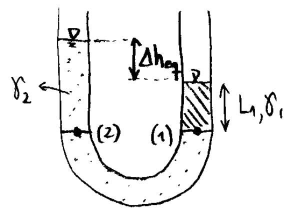
Pauta Auxiliar 6

CI31A - Mecánica de Fluidos

a)



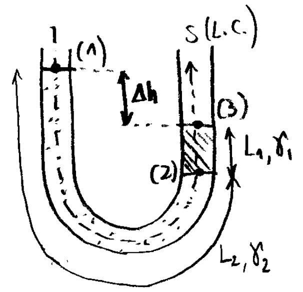
Ley hidrostática $\Rightarrow$ $p_1 = p_2$

$$p_1 = \gamma_1 \cdot L_1$$

$$p_2 = \gamma_2 \cdot (L_1 + \Delta h_{eq})$$

$$\Rightarrow \Delta h_{eq} = \left(\frac{\gamma_1 - \gamma_2}{\gamma_2} \right) L_1$$

b) Ec. Euler



Entre (1) y (2)

$$\frac{L_2}{g} \frac{dU}{dt} + B_2 - B_1 = 0$$

Entre (2) y (3)

$$\frac{L_1}{g} \frac{dU}{dt} + B_3 - B_2 = 0$$

$$B_i = z_i + \frac{p_i}{\gamma} + \frac{v_i^2}{2g} \qquad v_i = U \quad \forall i$$

Euler entre 1 y 2:

$$\frac{L_2}{g} \frac{dU}{dt} + \underbrace{h_2 + \frac{p_2}{\gamma_2} + \cancel{\frac{U^2}{2g}}}_{B_2} - \underbrace{\left(h_1 + \overset{0}{\cancel{\frac{p_1}{\gamma_2}}} + \cancel{\frac{U^2}{2g}} \right)}_{B_1} = 0$$

Euler entre 2 y 3:

$$\frac{L_1}{g} \frac{dU}{dt} + \underbrace{h_3 + \overset{0}{\cancel{\frac{p_3}{\gamma_1}}} + \cancel{\frac{U^2}{2g}}}_{B_3} - \underbrace{\left(h_2 + \frac{p_2}{\gamma_1} + \cancel{\frac{U^2}{2g}} \right)}_{B_2} = 0$$

$$(1) \quad \frac{L_2}{g} \frac{dU}{dt} + h_2 + \frac{p_2}{\gamma_2} - h_1 = 0$$

$$(2) \quad \frac{L_1}{g} \frac{dU}{dt} + h_3 - \frac{p_2}{\gamma_1} - h_2 = 0$$

Sumando (1) y (2):

$$\frac{L_1 + L_2}{g} \frac{dU}{dt} + \underbrace{h_3 - h_1}_{-\Delta h} + p_2 \left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1} \right) = 0$$

$$\Rightarrow \frac{L_1+L_2}{g}\frac{dU}{dt} - \Delta h + p_2\left(\frac{\gamma_1-\gamma_2}{\gamma_1\gamma_2}\right) = 0 \qquad (3)$$

De (2) $\Rightarrow \dfrac{L_1}{g}\dfrac{dU}{dt} + \underbrace{h_3 - h_2}_{L_1} - \dfrac{p_2}{\gamma_1} = 0$

$$\Rightarrow \frac{L_1}{g}\frac{dU}{dt} + L_1 = \frac{p_2}{\gamma_1} \qquad (4)$$

Utilizando $\dfrac{p_2}{\gamma_1}$ de (4) en (3)

$$\frac{L_1+L_2}{g}\frac{dU}{dt} - \Delta h + \left[\frac{L_1}{g}\frac{dU}{dt} + L_1\right]\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)$$

$$\frac{1}{g}\underbrace{\left[L_1+L_2+L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)\right]}_{Leq}\frac{dU}{dt} + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right) - \Delta h = 0$$

$$\therefore \frac{Leq}{g}\frac{dU}{dt} + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right) - \Delta h = 0$$

Pero $U = -\dfrac{dh_1}{dt}$ y $U = \dfrac{dh_3}{dt}$

$$\Rightarrow 2U = \frac{d}{dt}(h_3 - h_1) = -\frac{d(\Delta h)}{dt}$$

$$\Rightarrow U = -\frac{1}{2}\frac{d(\Delta h)}{dt} \Rightarrow \frac{dU}{dt} = -\frac{1}{2}\frac{d^2(\Delta h)}{dt^2}$$

$$\Rightarrow \frac{d^2(\Delta h)}{dt^2} + \underbrace{\frac{2g}{Leq}}_{A}\Delta h = \underbrace{\frac{2g}{Leq}L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)}_{B}$$

$$\frac{d^2(\Delta h)}{dt^2} + A\,\Delta h = B$$

$$\Rightarrow \Delta h(t) = C_1 \operatorname{sen}\left(\sqrt{\frac{2g}{Leq}}\,t\right) + C_2\cos\left(\sqrt{\frac{2g}{Leq}}\,t\right) + L_1\left(\frac{\gamma_1-\gamma_2}{\gamma_2}\right)$$

Cond. iniciales: $(t=0)$

$$\left.\begin{array}{l} \Delta h = \Delta h_{eq} + 2\Delta h_0 \\ U = -\dfrac{1}{2}\dfrac{d(\Delta h)}{dt} = 0 \Rightarrow \dfrac{d(\Delta h)}{dt} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} C_1 = 0 \\ C_2 = 2\Delta h_0 \end{array}$$

$$\Rightarrow \Delta h(t) = L_1\frac{(\gamma_1-\gamma_2)}{\gamma_2} + 2\Delta h_0\cos\left(\sqrt{\frac{2g}{Leq}}\,t\right)$$