P2]

Ec de Navier-Stokes:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{\rho}\nabla\hat{p} + \frac{\mu}{\rho}\nabla^2\vec{v}$$

Coordenadas cilíndricas:

$$\vec{v} = v_r\,\hat{r} + v_\theta\,\hat{\theta} + v_z\,\hat{k}$$

$$\nabla\vec{v} = \frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

Navier-Stokes en la dirección $\hat{k}$:

$$\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial z} +$$

$$+ \frac{\mu}{\rho}\left\{\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right\}$$

Si consideramos que el flujo es sólo en la dirección $\hat{k}$ $(v_r = v_\theta = 0)$, además es axisimétrico $(\partial v_z/\partial\theta = 0)$ y permanente $(\partial/\partial t = 0)$

$$v_z\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial z} + \frac{\mu}{\rho}\left\{\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right\}$$

Finalmente, con la ecuación de continuidad

$$\nabla\cdot\vec{v} = 0 \Rightarrow \frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \Rightarrow \frac{\partial v_z}{\partial z} = 0$$

$$\Rightarrow \frac{1}{\mu}\frac{\partial \hat{p}}{\partial z} = \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} \qquad \frac{\partial \hat{p}}{\partial z} = \text{cte.}$$

$$\frac{1}{\mu}\frac{d\hat{p}}{dz} = \frac{d^2 v_z}{dr^2} + \frac{1}{r}\frac{dv_z}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{\partial r}\right)$$

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{1}{\mu}\frac{d\hat{p}}{dz}\,r$$

$$r\frac{dv_z}{dr} = \frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{r^2}{2} + C_1$$

$$\frac{dv_z}{\partial r} = \frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{r}{2} + \frac{C_1}{r}$$

$$\Rightarrow v_z = \frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{r^2}{4} + C_1\ln r + C_2$$

Para el flujo que pasa entre las paredes de ambos tubos:

$$v_z\left(r = \frac{D}{2}\right) = 0 \quad ; \quad v_z\left(r = \frac{d}{2}\right) = 0$$

$$\Rightarrow \quad \emptyset = \frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{D^2}{16} + C_1 \ln\left(\frac{D}{2}\right) + C_2$$

$$\emptyset = \frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{d^2}{16} + C_1 \ln\left(\frac{d}{2}\right) + C_2$$

$$\Rightarrow \quad C_1 = \frac{1}{16\mu}\frac{d\hat{p}}{dz}\frac{D^2-d^2}{\ln(d/D)} = -0{,}0489\,\frac{d\hat{p}}{dz}$$

$$C_2 = -\frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{D^2}{16} - C_1 \ln\left(\frac{D}{2}\right) = -0{,}2196\,\frac{d\hat{p}}{dz}$$

$$\Rightarrow \quad v_z = 62{,}5\,\frac{d\hat{p}}{dz}r^2 - 0{,}0489\,\frac{d\hat{p}}{dz}\ln r - 0{,}2196\,\frac{d\hat{p}}{dz}$$

Para el flujo que pasa por el interior del tubo central:

$$v_z\left(r = \frac{d}{2} - e\right) = 0$$

$$v_z(r=0) = \text{cte.} < \infty$$

$$v_z = \frac{1}{\mu}\frac{d\hat{p}}{dz}\frac{r^2}{4} + C_1' \ln r + C_2'$$

con la 2^{da} condición de borde $\Rightarrow C_1' = 0$

$$\emptyset = \frac{1}{4\mu}\frac{d\hat{p}}{dz}\left(\frac{d}{2}-e\right)^2 + C_2'$$

$$\Rightarrow \quad C_2 = -\frac{1}{4\mu}\frac{d\hat{p}}{dz}\left(\frac{d}{2}-e\right)^2 = 0{,}00625\,\frac{d\hat{p}}{dz}$$

$$\Rightarrow v_z = 62{,}5\,\frac{d\hat{p}}{dz}r^2 + 0{,}00625\,\frac{d\hat{p}}{dz}$$

b) $\tau = \mu \dfrac{dv_z}{dr}$

$$\tau_{ext}\left(r = \frac{d}{2}\right) = \mu \left.\frac{dv_z}{dr}\right|_{r=\frac{d}{2}} = \frac{d\hat{p}}{dz}\left[125\,r - \frac{0{,}0489}{r}\right]_{r=\frac{d}{2}} \cdot \mu$$

$$= -0{,}00555\,\frac{d\hat{p}}{dz}$$

$$\tau_{int}\left(r = \frac{d}{2} - e\right) = \mu \left.\frac{dv_z}{dr}\right|_{r=\frac{d}{2}-e} = \frac{d\hat{p}}{dz}\left[125\,r\right]\cdot\mu$$

$$= 0{,}005\,\frac{d\hat{p}}{dz}$$

Equilibrio del tubo central

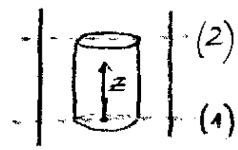
$$\sum F_z = 0$$

$$W = |\tau_{int}| \cdot 2\pi \left(\frac{d}{2} - e\right) \cdot L + |\tau_{ext}| \cdot 2\pi \frac{d}{2} \cdot L$$

$$W = 2\pi\left(\frac{d}{2} - e\right) \cdot L \cdot \left| -0{,}00555 \frac{d\hat{p}}{dz} \right| + 2\pi \frac{d}{2} \cdot L \left| 0{,}005 \frac{d\hat{p}}{dz} \right|$$

$$\Rightarrow W = 8{,}2 \times 10^{-4} \frac{d\hat{p}}{dz} \qquad W = 100 \, gr\text{-}f = 0{,}1 \, Kgf = 0{,}98 \, N$$

$$\Rightarrow \frac{d\hat{p}}{dz} = 1195{,}12 \left[\frac{N}{m^3}\right]$$



$$\frac{d\hat{p}}{dz} = \frac{\Delta p}{\Delta z} = \frac{(p_2 + \rho g L) - p_1}{L}$$

$$= \frac{p_2 - p_1}{L} + \rho g = \frac{dp}{dz} + \rho g$$

$$\Rightarrow \frac{dp}{dz} = 1195{,}12 \left[\frac{N}{m^3}\right] - 1000 \left[\frac{Kg}{m^3}\right] \cdot 9{,}8 \left[\frac{m}{s^2}\right]$$

$$\boxed{\frac{dp}{dz} = -8604{,}9 \left[\frac{N}{m^3}\right]}$$