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{r^2}{2} + C_1$$

$$dv_z = \frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{r}{2}\,dr + \frac{C_1}{r}\,dr$$

Finalmente, $$\boxed{v_z(r) = \frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{r^2}{4} + C_1 \ln r + C_2}$$

Aplicando las condiciones de borde, tenemos:

$r = 0$ $\quad v_z$ finita

$r = d/2$ $\quad v_z = V = cte$

Evaluando: $$v_z(0) = \frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{(0)^2}{4} + C_1 \ln(0) + C_2$$

como $\ln(0)$ no existe $\Rightarrow C_1 = 0$ (distribución parabólica)

$$v_z(d/2) = V = \frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{(d/2)^2}{4} + C_2$$

$$\Rightarrow C_2 = V - \frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{(d/2)^2}{4}$$

$$\therefore \boxed{v_z(r) = \frac{1}{4\mu}\frac{d\hat{P}}{dz}\left(r^2 - (d/2)^2\right) + V}$$

b) El caudal neto está dado por:

$$Q = \int_A v_z\, dA = \int_0^{2\pi}\int_0^{d/2} v_z(r)\, r\,dr\,d\theta$$

$$Q = 2\pi\int_0^{d/2}\left(\frac{1}{4\mu}\frac{d\hat{P}}{dz}\left(r^2 - (d/2)^2\right) + V\right) r\,dr$$

$$Q = 2\pi \int_0^{d/2} \left\{ \frac{1}{4\mu}\frac{d\hat{P}}{dz} r^3 dr - \frac{1}{4\mu}\frac{d\hat{P}}{dz}\left(\frac{d}{2}\right)^2 r\,dr + V r\,dr \right\}$$

$$Q = 2\pi \left\{ \frac{1}{4\mu}\frac{d\hat{P}}{dz}\frac{r^4}{4}\Big|_0^{d/2} - \frac{1}{4\mu}\frac{d\hat{P}}{dz}\left(\frac{d}{2}\right)^2 \frac{r^2}{2}\Big|_0^{d/2} + V\frac{r^2}{2}\Big|_0^{d/2} \right\}$$

$$Q = 2\pi \left\{ \frac{1}{4\mu}\frac{d\hat{P}}{dz}\frac{(d/2)^4}{4} - \frac{1}{4\mu}\frac{d\hat{P}}{dz}(d/2)^2\frac{(d/2)^2}{2} + V\frac{(d/2)^2}{2} \right\}$$

$$Q = 2\pi \left\{ \frac{1}{4\mu}\frac{d\hat{P}}{dz}\left[\frac{(d/2)^4}{4} - \frac{(d/2)^4}{2}\right] + V\frac{(d/2)^2}{2} \right\}$$

$$Q = 2\pi \left\{ \frac{1}{4\mu}\frac{d\hat{P}}{dz}\left(\frac{1}{4} - \frac{1}{2}\right)(d/2)^4 + V\frac{(d/2)^2}{2} \right\}$$

$$Q = 2\pi \left\{ -\frac{1}{4\mu}\frac{d\hat{P}}{dz}\frac{1}{4}(d/2)^4 + V\frac{(d/2)^2}{2} \right\} = \frac{\cancel{2}\pi (d/2)^2}{\cancel{2}}\left\{ V - \frac{1}{8\mu}\frac{d\hat{P}}{dz}(d/2)^2 \right\}$$

$$\boxed{Q = \frac{\pi d^2}{4}\left(V - \frac{1}{8\mu}\frac{d\hat{P}}{dz}(d/2)^2\right)}$$

c) Como sobre la integral de línea el caudal es cero $\Rightarrow$

$Q = 0$ (continuidad) $\Rightarrow V = \frac{1}{8\mu}\left(\frac{d\hat{P}}{dz}\right)\frac{d^2}{4} \Rightarrow \boxed{\frac{d\hat{P}}{dz} = 32\mu\frac{V}{d^2}}$

d) $\tau_{rz} = \mu\frac{\partial v_z}{\partial r} = \mu\frac{1}{\mu}\frac{d\hat{P}}{dz}\frac{r}{2} \Rightarrow \tau_{rz} = \frac{d\hat{P}}{dz}\frac{r}{2}$

para $r = d/2 \longrightarrow \tau_{rz} = \tau_0$

$\Rightarrow \tau_0 = \frac{d\hat{P}}{dz}\frac{d}{4} \Rightarrow \tau_0 = 32\frac{\mu V}{d^2}\frac{d}{4} \Rightarrow \boxed{\tau_0 = 8\frac{\mu V}{d}}$

e) Finalmente, $F = \tau_0 \pi l d \Rightarrow$ $\boxed{F = 8\pi\mu V l}$