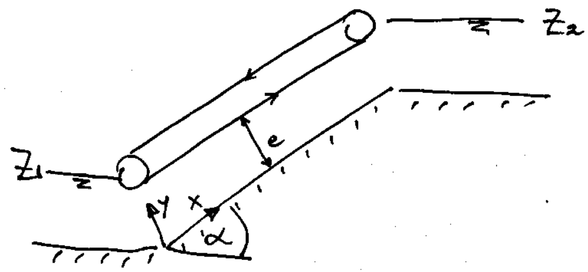
Distribución de Velocidades:



continuidad:

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial z}} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} = 0$$

i) Navier-Stokes:

s/x: $\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + \cancel{v}\frac{\partial u}{\partial y} + \cancel{w}\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial \hat{P}}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$

$$\therefore -\frac{1}{\rho}\frac{\partial \hat{P}}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} = 0$$

s/y: $\cancel{\frac{\partial v}{\partial t}} + u\cancel{\frac{\partial v}{\partial x}} + v\cancel{\frac{\partial v}{\partial y}} + w\cancel{\frac{\partial v}{\partial z}} = -\frac{1}{\rho}\frac{\partial \hat{P}}{\partial y} + \nu\left(\cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}} + \cancel{\frac{\partial^2 w}{\partial z^2}}\right)$

$$\therefore \frac{\partial \hat{P}}{\partial y} = 0 \qquad \therefore \frac{\partial}{\partial x}\left(\frac{\partial \hat{P}}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial \hat{P}}{\partial x}\right) = 0 \Rightarrow \frac{\partial \hat{P}}{\partial x} \text{ no dep. de } y.$$

ii) Cond. borde:

$u(y=0) = 0$

$u(y=e) = \omega R$

$$\therefore \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial \hat{P}}{\partial x} \qquad \Rightarrow u = \frac{1}{2\mu}\left(\frac{\partial \hat{P}}{\partial x}\right)y^2 + C_1 y + C_2$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\left(\frac{\partial \hat{P}}{\partial x}\right)y + C_1$$

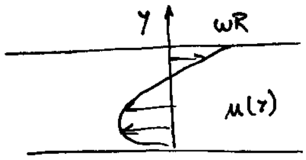
$u(y=0) = 0 \Rightarrow C_2 = 0$

$u(y=e) = \omega R$

$$\omega R = \frac{1}{2\mu}\left(\frac{\partial \hat{P}}{\partial x}\right)e^2 + C_1 e \Rightarrow C_1 = \frac{\omega R - \frac{1}{2\mu}\left(\frac{\partial \hat{P}}{\partial x}\right)e^2}{e}$$

$$\therefore \boxed{u = \frac{\omega R}{e}y + \frac{1}{2\mu}\left(\frac{\partial \hat{P}}{\partial x}\right)(y^2 - ye)}$$

$$\left.\begin{aligned}\hat{P}_1 &= \rho g z_1 \\ \hat{P}_2 &= \rho g z_2\end{aligned}\right\} \quad \frac{\partial \hat{P}}{\partial x} = \frac{\rho g (z_2 - z_1)}{L}$$



$\therefore$ distrib. de velocidades:

$$\boxed{u(y) = \frac{\omega R}{e}\, y + \frac{1}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right](y^2 - ye)}$$

b) Determinación de y^* tq. $u(y^*) = 0$

$$\frac{\omega R}{e} y^* + \frac{1}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right](y^{*2} - y^* e) = 0$$

$$A y^{*2} + B y^* = 0 \qquad A = \frac{1}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right]$$

$$B = \frac{\omega R}{e} - \frac{e}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right]$$

$$y^* = \frac{-B \pm \sqrt{B^2}}{2A} = \frac{-B \pm B}{2A} \Rightarrow y^* = -\frac{2B}{2A} = -\frac{B}{A}$$

$$\therefore \quad y^* = \frac{\frac{e}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right] - \frac{\omega R}{e}}{\frac{1}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right]} = e - \frac{\omega R}{2\mu e}\left[\frac{\rho g (z_2 - z_1)}{L}\right]$$

$\therefore$ Caudal ascendente Q_1:

$$Q_1 = \int_{y^*}^{e} u(y) = \left\{ \frac{\omega R}{e}\frac{y^2}{2} + \frac{1}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right]\frac{y^3}{3} - \frac{e}{2\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right]\frac{y^2}{2} \right\}\Bigg|_{y^*}^{e}$$

$$\boxed{Q_1 = \frac{\omega R}{2e}(e^2 - y^{*2}) + \frac{1}{6\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right](e^3 - y^{*3}) - \frac{e}{4\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right](e^2 - y^{*2})}$$

Caudal descendente Q_2:

$$\boxed{Q_2 = \int_0^{y^*} u(y) = \frac{\omega R}{2e} y^{*2} + \frac{1}{6\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right] y^{*3} - \frac{e}{4\mu}\left[\frac{\rho g (z_2 - z_1)}{L}\right] y^{*2}}$$

Caudal neto Q:

$$\boxed{Q = \int_0^e u(y) = \frac{\omega R}{2e} e^2 + \frac{1}{6\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right] e^3 - \frac{e}{4\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right] e^2}$$

c) ω tq $Q = 0$

$$\Rightarrow \frac{\omega R}{2e} e^2 + \frac{1}{6\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right] e^3 - \frac{e}{4\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right] e^2 = 0$$

$$\Rightarrow \omega = \frac{e^3}{12\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right] \cdot \frac{2}{Re}$$

$$\therefore \boxed{\omega = \frac{e^2}{6R\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right]}$$

d) Esfuerzo de Corte: $\left(\tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \mu \frac{\partial u}{\partial y}\Big|_{y=e}\right)$

$$\frac{\partial u}{\partial y} = \frac{\omega R}{e} + \frac{1}{2\mu}\left[\rho g \frac{(z_2 - z_1)}{L}\right](2y - e)$$

$$\therefore \tau_{yx}\Big|_e = \mu \frac{\omega R}{e} + \frac{1}{2}\left[\rho g \frac{(z_2 - z_1)}{L}\right] e$$

$$\boxed{\tau_{yx} = \mu \frac{\omega R}{e} + \frac{e}{2}\left[\rho g \frac{(z_2 - z_1)}{L}\right]}$$ esfuerzo de corte que se ejerce sobre la cinta.

e) Torque sobre cilindros:

Fza neta que ejerce el fluido sobre la cinta.

$$F_{tot} = \int_A \tau_{yx}\, dA = \int_0^L \left\{\mu \frac{\omega R}{e} + \frac{e}{2}\left[\rho g \frac{(z_2 - z_1)}{L}\right]\right\} dx$$

$$\therefore F_{tot} = \frac{\mu \omega R L}{e} + \frac{e \rho g (z_2 - z_1)}{2}$$

∴ Torque total que se debe aplicar sobre el sistema $= F \cdot r$

$$= \frac{\mu L \omega r^2}{e} + \frac{e \rho g r}{2}(z_2 - z_1)$$

∴ el torque que se debe aplicar sobre los cilindros es:

$$\boxed{T = \frac{\mu L \omega r^2}{e} + \frac{e \rho g r}{2}(z_2 - z_1)}$$